VALIDATION OF A NEW COMPUTATIONAL MODEL FOR MULTIPLE SCATTERING OF LASER RADIATION IN SPRAYS

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ABSTRACT Optical diagnostics of sprays, namely diffraction methods, Phase Doppler Anemometry, and laser sheet imaging, encounter difficulties in the dense region due to the multiple scattering of the incident laser radiation with the surrounding droplets. By understanding the radiative transfer within such polydisperse inhomogeneous turbid media, the accuracy and efficiency of modern optical techniques can be improved. A new model for simulating the propagation of laser radiation through complex 3D scattering structures like sprays is presented. The code, based on the Monte Carlo (MC) technique, is able to consider strong variations of both droplets size distribution and concentration with location. Droplets are assumed spherical, and the appropriate Mie phase function is used to describe each single scattering process. The model is experimentally validated by using several homogeneous water sample cells containing monodisperse polystyrene spheres. Various particle sizes (1 and 2 μm in diameter) and concentrations (optical depths varying from 2 to 10) are considered such that the complete scattering process resulting from each sample remains comparable to the one occurring either in the dilute or dense spray region. The light intensity profiles are measured experimentally and simulated for both the side and forward scattering. In this paper, we demonstrate very good agreement between experimental and simulated results. In one case, we show the influence of the size and the concentration of the spheres on the entirety of the resultant scattering process, and in another case the effect of the detection acceptance angle by considering respectively $\theta_a=8.6^\circ$ and $\theta_a=1.5^\circ$. Finally, we demonstrate the capabilities and application of the code for the real case of planar Mie imaging through a hollow cone water spray.

Keywords: Multiple scattering, Mie theory, dense sprays, Monte Carlo modeling, polydispersity.

INTRODUCTION

The number of laser diagnostics developed for spray characterization is very large. Each of them provides either various descriptions of the spray structure in a macroscopic scale (spray angle, distance of penetration, liquid core length and other geometrical features) or specific information in a microscopic scale (droplet size, diameter, velocity, liquid flux etc). Nowadays, three major techniques are well established and commonly used in laser diagnostics of sprays:

The first technique is Fraunhofer diffraction instrument commercially available as the Malvern Particle Size Analyzer. Through detection of the low angle light scattering in the far field region, this instrument allows fast and inexpensive line-of-sight determination of drop size distribution [1-2].

The second technique is based on interferometry point measurements. Its evolution started in 1964 with the creation of Laser Doppler Anemometry (LDA) [3] which provides local determination of droplet velocity. In 1975 the technique was improved by deducing also the droplet size from two Doppler signals detected simultaneously. This led to the creation of the well known Phase Doppler Anemometry (PDA) [4]. Since its first appearance the PDA has been largely improved [5] and the last version of the system is the so-called Dual-Mode PDA [6]. This system is at the present time the most reliable for accurate characterization of sprays and offers simultaneous measurements of droplet size, velocity, flux and concentration. However only local information can be extracted and complete characterizations are time consuming.

The third widely used technique is a whole field detection technique known as laser sheet imaging. Depending on the scattering and detection process involved, velocity vectors can be found with Particle Image Velocimetry, liquid volume fraction determined through Planar Laser Induced Fluorescence [7], and droplet sizes measured through droplet lasing [8] and Laser Sheet Dropsizing [9], or Planar Drop sizing techniques [10].

Although these three main techniques use different approaches, they are all based on the single scattering approximation; they all assume that the detected photons have experienced only one scattering event prior to arrival at the detector. This assumption remains valid when droplet number density is low (e.g. in the dilute spray) and when total photon path length within the spray is short (e.g. in spray peripheries). However, in dense spray regions and/or inside the spray, a large amount of multiply scattered light is detected and the single scattering assumption is no longer valid.

In laser sheet imaging techniques, multiple scattering blurs and attenuates the recorded images; in diffraction and interferometry point methods, it introduces uncertainty and ambiguity to the detected signal leading to errors on the measurement. Quantifying and reducing multiple scattering induced error is extremely complex requiring knowledge of the radiative transfer of the laser beam through the spray. While the number of papers related to light attenuation and multiple scattering in dense sprays has increased [11-12] over the last decades, there have been no convincing corrective solutions offered.

In this article, we present a new computational model for accurate simulation of the propagation of laser light beams through sprays. The model allows consideration of the strong variation of both droplet size and concentrations.
with location in 3D. This is achieved through division of the entire spray structure into small unitary homogeneous cubic cells [13]. Each cell is characterized by three optical properties: the extinction coefficient, the albedo and the scattering phase function.

In the first part of the paper, the propagation of a laser beam through a spray is detailed with appropriate terminology. The second section describes the modeling approach and the Monte Carlo technique. The validation of the model is discussed in section 3 with a comparison of the experimental and simulated intensity profile images. Finally, an example of an application is detailed for the case of Mie laser sheet imaging of a hollow cone water spray.

1. PROPAGATION OF LASER RADIATION IN SPRAYS

1.1 Extinction, scattering and absorption

When a beam of light enters a medium containing a collection of scattering particles (such as droplets), the incident light intensity is attenuated. The complete attenuation of light intensity is called “extinction” and is equal to the sum of the “scattering” and “absorption” processes. By definition, the change of intensity $dI(r, s)$ of a laser beam of initial intensity $I_i(r, s)$ (also called radiance or specific intensity and of units $W/\text{sr/cm}^2$) crossing an elementary volume of length $dx$ depends on the extinction coefficient $\mu_s$ as:

$$dI(r, s) = -I_i(r, s) \cdot \mu_s(r) \cdot dx$$

and

$$\frac{dI(r, s)}{I_i(r, s)} = -\mu_s(r) \cdot dx$$

where $dx$ is the elementary length, $r$ is the vector position and $s$ is the direction of propagation.

The Beer-Lambert law is given by an integration of Eq.2 over a distance $l$. By assuming a homogeneous medium with constant extinction coefficient, $\mu_s$, it is deduced that the initial light intensity is exponentially reduced along a line-of-sight of length $l$ such as:

$$I_f = I_i \cdot e^{-\mu_s l}$$  

Here $I_i$ and $I_f$ are respectively the initial and final light intensity along the same direction of propagation $s$.

As shown in Eq.2, the attenuation equals the ratio of the light intensity extinction with the initial light intensity. The attenuation can also be written (from Eq.3) under the form $1 - (I_f / I_i)$ where the term $(I_f / I_i)$ is the transmission.

In optical characterization of sprays, “attenuation” is often referred to as “obscuration”. However, for ease and clarity, we won’t use this term in this article.

The extinction coefficient equals to the sum of the scattering coefficient and the absorption coefficient:

$$\mu_s = \mu_s + \mu_a$$  

Each of these coefficients (in cm$^{-1}$) is proportional to the number density $N$ (# of droplets per cm$^3$) and to its respective extinction, scattering and absorption cross-section $\sigma_s$, $\sigma_a$ and $\sigma_a$ (in cm$^2$) as:

$$\mu_s = N \cdot \sigma_s ; \mu_a = N \cdot \sigma_a; \mu_s = N \cdot \sigma_a$$  

The importance of scattering over the extinction is quantified by the albedo:

$$\Lambda = \frac{\mu_s}{\mu_s + \mu_a} = \frac{\sigma_s}{\sigma_s + \sigma_a}$$  

For a non-absorbing medium, the albedo equals 1 and the extinction of the light is governed only by scattering. In fuel sprays, natural absorption occurs generally at very low levels. The light-droplets interaction is then dominated by the elastic scattering of the incident laser radiation. When fluorescence occurs, by adding a given tracer in the injected liquid, the incident wavelength is absorbed and reemitted into a wavelength of lower energy. During this inelastic-scattering process, the albedo is largely reduced and absorption must be considered.

1.2 Scattering orders, optical depth and scattering regimes

The “scattering order” corresponds to the number of times that an individual photon interacts with the droplets (or other scattering particles), prior to spray exit (or other turbid medium). By adding the scattering processes produced by each scattering order, the complete scattering process is obtained.

The distance of propagation between two scattering or absorbing events is called the free path length $l_f$. The mean free path length is calculated from the extinction coefficient such as: $l_f = 1 / \mu_s$. This relationship shows that the average distance traveled by a photon between two droplets is inversely proportional to the extinction coefficient. By dividing the total length $l$ traversed by a light beam, by the mean free path length $l_f$, the number of average path lengths along $l$ is deduced. The resultant term is the optical depth, $OD$:

$$OD = l / l_f = l \cdot \mu_s$$  

The optical depth is an approximation of the mean number of scattering events occurring through a scattering medium of distance $l$. It provides a crucial indication of the optical thickness of a probed spray.

Depending on the value of the average scattering order and optical depth of a spray, the scattering of light within the spray can be classified into 3 regimes:

The single scattering regime is defined when the average scattering order is inferior than 2. In this regime, ballistic photons are clearly dominant and the single scattering approximation can be applied.

The intermediate single-to-multiple scattering regime operates when the average of scattering events is comprised from 2 up to 9. In this case, the mean scattering order dominates all other orders.

In the final, multiple scattering, regime the average number of scattering events is greater or equal to 10. In this regime, the relative amount of each scattering order tends to be equal and no dominant scattering order is clear. It is important for the reader to differentiate the regime of “multiple scattering” from the process of “multiple scattering” which occurs also in the intermediate regime.

Current laser diagnostics of sprays fall into the intermediate scattering regime where the average number of light scattering events is too great for the single scattering
are the characteristics of the incident light scattered into various directions in space with preferential scattering angles. The probability distribution related to each scattering angle is the scattering phase function \( f(\vec{s}, \vec{s}') \). Note that this is a misleading term as \( f(\vec{s}, \vec{s}') \) has nothing to do with the phase of a light wave. The phase function describes the "possibility" for a photon with an initial direction \( \vec{s} \) to be scattered into the direction \( \vec{s}' \) after a scattering event. The parameters governing \( f(\vec{s}, \vec{s}') \) are the characteristics of the incident light (wavelength, polarization state, intensity profile), the optical properties of the surrounding medium (external refractive index) and the droplet characteristics (size, shape, refractive index, orientation). The intensity of light \( I_s(\vec{r}, \vec{s}') \) scattered into \( \vec{s}' \) is described as a function of \( f(\vec{s}, \vec{s}') \) as:

\[
I_s(\vec{r}, \vec{s}') = \mu_s(\vec{r}) \cdot \Delta x \int_{4\pi} f(\vec{s}, \vec{s}') \cdot I_s(\vec{r}, \vec{s}) \cdot d\Omega \tag{8}
\]

where \( d\Omega \) is the solid angle around \( \vec{s} \). The scattering phase function is dimensionless and is generally given under its normalized form (the integration over all directions equals 1). The incident direction \( \vec{s} \) and final direction \( \vec{s}' \) are defined by their polar and azimuthal angles \( \phi \) and \( \Psi \) in the Cartesian coordinate system (XYZ). When a scattering event occurs, the scattering phase function is defined in a locale coordinate system (UVW) such that each new direction of propagation is described by the scattering angles \( \phi_s \) and \( \Psi_s \). This change of coordinate systems is shown in Fig.1 for a single photon scattered by a droplet.

The shape of the scattering phase function can be extremely complex especially for non-spherical particles. Depending on their location within the spray, droplets produced by atomization have various geometrical forms and size distributions. In the near injector region, the spray is characterized by ligaments along the liquid core (which are subjected to primary atomization), large droplets of high velocity (which are subsequently deformed and subjected to secondary atomization), and other irregular liquid elements. The scattering of light from such arbitrarily shaped drops is extremely complex and dependent on the particle’s orientation to the incident light. After secondary atomization, the velocity of the droplets is reduced considerably. In this case, the aerodynamic pressure applied to the droplet is small in comparison to the internal pressure due to surface tension forces. The droplets formed on the periphery of the dense spray region (and in the dilute spray) have finally a quasi-perfect specificity. The droplets characteristics in the dense region of an atomizing spray are depicted in Fig. 2.

The physical phenomenon of light scattering by spherical particles has been initially described in the classical Lorentz-Mie Theory (LMT) where the incident light is assumed to be plane waves [14]. Another assumption of the Lorentz-Mie theory requires that the size of the particles is on the order of the incident light wavelength. In sprays, this assumption is respected as the diameter of the spray droplets is generally comprised from 1 to \( \sim 200 \mu m \) (for fuel sprays the common droplet sizes remain around 15-20 \( \mu m \)) and the laser light wavelength generally used ranges from the ultraviolet \( \sim 266nm \) (for PLIF measurements) to the near-infrared \( \sim 1\mu m \). In some techniques \( \lambda \) can almost reach 10\( \mu m \) (2\( \lambda \) technique [12]).

Depending on the droplet size parameter \( p \) (\( p = (\pi \cdot d) / \lambda \)), the scattering phase function of spherical drops can be either highly anisotropic with forward scattering dominant \((p>10)\) or more isotropic with a homogeneous distribution of the scattered radiation \((p<1)\). At very small \( p \) \((p<1)\), Rayleigh scattering theory is assumed. The scattering probability of spherical droplets over the angle \( \phi \) is homogeneous over \( 2\pi \), and the scattering phase function depends only on its scattering angle \( \phi \), defined between 0 and \( \pi \) and:

\[
f(\vec{s}, \vec{s}') = f(\phi_s) \tag{9}
\]

A convenient mathematical expression to describe the forward nature of the droplet phase function is the anisotropy factor \( g \).

\[
g = \int_{4\pi} \cos(\phi_s) \cdot f(\phi_s) \cdot d\Omega
\]

g equals 0 for isotropic scattering and \( g \) tends to 1 for forward scattering phase functions. In sprays, the factor of anisotropy ranges from 0.65 to 0.95. For a 20 \( \mu m \) water droplet in air illuminated at 532nm incident light, \( g=0.86 \).

1.4 Attenuation and multiple scattering in spray

Laser beam propagation through a spray is subject to attenuation and multiple scattering. These processes introduce errors in the measurement of droplet size and concentration particularly in the intermediate scattering

Figure 1: Trajectory of a photon scattered by a droplet

Figure 2: Droplets formation in an atomizing spray
regime. These processes are described as follows:

1- Light beam attenuation along incident direction: The probe beam is attenuated as it traverses the spray due to both scattering and absorption. Dependent on position along the laser line-of-sight, not all droplets are illuminated with the same intensity (see Fig.3a).

2- Attenuation from the incident laser line-of-sight (or sheet) to the detector (called also out-of-plane attenuation): This corresponds to “secondary scattering” from droplets lying between the probe beam and the detector (see Fig.3a).

3- Multiple scattering: “Extraneous light” is detected after being multiply scattered by a number of the surrounding droplets (see Fig.3b).

Figure.3: Illustration of attenuation (a) and multiple scattering (b) processes when probing a spray with a laser beam.

The magnitude of error introduced by each process varies with position, in a manner dependent on the spray geometry. Corrective solutions are unique for each source-detector configuration and for each spray structure. The most flexible way to understand and quantify the amount of error introduced by attenuation and multiple scattering in an optical diagnostics of a spray is to simulate the problem.

2. MODELLING OF PHOTONS TRANSPORT IN SPRAYS

2.1 Radiative transport theory

The radiative transfer theory is a theoretical model for the transport of photons through a scattering medium. The method ignores the behavior of the component wave amplitudes and phases, and treats photons as point particles. The theory is based on the central Radiative Transfer Equation (RTE) (or equation of radiative transfer). The RTE is a balance of energy between the incoming, outgoing, absorbed, scattered and emitted photons through a volume element. For the case of laser propagation in sprays, the RTE is given as:

\[
\frac{1}{c} \frac{\partial I(\vec{r}, \vec{s}, t)}{\partial t} = -\mu_I(\vec{r}, \vec{s}, t) + \mu_s(\vec{r}, \vec{s}, t) \int f(\vec{s}', \vec{s}) I(\vec{r}, \vec{s}', t) \, d\Omega
\]

(1)

with

\[
\mu_s(\vec{r}, \vec{s}, t) = \mu_s(\vec{r}, \vec{s}, t) + \mu_a(\vec{r}, \vec{s}, t)
\]

(3)

where \( t \) is time and \( c \) is the speed of the light in the surrounding medium. A description of the RTE includes: The change of radiance along a line of sight (term (1)) corresponds to the loss of radiance due to the extinction of the incident light (term (2)) plus the amount of radiance that is scattered into all other directions \( s' \) from the incident direction \( s \) (term (3)). The total extinction represented by term (2) equals the radiance lost due to scattering of the incident light in all other directions, minus the radiance that is absorbed at each light-droplet interaction.

Although the RTE is applicable for a wide range of turbid media, analytical solutions are only available in rather simple circumstances, where assumption and simplification are introduced to reduce the equation to a more tractable form. The main reason for the formidability of the RTE equation is its integro-differential nature. Since there are no analytical solutions available to the transport equation in realistic cases, numerical techniques have been developed and utilized. The most widely used numerical solution is based on the Monte Carlo technique.

2.2 The Monte Carlo technique

Nowadays, the Monte Carlo technique is the most versatile approach to solve the RTE in complex 3D structures. The technique has been extensively applied in the last decade in biomedical-optics research for the case of light propagation through tissues [15]. Recently, new MC models have been adapted by the authors [13] for the investigation of light scattering in sprays. The main assumption of the MC technique is to define the source light as point entities (photon packets); these point entities are called photons here for simplicity. Each photon enters the medium containing scattering and absorbing centers (droplets) with an initial direction and each photon is tracked as it travels through the medium. The photon trajectory is governed by probability density functions defined beforehand: the probability that a photon is scattered, the probability that it is absorbed and the probability to follow a new direction of propagation after a scattering event. The principle steps of the MC technique are as follows:

The free path length \( l \) before each light-droplet interaction is derived from the Beer-Lambert law and calculated as a function of the extinction coefficient using a random number \( \xi \) uniformly distributed between 0 and 1:

\[
l = -\ln(\xi) / \mu_a
\]

At each interaction with a particle, photons can be
either absorbed or scattered depending on the medium albedo, \( \Lambda \), as defined in Eq.6. In the MC technique, the scattering phenomena are assumed independent of each other. This assumption requires a distance between particles of greater than three times the radius [16].

After each scattering event, the photon’s direction is selected with a random number and a Cumulative Probability Density Function (CPDF) calculated from the appropriate Mie phase function \( f(\theta) \). The polar scattering angle \( \theta \) defined between 0 and \( \pi \) is found from the inverse of the CPDF of \( f \) by \( \theta = \text{CPDF}^{-1}(\xi) \) (where \( \xi \) is a random number between 0 and 1). The azimuthal scattering angle \( \phi \), is uniformly distributed between 0 and \( 2\pi \). As shown Fig.3, a change of coordinate system is performed at every scattering event.

When a new propagation direction is defined, the position of the next scattering point is re-calculated and the process repeated until the photon is either absorbed or exits the medium. The total number of photons transmitted depends on the accuracy desired and on the detector characteristics. The final propagation direction, position, number of scatters, and the total path length are calculated at the end of each photon’s journey. If the detection conditions are satisfied (e.g. the photon lies within the detector field of and its trajectory is within the acceptance angle), these data are recorded. The process is repeated for a large number of photons such that the distribution of all light intensity impinging on the detector has been found in the 3D coordinate system. If an infinite number of photons were sent, the exact solution of the RTE would be obtained.

The MC technique can handle all conceivable geometrical configurations including source, medium, and detector and it is the most flexible method for reaching a reasonable estimate. Thus the MC technique is a powerful tool to understand and simulate scattering processes in different turbid media and in particular in sprays.

### 2.3 Polydisperse and inhomogeneous turbid media

The complexity of a turbid medium has a direct bearing on the complexity of the MC model required for the simulation. By definition, inhomogeneous media are characterized by a variation of the number density \( N \) of scattering particles at various locations. Working with such media requires the scattering medium to be decomposed into a large number of elementary volumes assumed homogeneous (with constant \( N \)). In the presented model, these elementary volumes are cubic cells of constant size. The number and the size of the cells are chosen based on the accuracy required and on the geometry of the medium. Figure 11 shows the decomposition of a spray into unitary cells. Inhomogeneous media can be “monodisperse” (only one size of particle present), “uniformly polydisperse” (constant particle size distribution with location) or “polydisperse” (changes of particle size distribution with location). For monodisperse and uniformly polydisperse inhomogeneous media, a unique scattering process can be assumed through the entire spray and only one phase function is used. The variation of the extinction coefficient, \( \mu_e \), is in this case simply related to the variation of the number density, \( N \), of particles. For inhomogeneous polydisperse media, both number density of particles and particle size distribution vary with location. The extinction cross section, \( \sigma_r \), and the scattering phase function must therefore be defined in each cell. This constitutes the most complex scattering medium, and is characteristic of sprays. In the model described here, 25 different phase functions can be used and the entire spray structure can be modeled via at least 100x100x100 cubic cells.

### 3. VALIDATION AND APPLICATION OF THE CODE

#### 3.1 Experimental set-up

The basic of the experiment consists to send a laser beam through a homogeneous monodisperse scattering medium of known optical properties and to detect the intensity profiles of the scattered light. The incident laser light is produced by a Spectra-Physics Tsunami Ti:Sapphire mode locked laser. Light pulses of \( \approx 80 \) fs duration (FWHM \( \approx 11 \) nm centered at \( \approx 800 \) nm) and 1mJ energy is transmitted through a 10nmx10nmx45mm cuvette containing scattering suspension of polystyrene spheres immersed in distilled water. Various solutions are considered by varying the concentration and the size of the polystyrene spheres. The number density of spheres in each cell has been adjusted to provide optical depths of 2, 5 and 10 for two sphere diameters \( d \) equal to 1 and 2 \( \mu m \). The light intensity profiles were detected on both the front and side face of the cell using an Electron Multiplying CCD camera (Andor iXon DV887). A surface of 10mmx10mm is imaged on to 200x200 CCD pixels resulting in an image resolution of 50\( \mu m \). The detection acceptance angle \( \theta_i \) of the collection optics has been fixed to 8.6° and 1.5°. Due to the high pulse repetition rate (86 MHz) and the long detection time aperture (0.015s), the detected light is perceived by the camera as a continuous laser source. For each measurement, 10 images are recorded and averaged. An illustration of the experiment is shown in Fig.4.

#### 3.2 Description of the simulation

In the simulation, the laser wavelength is assumed monochromatic and equal to 800nm. The dimensions of the experimental cell are assumed by considering a cubic volume of 10mm length. The source of light, \( S \), is modeled via the initial experimental matrix. Using this technique, the exact Gaussian profile of the experimental laser beam can be modeled (see Fig.5). Computed photons are recorded at the exit position provided the detection conditions are met. This implies that the angle between the vector normal to the detection face (front face or side face) and the vector direction of the photons must be within the acceptance angle defined.
Figure.5: Simulation configuration: The laser source $S$ is modeled from the experimental image matrix. Photons are sent from $S$ into a scattering cubic cell of length $L=10\text{mm}$.

According to Ma et al. [17], polystyrene spheres illuminated at 800nm have a refractive index of $n=1.578-0.0007i$. Due to the negligible part of the absorbing component when comparing to the scattering component ($\Lambda$ tends to 1), the spheres have been assumed non-absorbing in the model, with $n=1.578+0.0i$. The refractive index of the surrounding medium is for distilled water: $n=1.33+0.0i$. The resulting Mie phase functions used in the model for $d=1\mu m$ and $d=2\mu m$ are shown in Fig.6. One billion photons are transmitted in each simulation. For side detection, 3 billion photons have been sent in order to increase the spatial resolution of the MC images.

Figure.6: Mie scattering phase function for polystyrene spheres of diameter $d=1\mu m$ and $d=2\mu m$, in a water suspension. The incident light is unpolarized with $\lambda=800\text{nm}$. Each division represents a change 10:1 in intensity.

3.3 Comparison of experimental and simulated results

The first set of comparisons is based on the forward face detection at large detection acceptance angle ($\theta_a=8.6^\circ$) as illustrated in Fig.8. These results show that the simulated data agree both qualitatively and quantitatively with the experimental. By increasing the optical depth, the light intensity transmitted through the scattering sample is reduced and the laser beam profile diffuses. These characteristics are observed, both experimentally and via simulation, particularly at $OD=5$ and $OD=10$. The amount of light crossing the sample reaches a maximum value of $\approx 18.2\%$ when assuming $d=1\mu m$ at $OD=2$. By holding parameters identical but considering the $2\mu m$ diameter scattering spheres, it is seen that the detected transmitted light reaches $\approx 30\%$. This divergence, noticeable at all $OD$, is caused by the difference in the forward scattering lobe between the two particle sizes. As mentioned previously and seen in Fig.6, the forward scattering lobe for $d=2\mu m$ is more developed than the lobe corresponding to $d=1\mu m$. This implies more light is scattered into the forward direction at each scattering event.

Figure.7: Comparison between the front face experimental and simulated images at detection acceptance angle $\theta_a=8.6^\circ$. Solutions of polystyrene spheres of 1 and 2 $\mu m$ diameter are considered at various optical depths $OD$. 
The simulated light intensity is of the same order as the light intensity recorded experimentally. However, discrepancies increase at high optical depth (OD=10) with more photons detected in the simulation than experimentally. These divergences can also be noticed for side scattering detection (see Fig.7).

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Figure 7: Comparison between side scattered experimental and simulated images at detection acceptance angle $\theta_a$=8.6° with OD=5 and OD=10.

There are three reasons behind this phenomenon: Firstly, experimental errors are introduced at high optical depth due to the limitations of the Beer-Lambert law. Secondly, large optical depth decreases the amount of detected photons on the forward face. The statistics are thus reduced, introducing errors in the MC calculations and reducing the spatial resolution of the MC images. The problem of low statistics is also encountered for the side detection at every OD. Note, however that for the side detection, as OD is increased the detected light intensity increases. The last reason for the discrepancies is the absorption process. At very high OD, the absorption of the spheres starts to become significant and it may be necessary to consider this in the simulation. Despite the quantitative divergences, it is seen from Fig.7 that the simulated spatial light intensity distribution is comparable to the experimental. By increasing the OD, it can be seen that the cylindrical shape of the incident beam becomes wider. Furthermore, the detected light intensity is increased and the distance of photon penetration in the scattering medium is reduced. By the same token, both experimental and simulated results demonstrate that for a more forward scattering phase function (here for $d=2 \mu m$), the light tends to penetrate further into the scattering medium and to conserve the initial shape. When the detection acceptance angle is reduced, the light intensity is also reduced while the initial laser beam profile is conserved as seen Fig.8. Once again, the MC images are in good agreement with the experimental images when assuming $\theta_a = 1.5°$ for $d=1 \mu m$ at OD=2 and OD=5.

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Figure 8: Comparison between the front face experimental and simulated images at detection acceptance angle $\theta_a$ =1.5° for 1 $\mu m$ diameter polystyrene spheres.

3.4 Application of the model to a hollow cone spray.

A hollow cone water spray running at 4 bars injection pressure is imaged using a CCD camera via a laser sheet of 532nm wavelength. The spray is assumed symmetrical, and the full 3D structure is constructed in the model by rotating 2D mapping data (extinction coefficients and drop sizes) around a central vertical axis (see [13]). The dimensions of the full simulated volume are 20mm x 20mm x 15mm. The laser sheet (1mm wide and 20mm high) traverses the scattering medium in the middle of the spray. The wavelength of the laser is 532 nm. Drops are assumed spherical and non-absorbing with a refractive index of 1.4+0.0i. The detection area is the side faces of the scattering volume, parallel to the laser sheet (Fig. 1). The detector acceptance angle is $\theta_a$=2.5°. With this angle, a large number of photons are required to obtain good statistics: 5 billion photons are sent. Figure 9 (a) and (b) shows the divergence between the experimental Mie image and the MC image. The laser light sheet enters on the left hand side of the image and leaves on the right.
Figure 9: Illustration of Monte Carlo simulation in Mie laser sheet imaging of a Hollow cone spray, with the resultant experimental (a) and simulated (b) images.

It can be seen that the basic spray structure of the simulated image agrees well with the experimental image even if some differences on the light intensity distribution can be noticed. These differences are explained by several factors. Firstly data used in the simulation are symmetrical around the spray axis, whereas real sprays of this type are known to be asymmetric by up to 15% in mass flow rate. Secondly, MC data corresponding to the scattering coefficient have not been fully corrected from attenuation errors and multiple scattering. Thirdly, the optical properties of the liquid sheet are not modeled in the MC code (only Mie scattering is considered).

5. CONCLUSION

A new computational model for investigating light scattering in sprays has been validated. The comparisons between experimental and simulated results demonstrate good agreement for different solutions of monodisperse polystyrene spheres suspended in distilled water. Various configurations have been investigated by changing the properties of the scattering medium, (various concentration and size of particles) and of the collection optics (forward or side detection, different detection acceptance angles). In all cases, the light intensity distribution provided by the MC code remains close to the experimental results. Using this technique, corrective solutions and information concerning multiple scattering in sprays can be accurately deduced. Therefore, the MC model of the type developed here presents a promising method for improving and developing new optical diagnostics for sprays.

The authors acknowledge the support of the EPSRC (project GR/R92653), the Royal Society (project 15298), Laserlab Europe (project Ilc001131), the Swedish Vetenskapsrådet and the European Union Large Scale Facility program.

6. REFERENCES