ELSA MODEL FOR ATOMIZATION: TO BENEFIT OF THE EULERIAN AND LAGRANGIAN DESCRIPTIONS OF THE LIQUID PHASE

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ABSTRACT In the context of the Euler – Lagrange Spray Atomization (ELSA) model [1,2,3] for two-phase flows, a Eulerian approach is used to describe the dense part of the spray and it is completed elsewhere by a standard Lagrangian approach. A recurrent issue of the Lagrangian approach is the difficulty to reach a number of stochastic samples large enough, in each mesh cell, to achieve statistical convergence. It is necessary to propose methods that can benefit of the Lagrangian formulation to describe the statistical dispersion but that are converged for certain key quantities like the mass of liquid. Such ideas are already developed under the so-called Direct Quadrature Method Of Moments (DQMOM) [4]. This complex method is not directly used here, a more practical approach that used the usual formalism of Lagrangian methods for spray is proposed. The method will take into account the originally unused Eulerian equation in the diluted part of the spray to transport the key mean quantities of the spray. In each mesh cell that contains Lagrangian stochastic particles the liquid mass fraction can be obtained from the Eulerian equation. Equivalently, in the mesh cell considered, a Lagrangian liquid mass fraction is defined. It is clear that big statistical fluctuations of this variable can be expected if the number of particles in the cell is not high enough. The least noisy Eulerian variable will be used to correct the Lagrangian one. There are numerous possible ways to correct the Lagrangian variable, one method will be presented. In the same manner, the Eulerian equation for the density of liquid surface is linked to the Eulerian equations. But the Eulerian equations have to be linked also to the Lagrangian phase, this is achieved through the liquid turbulent diffusion flux closure and through the source terms of surface density transport equation, i.e. those modeling break-up and coalescence effects. By this way, the Lagrangian phase, whenever it is available, enables to take into account the joint probability density functions of fluctuating variables such as droplet diameter or droplet velocities.

Keywords: Eulerian, Lagrangian, Spray, Atomization, Model

1. INTRODUCTION

Liquid injection is used in several industrial devices, this work concerns the atomization occurring with high velocity that can be employed for instance in engines. Nowadays, many constraints act on the design of new engines. They are due to the need of a high efficiency together with a very low level of pollutant emission. For instance in Diesel engine, a key factor is the atomization process. Many studies are devoted to this problem in order to get accurate modelling of the atomization phenomena. One of the results is the importance of the design of the injector itself. The classical Lagrangian models used to describe the spray atomization are generally based on the assumption of a dilute spray together with the idea that the liquid flow is a set of separated parcels. These assumptions do not work in the vicinity of the injector. As a consequence, the flow can not be represented with such models in the dense part of the spray. Generally, a unique mesh cell of the size of the injector diameter is used at the injector exit. Clever models are used to describe the unresolved phenomena happening in the so-called primary break-up region. But for instance, all the injector designs cannot be represented with a unique model. There is a need of suitable models adapted to this zone that allow the complex exit flows to be described with several mesh cells in the injector diameter. The ELSA (Eulerian Lagrangian Spray Atomization) model is devoted to this problem for injection with high Reynolds and Weber number [1,2,3].

Figure 1: Typical picture of an atomizing spray obtained with the ELSA model. The dark part represents the transition zone from the Euler description to the Lagrange description. This transition zone is computed dynamically, hence it evolves during the injection.

Basically an Eulerian approach is used in the dense part of the flow and when the main assumptions of Lagrangian model are reached a classical Lagrangian method is initiated dynamically. Both Eulerian and Lagrangian parts are computed by the same code at the same time and they interact together, see Figure 1. The transition criterion is based for instance on the level of dilution represented by the liquid volume fraction that is computed everywhere in the computational domain. Once the transition is set, the part of the computational domain that is described by Lagrangian method contains also the Eulerian unused equations that are always computed on the whole domain.
It appears that it is possible to benefit of these unused equations in the Lagrangian part in order to improve the spray representation. In this paper, we discuss the way to couple these two approaches and the improvement that can be expected. In the first part, the ELSA model is briefly reviewed. Then, the method to switch between Eulerian and Lagrangian approaches is analysed. In the third part, a simple way to couple Eulerian equations to Lagrangian spray representation is proposed. Finally, the two-way coupling is considered, that is to say how the Lagrangian particles act on the Eulerian equations.

2. OVERVIEW OF THE ELSA MODEL

The ELSA model has been developed for several years and it is originally dedicated to rocket and Diesel engines [1-3]. In such flows, the Weber and Reynolds numbers are supposed to reach “high” values. The idea proposed par Vallet and Borghi [5] is that the dense part of the spray can be studied as a single-phase flow composed of a liquid and a gas mixture. A mean approach has to be used in combination with a turbulence model; here, the standard \( (k-\varepsilon) \) model is used. The initial dispersion of the liquid is represented with a turbulent flux. To recover the size of the droplets finally formed, a length scale is introduced thanks to the liquid surface density \( \Sigma \). This variable is transported with a model equation inspired from flame surface density equation. Finally, the Lagrangian representation of the spray is initiated [6-7]. The schematic of the atomisation processes is represented on Figure 2.

2.1 Model for Liquid Dispersion

The standard equation for velocity and the turbulence modelling are used based on the \( (k-\varepsilon) \) model. The ability of this simple approach to reproduce the correct behaviour of the flow at the exit of the injector is questionable. There is not so much data in the dense part of the spray due to the difficulty to perform measurements. Another configuration, closer to the rocket engine injections, was considered in [8]. It is shown that the standard \( (k-\varepsilon) \) model allows the main characteristics of the flow to be recover without any change in the \( (k-\varepsilon) \) model but with a correction on the turbulent mass liquid flux due to density ratios. This quantity appears in the equation concerning the liquid mass fraction:

\[
\frac{\partial \rho \tilde{Y}_l}{\partial t} + \frac{\partial \rho u_j\tilde{Y}_l}{\partial x_j} = \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial \tilde{Y}_l}{\partial x_i} \right)
\]  

(4)

The turbulent diffusion flux of liquid (RHS of the previous equation) here must represent correctly the liquid dispersion as it can be shown in the exact following equation:

\[
\rho \nu' \tilde{Y}_l = \rho \tilde{Y}_l (1-\tilde{Y}_l)(\tilde{p}_{ij} - \tilde{p}_{e,j}) = \rho \tilde{Y}_l \left[ \tilde{p}_{ij} - \tilde{u}_j \right]
\]

(5)

where \( \tilde{p}_{ij} \) and \( \tilde{p}_{e,j} \) stands respectively for the mean velocity conditioned to be in the liquid phase and to the mean velocity conditioned to be in the gas phase. It is very clear that for two phase flows the modelling of this term is crucial since it drives the liquid mass dispersion and it has to be considered carefully since its modelling represents the slipping velocity between both phases. This is not certain that the classical gradient law is able to represent all the phenomena involved by the liquid dispersion, especially when the spray becomes diluted enough and the liquid parcels become spherical droplets. These considerations allow an approach called the Quasi-Multiphase flow Model to be built. In this approach using the previous equation a multiphase flow equivalent method can be obtained with a single flow code, see [2]. Nevertheless, for this work, the gradient law model is only used in the very dense part of the flow, where the transformation of the liquid as a set of

\[ \frac{\partial \rho \tilde{Y}_l}{\partial t} + \frac{\partial \rho u_j \tilde{Y}_l}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \frac{\partial \tilde{Y}_l}{\partial x_i} \right) (1 - R_{il}) \]

(1)

where \( R_{il} = \frac{\rho u_i \tilde{Y}_l}{\rho} \) is the turbulent liquid mass flux that has to be modelled and \( \tilde{Y}_l \) is the liquid mass fraction.

This equation is of greatest importance because the dispersion of the liquid is represented by this way. There is no source term due to the vaporization, since, as a first step, the vaporization is neglected in the dense part of the spray. In order to complete these two equations, the mean density is given by this relationship:

\[
\frac{1}{\rho} \frac{\partial \tilde{Y}_l}{\partial t} + \frac{1 - \tilde{Y}_l}{\rho_g} \rho = \frac{\bar{P}(1-\tilde{Y}_l) R_{e,T_g}}{1 - \tilde{Y}_l \rho_i / \rho_g}
\]

(2)

And the pressure law takes into account the presence of the liquid:

\[
\bar{P} = \lambda (1 - \tilde{Y}_l) R_{e,T_g}, \quad \frac{1}{\tilde{Y}_l \rho_i / \rho_g}
\]

(3)

Figure 2: Schematic of atomization
droplets is not completely achieved. Its modelling will be modified in the diluted part thanks to the use of Lagrangian particles information.

2.2 The Liquid Surface Density as a Generalised Interpretation of the Droplet Diameter

Once the equations for the liquid phase dispersion are solved, the size of the newly formed droplets at the interface Euler/Lagrange has to be computed. In order to determine them, a transport equation for the mean liquid density surface is used:

$$\frac{\partial \Sigma}{\partial t} + \frac{\partial \tilde{w} \Sigma}{\partial x_j} = \frac{\partial}{\partial x_j} \left( D_{w} \frac{\partial \Sigma}{\partial x_j} \right) + \alpha \frac{\Sigma (1 - \Sigma)}{\tau}$$  \hspace{1cm} (6)

In this equation, a first order closure is also used here in order to model the diffusion term at least in the dense part where the liquid is not a set of liquid droplets. As far as the production and the destruction terms of liquid surface density are concerned, they are modeled by:

$$\frac{\Sigma}{\tau}$$ corresponds to the production of surface due to the turbulent flow stretching, the effect of droplets collisions, the liquid surface instability..., but here, all these phenomena are supposed to be represented by a global turbulent time scale. A finer approach considering several time scales (one for each phenomenon) should be more suitable. But, at the moment, the lack of experimental or DNS data at the exit of Diesel injectors does not permit the establishment of a complex model. Hence, up to now, we have chosen to use the simplest model, just saying that the liquid surface density increases due to a global turbulence.

$$\frac{\Sigma^3}{\tau \Sigma_{\text{corr}}}$$ corresponds to the destruction of surface due to coalescence effects; this is supposed to also be driven by turbulence. But, the destruction term is defined also in order to reach, at the equilibrium, a critical liquid surface density that corresponds to a critical droplet diameter. This diameter corresponds roughly to the diameter of a stable droplet under the considered flow conditions. In this critical diameter, the small scales of the flow are involved hence the small scale forces, such as the surface tension, are important. Several formulations taking into account surface tension and other parameters can be done [1], here we used the same as used by Blokkeel et al [6-7].

$$\Sigma_{\text{corr}} = \frac{3}{\alpha_{\Sigma}} \frac{\rho l^{2/3} \tau_{l}^{3/5}}{k_{l}^{1/5}}$$ \hspace{1cm} (7)

3. TO SWITCH FROM EULERIAN TO LAGRANGIAN DESCRIPTION

3.1 Standard Method

As described in the introduction, the Eulerian single-phase approach has to be coupled with a Lagrangian formulation once the spray is considered to be diluted enough. This is done taking advantage of both the Eulerian formulation for the dense part of the spray and the extensive knowledge developed with the Lagrangian formulation for the diluted region, a unified formalism coupling both approaches may be put forward. Initially, in order to switch from Eulerian to Lagrangian formulation, a criterion must be introduced. Both methods are applicable to the entire atomization process, but practically the modelling is more straightforward considering the Eulerian method for the dense part of the flow, and a lot of experience exists to apply the Lagrangian method for the diluted part. Moreover, it can be argued that, at least in principle, there will probably be a transition zone between the dense and the diluted parts where both methods are suitable. Up to now, the criterion is based on the ratio between the mean distance between droplets and the mean droplet diameter. In fact, this corresponds to the definition of a dilution factor:

$$f = 1 / d_{23} = n^{-3/5} / d = 0.806 \rho_{l}^{-1/5}$$ \hspace{1cm} (8)

Once this critical value is reached, stochastic particles representing the spray must be created and initial values have to be defined. To this end, a method to extract the liquid velocity and the liquid turbulence intensity from their corresponding variables in the Eulerian representation has been developed in [6-7]. The size of the newly created droplets is chosen equal to the local value of the mean Sauter diameter as given by $$d_{s} = 6 \rho_{l}^{3/5} / \rho_{l} \Sigma$$. The droplets are homogeneously dispersed in the mesh cell considered, with an initial velocity given by the mean velocity of the liquid and modulated by a random velocity fluctuation that describes the liquid kinetic energy.

The mean liquid velocity is deduced from the mean mixture velocity by:

$$\bar{u}_l = \bar{u} + \rho u' Y_{u} / \rho_{l}$$ \hspace{1cm} (9)

and the mean liquid kinetic energy by:

$$k_l = Y_{l} k_{l} + (1 - Y_{l}) k_{g} + \frac{1}{2} \rho_{l} u' Y_{u} / \rho_{l}^{3/5} \rho_{l}^{5/3} \left( \frac{1}{Y_{l}} + \frac{1}{1 - Y_{l}} \right)$$ \hspace{1cm} (10)

together with the hypothesis that $$\rho_{l} k_{l} = \rho_{g} k_{g}$$.

This hypothesis for the kinetic energy of the liquid and the use of a single value for the droplet diameter within each cell are probably the two points that need further improvements. However, it can be expected that the mixing induced by turbulence in the Lagrangian description will rapidly cancel the effects of these two choices.

Despite these drawbacks, the method has been proved to give satisfactory results and to improve the reliability of computational results when varying several parameters [3]. The figure 3 presents the vapour and liquid penetrations for the same injection characteristics but with two different temperatures in the chamber where the spray is injected. The DDM model corresponds to the classical Lagrangian model that has been carefully fitted for this injector when
varying other parameter than the temperature such as the gas density.

Figure 3: Ability to recover liquid and vapour penetration with ELSA standard model when varying gas temperature.

The IFP results are experimental data [9]. The effect of the increase of temperature is the decrease of the liquid penetration due to a higher vaporisation rate. At the same time, the vapour penetration is unchanged. This behaviour is actually reproduced by the ELSA model.

Such good results demonstrate the potentiality of the ELSA approach but many improvements are still possible. In particular the Eulerian-Lagrangian transition as to be improved.

3.2 Possible Improvements

- Mass conservation at the transition

The best way to conserve the mass flux at the transition is to compute the incoming flux coming from the Eulerian part and to transfer it to a Lagrangian flux. In practice this simple idea is difficult to achieve everywhere in the computational field. The Eulerian liquid mass flux for instance can be computed as the sum of the convective flux plus the turbulent diffusive flux:

\[
\Phi^E = \int_{\Omega_E} \left( \rho_{\text{E}} Y_{\text{E}} - D_{\text{E}} \frac{\partial Y_{\text{E}}}{\partial x} \right) \cdot nd\Omega
\]

(11)

where \( \Omega_E \) is the Eulerian surface and \( \Phi^E = \Phi_{\text{in}}^E - \Phi_{\text{out}}^E \) is the net Eulerian liquid flux to the Lagrangian zone. This flux has to be balanced by the Lagrangian flux that is \( \Phi^L = \Phi_{\text{m}}^L - \Phi_{\text{out}}^L \). This is simply the difference between the droplet that enters into the Lagrangian domain (\( \Phi_{\text{in}}^L \), the unknown variable) minus the droplet that enters into the Eulerian zone. These last droplets are deleted because Lagrangian droplets are not considered in the Eulerian zone. They forms the flux \( \Phi_{\text{out}}^L \), this flux is actually difficult to compute and to take into account.

In fact, for each particle coming from the Lagrangian zone to the Eulerian zone, the huge challenge is to determine what is the transition cell that is affected by this particle especially when the transition evolved during the same time step. The problem becomes nearly without solution when considering CFL number bigger that the unity.

Thus, instead of using the flux, we simply imposed in the transition cell the value of the Eulerian field to the Lagrangian representation. This is done by adding or deleting liquid Lagrangian particles in order to respect the Eulerian liquid volume fraction, the Sauter mean diameter and the velocity. In this case, the flux of liquid is due to the turbulent motion of the liquid droplets that go out of the transition cell. We did several tests that globally show a good mass conservation.

- Lagrangian description

The Lagrangian description corresponds to perform a Monte Carlo simulation to solve the joint PDF of the liquid spray for variables like diameter, liquid velocity, temperature. The sum of all the stochastic samples give the liquid volume fraction. In this sense, this approach is very complete and doing the same using Eulerian equations is still out of reach. The well know problem of this Monte Carlo method is its slow degree of convergence when increasing the number of particles, i.e. stochastic samples. In many practical applications, the statistical convergence is far to be reached. Moreover, to decrease the Eulerian cell mesh, it is necessary in principle to increase the number of samples to keep the number of particles per cell constant. This problem of statistical convergence plays an important role in the dependency of the Lagrangian method’s results on the mesh size.

The result, for instance when considering an established atomizing jet, is that the Lagrangian field can fluctuate introducing non stationery behaviour on the Eulerian field in case of two-way coupling. This may be look like LES, but this is not and coming to LES is not the solution since mesh cell size has to decrease. Then, diminishing the mesh size has the consequence either to increase the stochastic particle number either to reduce the statistical convergence.

- Unused Eulerian equations

The Eulerian equations have to be computed in the whole computational domain even if they are not used in the Lagrangian part. The Eulerian equations of the ELSA model concerned by this point are the equation for the liquid mass fraction and the liquid surface density. This is of course a waste of computational efficiency. Moreover, in the Lagrangian part it is supposed that the Lagrangian equation

Figure 4: Schematic of the Euler-Lagrange transition with liquid mass fluxes.
particles transport the liquid mass fraction and the liquid surface in a better way than the Eulerian equations. Hence, those Eulerian equations if they are not in accordance with the Lagrangian results are probably wrong. The problem is that they are coupled with the Eulerian zone through for instance turbulent diffusion. A possible way would be to correct them thanks to the Lagrangian computation in the Lagrangian domain.

Finally, it seems quite natural to couple both Lagrangian and Eulerian methods in order to:

1. benefit of the mass conservation ensured by Eulerian equations;
2. reduce the effect of the lack of convergence in Lagrangian statistics.

allow Eulerian equations to benefit of the complete simulation of the joint PDF of the spray by the Lagrangian approach.

### 3.3 Coupling Euler to Lagrange

In this section, we describe how the Eulerian equations are used in the diluted part to correct the Lagrangian variables transported by the stochastic particles. The aim is to benefit of the Lagrangian formulation to describe the statistical dispersion but that are converged for certain key quantities. Such ideas are already developed under the so-called Direct Quadrature Method Of Moments (DQMOM) [4].

Instead of this more complex method, a simple and practical method is used here. The liquid mass fraction and the liquid surface density have to be computed everywhere thanks to Eulerian equations. Doing so, the classical properties, like mass conservation for instance, of such equations are solved by finite volume method. It is also requested that the fields of such important variables have to be smooth without any statistical noise. The figure 5 shows the local Lagrange surface density based on droplets in each computational cell. This field is really noisy due to a lack of statistical convergence, although that around fifty thousand stochastic particles are used, number that is relevant for practical applications.

At the contrary, when the complete coupling method is applied, the field of surface density is smooth and well represented, see figure 6. It can be argued that the cell sizes are actually small but when looking at the gradient of such a key value like the surface density, such cell sizes seem to be necessary.

\[ \Sigma = \rho \frac{Y^*}{\rho} \]

Figure 6: Smooth field of liquid surface density obtain after the complete coupling method \( \Sigma / \rho \)

The simplest method is used to impose the Euler variables on the Lagrangian variables. The Lagrangian spray is described by a set of stochastic particles \( k \), each particle \( k \) contains \( n_k \) droplets of diameter \( d_k \) and with a velocity vector \( \vec{v}_k \). The density of the liquid is supposed to be constant and is referred as \( \rho_L \). One mesh cell of volume \( V \) that contains \( n_p \) stochastic particles is considered. Equivalently, in the mesh cell considered, a Lagrangian liquid mass fraction \( \tilde{Y}^*_k \) is defined. It is clear that big statistical fluctuations of \( \tilde{Y}^*_k \) can be expected if the number of particles \( k \) in the cell is not high enough. Considering this key variable, it would be advisable to use the least noisy Eulerian variable \( \vec{Y} \) instead of the Lagrangian one \( \tilde{Y}^*_k \). There are numerous possible ways to correct the Lagrangian variable. The simplest method would consist in changing proportionally either all the \( n_k \) or all the \( d_k \).

To choose between both possibilities, it is possible to take into account the fact that the Sauter Mean Diameter \( d_{32} \) also has to respect the Eulerian phase, arguing that the statistical noise has to be reduced too. The surface density equation gives the Eulerian value \( \Sigma \) that has to be compared to the Lagrangian one \( \Sigma^* \). Hence, the modified value of \( n_k \) is \( n_k^* = \varepsilon_n n_k \) and the modified diameter is \( d_k^* = \varepsilon_d d_k \) for each particle in the mesh cell considered. Thus, to ensure that \( \tilde{Y}^*_k = \vec{Y} \) and \( \Sigma^* = \Sigma \), \( \varepsilon_d \) and \( \varepsilon_n \) are determined through the following equation:

\[
\begin{align*}
\varepsilon_d &= \frac{\vec{Y}}{\tilde{Y}^*_k} \\
\varepsilon_n &= \left( \frac{\Sigma}{\Sigma^*} \right) \left( \frac{\tilde{Y}^*_k}{\vec{Y}} \right)^2
\end{align*}
\]

### 4. TWO WAY COUPLING: LAGRANGE TO EULER

Hence, the Lagrangian phase is linked to the Eulerian equations. But the Eulerian equations also have to be linked to the Lagrangian phase. This step is important: it allows to benefit of the complete representation that can be expected from the Lagrangian stochastic particles of the spray PDF.
Without this two-way coupling, the Lagrangian phase would be useless and would not have to be used. In this case, a complete Eulerian approach can be applied. Though in such an approach the representation of the fluctuation of variables like liquid diameter, liquid velocity and droplet temperature would be neglected or approximated by another means. Neglecting these fluctuations is in principle questionable since such variables act in source terms that are not linear like drag force, vaporisation and heat transfer. The mass fraction of liquid is computed with an Eulerian equation but in this equation, as outlined previously, the turbulent liquid flux is directly related to the slip velocity between the gas and the liquid phase or equivalently to the difference between the mean velocity and the liquid velocity. Hence the proposition is that the liquid velocity is obtained from Lagrangian particles, \( \bar{u}_l^* \), to rebuild the turbulent flux.

This has to be done wherever, but in some place the statistical convergence even on liquid velocity is not reached because of a lack of stochastic particles. Even, there is some cells where there is not particles at all. Another possibility to compute the liquid velocity and the turbulent liquid flux is then to use the standard Eulerian model, here this is the liquid velocity obtained using the gradient law.

Finally an Eulerian liquid velocity is obtained thanks to equation (9). Everywhere the actual turbulent liquid flux is obtained thanks to:

\[
R_{li} = \frac{1}{\alpha} (\alpha \bar{u}_l^* + (1 - \alpha) \bar{u}_l - \bar{u}_i)
\]

The last point concerns the determination of the parameter \( \alpha \), considering that the statistical error of Monte Carlo method evolves like the inverse of the square root of the number of sample \( n \) the following rule is used.

\[
\begin{cases} 
\text{if } np = 0, \alpha = 0 \\
\text{else: } \alpha = \left(1 - \frac{1}{\sqrt{np}}\right)
\end{cases}
\]

Figure 7: Level of influence \( \alpha \): \( 1/\sqrt{np} \) if there is particles, else the value is set to 1

Figure 7 shows the evolution of \( 1/\sqrt{np} \), the value is set to 1 where there is no particles. The method shows that the influence of the Lagrangian phase is strong in the area where the gradients are intense and where the amount of liquid is quite high at the exception of the dense zone of the spray near the injector exit. In the dense zone, since the conditions are not in accordance with the classical Lagrangian requirements, only the dense Euler approach is used.

Figure 8 shows the level of correction that has to be applied due to the coupling with the Eulerian liquid mass fraction. Such correction factors are in order of the unity except in area close to the border of the Lagrangian field. In these places, the number of particles per mesh cell is very low and an important correction may be necessarily due to the huge statistical noise. This correction may seems important if particles are considered as liquid droplets. But, it has to be kept in mind that they are only statistical samples that represent the PDF of the spray. Thus particles in each mesh cell, in this coupled approach, are representative of the level of fluctuation around the mean value that are given by Eulerian equations. These characteristics of the joint spray PDF are used, when available, to improve the modelling of unclosed terms in the Eulerian equations.

Figure 8: Field of \( E_a \) that described the Eulerian correction applied to the Lagrangian stochastic particles.

Such characteristics concern, for instance, the mean conditional velocity that is used to represent the liquid dispersion. The contribution of Lagrangian particles on the modelling of the liquid turbulent flux for the Euler liquid mass fraction is represented on figure 9.

It can be seen that Lagrangian corrections are important mainly in the mixing layer around the spray. This is a good point, since this is where the difference between the mean liquid and the mean mixture velocity is the more important. Most probably in this zone the standard gradient law closure would not be sufficient.

Figure 9: Lagrangian particle correction field to the turbulent liquid mass flux
5. CONCLUSION

In the context of the ELSA model two methods to describe the spray are used. The first one is the Eulerian method that is applied in the dense part of the spray. The second one is the Standard Lagrangian method, applied once the spray is diluted enough. In the Lagrangian zone the Eulerian equations used to described the spray (Liquid mass fraction equation and Surface density equation) are still activated but unoccupied since all the information is originally taken from Lagrangian particles only. For different purposes (like mass conservation, reduction of statistical noise, reliability of Euler equations, ...) it appears quite natural to coupled both methods. The first way coupling (Euler to Lagrange) is relatively straight forward and a simple method is proposed that modified the Lagrangian particle characteristics all proportionally in a given mesh cell. Different methods must be proposed in future works to improve this point and test the influence of the chosen method. The second way coupling (Lagrange To Euler) is less straight forward, but a coupling is proposed through the unclosed terms of the Eulerian equations. A weighting factor $\alpha$ has to be defined to used the Lagrangian data only if the statistical convergence is achieved. From this two-way coupling the Lagrangian statistical noise is drastically reduced. This method could be used also in the feature to determine where to use the Lagrangian particles, for instance if the Lagrangian source term is identical to the Eulerian one, the Lagrangian particle may be deleted to optimize the computational efficiency. This work has to be extended also for vaporising and reacting spray.

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7. NOMENCLATURE

Reynolds average of a variable $a$ is expressed by $\overline{a}$, the fluctuation is $a'$; Favre average of a variable $a$ is expressed by $\bar{a}$, the fluctuation is $a''$;
'Subscript 'l' refers to liquid and subscript 'g' refers to gas.

Roman letters:

- $d_{32}$ Sauter mean diameter of droplet [m]
- $D_i$ turbulent diffusion coefficient [m$^2$/s]
- $k$ turbulent kinetic energy [m$^2$/s$^2$]
- $np$ number of Lagrangian particle per mesh cell [-]
- $R_{df}$ turbulent diffusion liquid mass flux [m/s]
- $\overline{u}_i$ velocity in the direction $i$ [m/s]'
- $Y$ mass fraction [-]

Greek letters:

- $\alpha$ weighting factor for Lagrangian coupling [-]
- $\alpha_1, \alpha_2$ modelling constants for the transport equation of

8. REFERENCES