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CONTRIBUTION TOWARDS THE DEVELOPMENT OF MODELING METHODS FOR TURBULENT SPRAYS

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ABSTRACT

A population balance approach with a novel cascaded breakage kernel was developed to study turbulent sprays. The proposed cascaded approach applies wave breakup theory to the primary liquid core and a turbulent breakup that recognizes multiple droplet breakups can occur due to one eddy in the far field of the spray. Two-dimensional (2-D) axi symmetric numerical simulations were performed to predict the evolution of a turbulent spray with the proposed cascaded population balance approach in a computational fluid dynamics (CFD) framework to visualize the predicted spray. The results were evaluated by comparing intact length of the liquid core and droplet Sauter mean diameter distributions in the far field of the spray with experimental droplet measurements and were found to be in good agreement.

INTRODUCTION

The fragmentation of a liquid volume by a high speed gas stream to form a spray is a phenomenon that embraces many chemical feed injection systems. There is a considerable interest in obtaining detailed knowledge of the structure of the atomized liquid, in particular, the distribution of droplet sizes as a function of fluid and dynamic properties. Droplet and velocity statistics and shape of the spray plume formed is a general approach for representing the complex nature of sprays.

The fundamental approach used for modeling poly-dispersed sprays has been limited to the discretization of the liquid flow field into groups of equi-sized droplets called "parcels" or "blob" that are tracked in a Lagrangian framework. The alternate approach is the solution of separate Eulerian conservation equations for a number of size ranges. Computational approaches developed for spray modeling are often derived from the spray equation developed by Williams (1958) for the droplet distribution function. Dukowicz (1980) outlined the Lagrangian Monte Carlo procedure o develop a coupled solution between the spray equation and computation of the gaseous phase. To complete the spray formulation, a model of droplets production is required.

The widely accepted modeling approach for atomization or droplet production is based on the use of a wave growth theory that predicts the spray characteristics such as droplet size distribution and spray angle. The wave breakup model of Reitz (1987) is the most widely used for modeling production of droplets. Reitz assumed that breakup results from a hydrodynamic instability caused by surface tension and that the newly formed droplets are characterized by a single Rayleigh mode (1879) of atomization. At the same time, the Rayleigh's type of breakup takes place when the liquid jet is injected into quiescent environment at a relatively low velocity. When the liquid jet is injected into the flowing motion of gas at high relative velocity, large Weber number, the influence of interfacial forces is less pronounced and the mechanism of breakup becomes more complex. A wide range of turbulent eddies may impact on the liquid jet resulting in its breakup. Air-assisted atomization is the preferred mechanism in a range of chemical engineering applications where independent control of gas and liquid flows is required and the determination of onset of nozzle blockage during operation is critical. The modeling of droplets formation under this type of breakup (Lefebvre, 1989, Lin and Reitz, 1993 and Chigier and Reitz, 1996) often referred to as the air-assisted or air-blast atomization, is the main subject of this work.

Atomization theories including wave breakup models i.e.: the surface wave breakup model by Reitz [1], the Kelvin-Helmholtz (K-H)and Rayleigh-Taylor (R-T)theory by Patterson and Reitz and the Taylor analogy breakup model (TAB) model by Amsden and O'Rourke[2] or turbulence and more recently shear models (Lasheras and Hopfinger, 2000 and Gorokhovski and Saviliev, 2003) are widely used. In atomization studies, the effect of infinitesimal wave growth on the jet surface and droplet turbulence are both critical for determining dominant length and time scale of atomization. Models that account for wave growth and turbulence effects at the exit of the nozzle are yet to be developed.

For primary atomization or fragmentation of the intact liquid jet, applying the K-H theory, the breakup of liquid jet occurs due to the viscous forces caused by relative motion and has been well-understood and the dispersion relation, provided by Reitz and Bracco (1979), obtained by the stability analysis of a cylindrical liquid surface subject to linear perturbations was applied. Secondary breakup is generally modeled using the Taylor analogy breakup [TAB] model (O'Rourke and Amsden, 1987) is based on analogy between oscillating droplets and a mass-spring system. The droplet distortion is given by a forced, damped harmonic oscillator. The forcing term is the aerodynamic drag, the dampening effect is provided by the viscous term and the restoring force is given by the surface forces (tension) on the droplet. The TAB model has several advantages since no input for spray angle is required, liquid viscosity effects are well-accounted for, and the explicit information of distortion

and oscillation effects on the inter-phase exchange rates of mass, momentum, and energy. The inherent limitations of the TAB model are (1) only one oscillation mode can be tracked, (2)droplet oscillation and breakup calculations require additional information for deformation and oscillation and(3) turbulent effects are ignored. Several experimental studies (Lefebvre, 1989, Lin and Reitz, 1993, Chigier and Reitz, 1996, Lasheras and Hopfinger, 2000 and Georijon and Reitz, 1999) have also noted the difficulty in clearly defining a dominant air-blast atomization mechanism. Each spray region produces droplets with a large spectrum of size, which is often independent of the breakup pre-existing properties.

Accurate knowledge of fluid mechanics is required for the quantification of spray processes. In recent year computational fluid dynamics (CFD) has emerged as a powerful tool for the understanding of the fluid mechanics prevailing in the reactors. Its success in single phase processes is significant while the presence of multiple phases makes the description of the flow difficult. This requires the correct distribution of disperses phases (liquid droplets) with regards to both the size and number in spatiotemporal coordinates. This can be obtained by using a population balance approach (PBM). The disperse phase distribution is governed by processes of convection and diffusion caused due to the continuous phase (atomizing gas, which is turbulent in nature) and coalescence and breakage of the dispersed phase, which again are essentially governed by the local flow characteristics. In view of this, both the PBM and CFD rely on the success of each other and hence either of them can not be used for the entire description of the multiphase flow behavior. Hence there is a need to couple these approaches for the accurate modeling of sprays at the cost of the lesser computational resources. Due to different nature of the governing equations involved in both the approaches, the methods to solve the resulting equations need to be developed.

Unlike other population balance processes, spray dynamic modeling is further complicated due to the existence of multiple cascaded physical mechanisms, such as primary ligament breakup and secondary droplet atomization that are responsible for spray formation. At the exit of a nozzle, a very weak residual turbulence (Villermaux et. al) has been noted whereas droplet turbulence downstream has been studied (Gorokhovski and Saviliev, 2003). The primary atomization of spray depends on the interaction of the jet structure with the ambient gas, leading to droplet fragmentation due to shear at the periphery of the jet and large ligament breakup close to the liquid core. The ensuing secondary breakup commences after primary droplets are created and depends on the initial droplet sizes formed, turbulence and physical properties of the fluids and ambient conditions of the spray.

In this work, a new model that considers both (1) wave growth on the jet surface using and (2) droplet –eddy and droplet-droplet interactions using a turbulence breakup and dispersion model is proposed to capture the effects of turbulence on droplet breakup as well as droplet trajectories. A cascaded breakup kernel approach for population balance formulation to describe the evolution of droplets in sprays is proposed in this work. Using population balance equations (PBE), the dispersed phase is represented as a population and the size distribution of the population is approximated as a summation of N Dirac Delta functions, represented as droplet phases and each phase is characterized by a diameter and volume fraction. Also, new cascaded kernel functions that transition from wave breakup of jets to turbulent breakup of droplets is presented. Extended source term parameterizations for the breakup mechanism are coupled with continuity and momentum equations in a computational fluid dynamics (CFD) model. This model eliminates the limitation of fixed size class discretization. Experimental validation of the model indicates that an extended model may lead to an improved description of the droplet size distribution, spray plume dimensions and also the volume-averaged droplet velocity resulting in its breakup.

POPULATION BALANCE MODEL FOR SPRAYS

The spray system is considered as a two-phase flow using a full multi-fluid Eulerian model. In this approach mass and momentum balance equations are solved for each phase. The coupling between phases is achieved through inter phase exchange terms. The governing equations are solved using a finite volume method. Spatial differences are formed on an axi-symmetric general coordinate with all gas field variables stored at the same grid point. Second-order accurate central differencing scheme is used for the diffusion terms, and a second order upwind scheme is used for the convection terms. The implicitly coupled pressure and velocity equations are solved by the PISO algorithm. The strong coupling terms between droplet and gas phases are evaluated by the same time-splitting technique. Implicit coupling procedures are used to treat momentum exchanges to avoid the small time steps.

The mass and momentum balance equations can be written as

$$\frac{\partial}{\partial t}\alpha_L\rho_L + \nabla.(\alpha_L\rho_L\overline{u_L}) = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}\alpha_G\rho_G + \nabla.(\alpha_G\rho_G\overline{u_G}) = 0 \qquad (2)$$

$$\frac{\partial}{\partial t}\alpha_L\rho_L u_L + \nabla.(\alpha_L\rho_L\overline{u_L}u_L) = -\alpha_L\nabla P + \overline{\overline{\nabla.\tau_L}} \qquad (3)$$

$$\frac{\partial}{\partial t}\alpha_G\rho_G u_G + \nabla.(\alpha_G\rho_G\overline{u_G}u_G) = -\alpha_G\nabla P + \overline{\overline{\nabla.\tau_G}} \qquad (4)$$

The turbulence was modeled using standard k – ε model (Launder and Spalding, 1972) suitably extended to spray flows.

The droplet phase is assumed to be composed of 12 discrete droplet bins and a discretized population balance equation is solved for the droplet number density along with birth and death terms due to breakup and coalescence. The equation for the i^{th} droplet class fraction f_i is written as

$$\frac{\partial}{\partial t}\alpha_L\rho_L f_i + \nabla.(\alpha_l\rho_l\overline{u_L})f_i = S_i \qquad (7)$$

where f_i is defined as the ratio of total volume of droplets of class i to the total volume of droplets of all classes. These functions are expressed as

$$S_i = B_{breakup} - D_{breakup} \qquad (8)$$

2.1 .Primary Atomization – Wave breakup kernel for Turbulence Induced Wave Growth

Primary atomization, or disintegration of the liquid core, is simulated by assuming a turbulence induced wave growth process, by performing a linear stability analysis of a cylindrical liquid surface subjected to small perturbations using first order wave theory (Reitz, 1980). An infinitesimal axi-symmetric displacement,η_0, applied to the surface of a liquid cylinder and represented by the displacement equation,

$$\eta = Real\left(\eta_0 e^{ikz+\omega t}\right) \qquad (7)$$

Reitz and Bracco, (1979) derived the dispersion relation relating the growth rate of the infinitesimal displacement to its wavelength and physical and dynamic parameters of the liquid and the ambient gas.

$$\omega^2 + 2\nu_1 k^2 \omega \cdot \left[\frac{I_1'(ka)}{I_0(ka)} - \frac{2k\ell}{k^2+\ell^2}\frac{I_1(ka)}{I_0(ka)}\frac{I_1'(\ell a)}{I_1(\ell a)}\right] = \frac{\sigma k}{\rho_1 a^2}\left(1-k^2a^2\right)\left(\frac{\ell^2-k^2}{\ell^2+k^2}\right)\frac{I_1(ka)}{I_0(ka)} + \frac{\rho_2}{\rho_1}\left(W-\frac{i\omega}{k}\right)^2 k^2\left(\frac{\ell^2-k^2}{\ell^2+k^2}\right)\frac{I_1(ka)K_0(ka)}{I_0(ka)K_1(ka)} \qquad (8)$$

where the primes denote differentiation.

Numerical solutions of the dispersion relation lead to a single maximum in the wave growth curve. Curve fitted solutions (Reitz, 1987) to determine the maximum growth rate (Ω) and fastest growing wavelength (Λ).

$$\Omega = \frac{0.34 + 0.38We^{1.5}}{(1+Z)(1.4T^{0.6})}\sqrt{\frac{\sigma}{\rho_L r^3}} \qquad (9)$$

$$\Lambda = \frac{9.02r(1+0.45\sqrt{Z})(1+0.4T^{0.7})}{(1+0.865We^{1.67})^{0.6}} \qquad (10)$$

$$u_r = \left|u_g - u_l\right| \qquad \text{(relative velocity)}$$

The daughter droplets formed from the parent blob are given as,

$$r_c = B_0\Lambda \qquad B_0 = 0.61 \qquad (11)$$

and the time for breakup of parent blobs is represented as

$$\tau = \frac{3.788B_1 r}{\Omega\Lambda} \qquad B_1 = 60 \qquad (12)$$

The length for breakup used in this model is the air-assisted breakup length suggested by Lin and Reitz (1993),

$$L_b / 2a = 0.5 * We_L^{-0.4} * Re_G^{0.6} \qquad (13)$$

Unlike other breakup kernels, this model does not have a probability distribution for the daughter bubbles. It is assumed that for any parent bubble, r, the daughter bubble radius is fixed at r_c. Thus to calculate the breakup rate, the daughter bubble radius for the breakup is determined and then breakup is based on daughter bubble distribution. The parent droplet class is assumed to have the characteristic diameter of its class, while a child droplet is in the class if it falls into the range of sizes attributed to the class.

2.2. Secondary Droplet Breakage Kernel for Turbulent Sprays

Inertial effects are relatively dampened during secondary atomization and the droplet breakup occurring in this region is analyzed in terms of droplet interactions with turbulent eddies. Droplet breakup parametrization consists of the product of the breakup probability and the collision frequency. Summarizing over the possible eddy sizes yields

$$\Omega_B(\mathrm{v}_i, \mathrm{v}_k) = \sum P_B(\mathrm{v}_i, \lambda_j, \mathrm{v}_k)\omega_B(\mathrm{v}_i, \lambda_j) \qquad (14)$$

In spray processes, because of the high number density of droplets near the spray nozzle, collisions of droplets occur frequently and then affect spray characteristics. The potential outcomes of binary droplet collisions can be viewed as, (1) coalescence to form a single droplet; (2) bouncing off of droplets with exchange of momentum between droplets (3) coalescence followed by droplet segregation; (4) Separation of droplets with the formation of secondary droplets. In this work, binary droplet/bubble coalescence model (Luo, 1993) is considered and the collision frequency, between eddies of size between λ_j and $\lambda_j + d_\lambda$ and droplets of volume size v_i, is a similar expression to the collision frequency between gas molecules and is given by

$$\omega_B(\mathrm{v}_i, \lambda_j) = \frac{\pi}{4}(d_i + \lambda_j)^2 \cdot u(\mathrm{v}_i, \lambda_j)n(\mathrm{v}_i)n(\lambda_j) \qquad (15)$$

Coalescence probability is given as:

$$P_C(d_i, d_j) = \exp\left\{-C_1 \cdot \frac{\left[0.75(1+\xi_{ij}^{\ 2})(1+\xi_{ij}^{\ 3})\right]^{0.5}}{(\rho_G/\rho_L+\gamma)^{1/2}(1+\xi_{ij})^3} We_{ij}^{\ 1/2}\right\} \qquad (16)$$

Droplet breakup is analyzed in terms of droplet interactions with turbulent eddies. The turbulent eddies increase the surface energy of the droplet through deformation. Breakup occurs if the increase in surface energy

reaches a critical value. A binary breakage is assumed. The kernel contains no adjustable parameters. The breakup rate (Luo, 1993) of droplets of size v into particle sizes of $v f_{BV}$ and $v(1-f_{BV})$.

The relative velocity between the colliding droplet and turbulent eddy is expressed as

$$u(v_i,\lambda_j)=(u_t(v_i)^2+u_t(\lambda_j)^2)^{\frac{1}{2}} \quad (17)$$

With turbulent velocity

$$u_t(\lambda_j)^2=\beta^{\frac{1}{2}}(\varepsilon\lambda_j)^{\frac{1}{3}} \quad (18)$$

This equation is also used for finding the turbulent velocity of the droplets by replacing the eddy length scale by the droplet diameter analogous to bubble breakup (Luo, 1993). The probability of obtaining one specific droplet daughter class, as a result of breakup of a parent droplet size, colliding with an eddy size is given as the sum over different eddy energy levels e_1.

$$P_B(v_i,\lambda_j,v_k)=$$

$$\sum P_B(v_i,\lambda_j,e_1,v_k)\omega(\lambda_j,e_1) \quad (19)$$

Where $\omega(\lambda_j,e_1)$ is the fraction of is eddies of size λ_j having energy level e_1. It is assumed that the turbulent kinetic energy probability distribution is

$$p_e(\chi)=\exp(-\chi) \quad (20)$$

Where $$\chi=\frac{e(\lambda_j)}{\bar{e}(\lambda_j)} \quad (21)$$

and mean turbulent kinetic energy of an eddy with size λ_j, $e(\lambda_j)$, was given as,

$$\bar{e}(\lambda_j)=\frac{\pi\beta}{12}\rho L\varepsilon^{\frac{2}{3}}(\lambda_j)^{\frac{11}{3}} \quad (22)$$

The function related to breakup due to the energy density causing breakage into a smaller daughter volume size k may be written as

$$F_d(d_k)=\max(e(\lambda_j)-$$

$$\pi\sigma d_i^{\,2}[\frac{d_k^{\,2}}{d_i^{\,2}}+(1-\frac{d_k^{\,3}}{d_i^{\,3}})^{\frac{2}{3}}-1,0)$$

Hence the probability of breakup

$$P_B(d_k,\lambda_j)=$$

$$\frac{w_d(\lambda_j)-\frac{6\sigma}{d_k}}{\int_{dk,\min}^{dk,\max}(w_d(\lambda_j)-\frac{6\sigma}{d_k})d(d_k)} \quad (23)$$

Where, $P_B(d_k,\lambda_j)=$ (17)

$$\frac{w_d(\lambda_j)-\frac{6\sigma}{d_k}}{\int_{dk,\min}^{dk,\max}(w_d(\lambda_j)-\frac{6\sigma}{d_k})d(d_k)} \quad (24)$$

The coupling between primary and secondary droplet breakup and CFD was achieved through the inter-phase exchange terms based on the Sauter mean droplet diameter (SMD).

APPLICATION OF CASCADED FOMULATION TO TURBULENT SPRAY

Equations (1) through (23) were solved numerically using Fluent software- a commercial finite volume CFD solver using user defined functions. The population balance approach using a cascaded breakup kernel approach has been applied for spray analysis of a non-evaporating spray from an air-assisted atomizer. Model validation is done for experimental results (Stepowski et al., 2001). Half-geometry (axi-symmetric) spatial grid was setup. The simulated round jet of water emits from a central tube (D_L = 1.8 mm) at low velocity and is atomized by a concurrent flow of air flowing at high velocity from an annular duct (D_G = 3.4 mm). The 2-dimensional computational grid used is axi-symmetric. The flow geometry was discretized into 270,000 hexahedral cells. Due to symmetry only one-half of the spray geometry was modeled. The dimensions of the grid are 0.003 and 0.012 m. respectively.

The population balance spray formulation using a cascaded breakup kernel approach was introduced into CFD through user defined sub-routines via inter-phase exchange terms. Model validation is done for experiments (Stepowski et al., 2001) performed on an air-assisted atomizer generated non-evaporating spray. A half-geometry (axi-symmetric) spatial grid was setup. The simulated round jet of water emits from a central tube (D_L = 1.8 mm) at low velocity and is atomized by a concurrent high speed air stream flowing at high velocity from an annular duct (D_G = 3.4 mm).

RESULTS AND DISCUSSION

Computed half-geometry (axi-symmetric) spatial distributions of secondary droplets in the spray, for We = 350 and Re_L = 13, 31 and 55 are displayed in Fig(s). 3-6. Numerical distributions in the primary atomization region can be assessed only qualitatively since jet breakup measurements, in the small but critical near-field liquid core, are not available in scientific literature. Fig(s) 3-5 represent the primary breakup, the radii of the daughter droplets formed by stripping the core and the maximum wave growth before breakup. The change in Levich jet breakup length with

increasing liquid flow is shown in Fig.(5) and is found to be in agreement with experimental results. Secondary droplets form early in the atomization process (at ~ 2.5 mm) from liquid inlet as a result of the wave breakup theory (Fig. 3) in the near field region of the nozzle orifice due to the high turbulent air stream turbulence accompanied by finer distribution of droplets downstream mainly due to secondary breakup. Spray simulations show that, droplet distributions broaden with increase in liquid flow rate (Fig. 6) proportional to the size of the injector orifice are broken down and new droplets are produced with mean diameters of 100-200 microns at low liquid Reynolds numbers. The spatial distribution of secondary droplets in Fig. 3 shows that a broad spectrum of droplet size is presented in the spray at high gas to liquid momentum ratios. The simulation at high liquid Reynolds number shows that larger droplets, with mean diameters as large as 500 microns, are scattered throughout the far field of the spray indicating the irregular nature of the breakup. These results are found to be in acceptable agreement with experimental measurements.

ACKNOWLEDGMENT

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NOMENCLATURE

a radius of droplet, m

$B_{breakup}$ birth breakup, 1/(m3 s)

$D_{breakup}$ death breakup, 1/(m3 s)

d_i droplet diameter for ith class, m

D_L hydraulic (liquid) diameter, m

D_G hydraulic (gas) diameter, m

$e(l)$ eddy energy level, J

$e(ì_j)$ eddy energy, diameter size ìj, J

$e(\lambda_j)$ turbulent kinetic energy of the eddy of size λj, J

$\bar{e}(\lambda_j)$ mean turbulent kinetic energy of the eddy of size λj, J

$F_d(d_k)$ breakup function due to energy den- sity into smallest daughter fragment

I_i Modified Bessel Function, 1st kind

K_i Modified Bessel Function, 2nd kind

f_i fraction of droplets of class i

f_{BV} volume breakup fraction

k $\text{wavenumber} = \left(\frac{2\pi}{\lambda}\right)$

L_b intact core length [m]

ℓ $\text{Wavenumber} = \sqrt{\frac{k^2+\omega}{\nu_L}}$ [m^{-1}]

n_i droplet number density of class i [m^{-3}]

$n(V_i)$ number density of droplets in class i [m^{-3}]

$n(\lambda_j)$ number density of eddies of diameter λ_j [m^{-3}]

P , p pressure, Pa

$P_B(V_i,\lambda_j)$ breakup probability of droplet V_i being hit by an eddy of size λj

$P_B(V_i,\lambda_\varphi,e_l,V_k)$ breakup probability of droplet (volume) V_i being hit by an eddy of size λ_j with energy level e_l breaking up into smallest daughter fragment V_k

$P_B(V_i,\lambda j, V_k)$ breakup probability of droplet V_i being hit by an eddy of size λ_j

r Drop radius [m]

r parent droplet radius [m]

r_c daughter droplet radius [m]

Re_L liquid Reynolds number = $\frac{\rho_L u_L r}{\mu}$

Re_G gas Reynolds number = $\frac{\rho_G}{\mu_G} u_r d_G$

S_i interphase exchange source term [kg/m^{3}s]

T Taylor number = $ZWe^{1/2}$

t time [s]

u_i velocity of phase i [m/s]

u axial velocity component

$u(V_i, V_j))$ relative velocity between droplets of volume sizes V_i and v_j [m/s]

$u(V_i,\lambda j))$ relative velocity between droplets of volume sizes *vi* and an eddy of diameter size V_j [m/s]

u_r , W relative velocity between liquid and gas [m/s]

$We = We_G = (\rho_G u_r^2 r)/\sigma$ droplet Weber number

We_L liquid Weber number = $\rho_L u_r^2 D_L/\sigma$

Z Ohnesorge number = $\frac{\sqrt{We_L}}{\text{Re}_L}$

z axial coordinate [m]

Subscripts

i, j, k droplet classes

L liquid phase

G gas phase

Greek letters

α_i volume fraction of phase i

β constant, 2.41

χ kinetic energy fraction for an eddy in turbulence

ε turbulent dissipation [m^2/s^3]

η surface wave amplitude [m]

η_0 initial surface wave amplitude [m]

ω wave growth rate [Hz]

λ_j eddy diameter class *j* [m]

Λ fastest growing wavelength [m]

ν dynamic viscosity [m^2/s]

ρ fluid density [kg/m^3]

σ surface tension coefficient [N/m]

τ breakup time [s]

τ stress term [N/m^2]

$\Omega_B(V_i, V_k))$ breakup rate of class V_i into smallest daughter class V_k, [1/(m^3 s)]

Ω maximum growth rate [Hz]

ξ_{ij} ratio of eddy size λ_j to droplet of class i

Constants:

B_0 0.61

B_1 60

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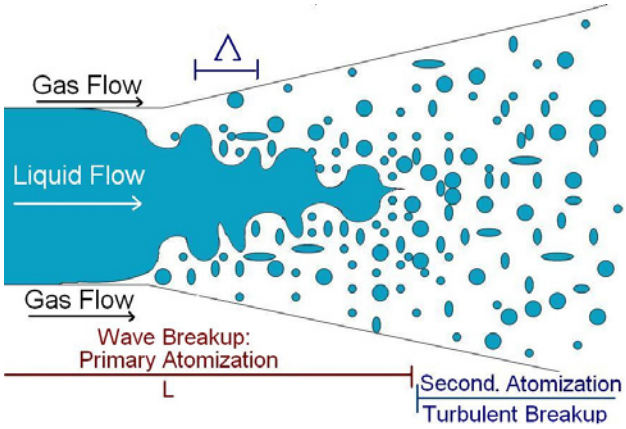

Figure. 1: Schematic representation of numerical approach used for turbulent spray analysis

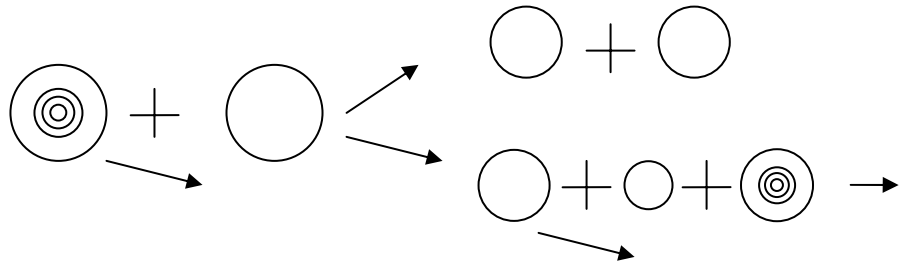

Figure. 2: An eddy colliding with a droplet may result in a collision that transfers the turbulent kinetic energy of the eddy to the droplet (top right) or daughter droplets may be formed along with a used eddy.

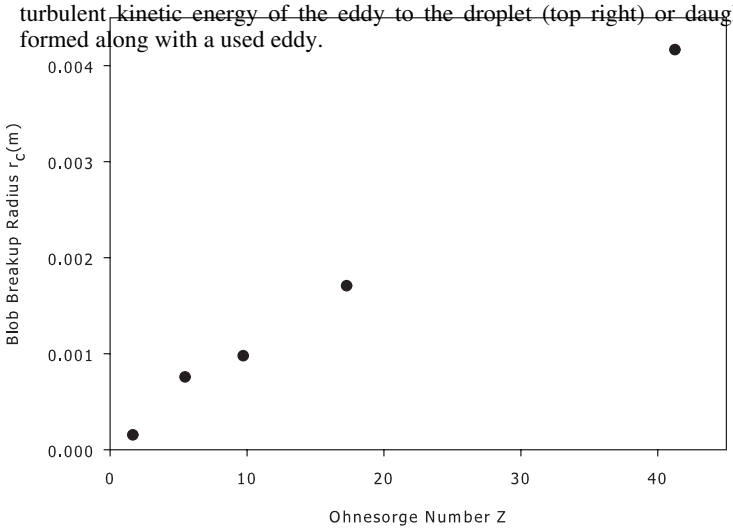

Figure 3: Daughter bubble radii formed using primary breakup model in air blast atomizer experiments.

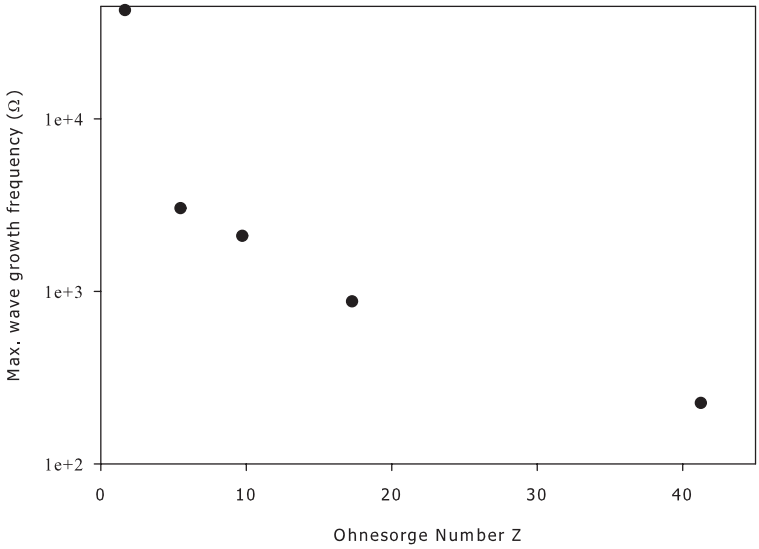

Figure 4: Maximum wave growth frequency achieved during primary breakup

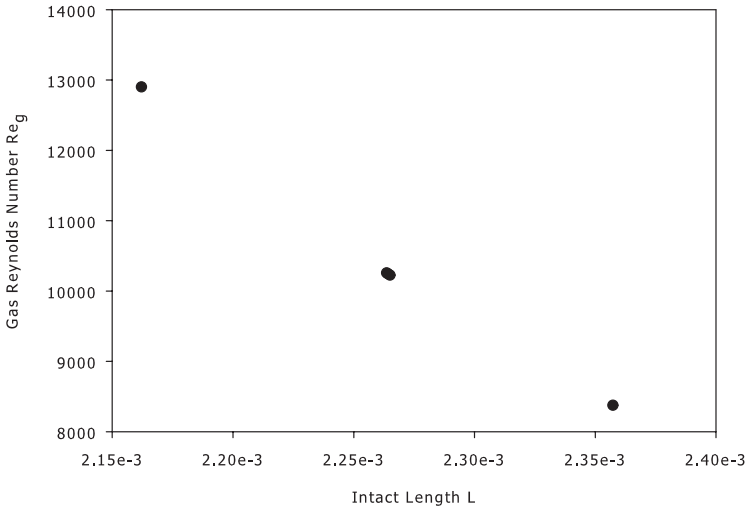

Figure 5: Jet intact or Levich breakup length in a high speed gas stream for atomizer experiments.

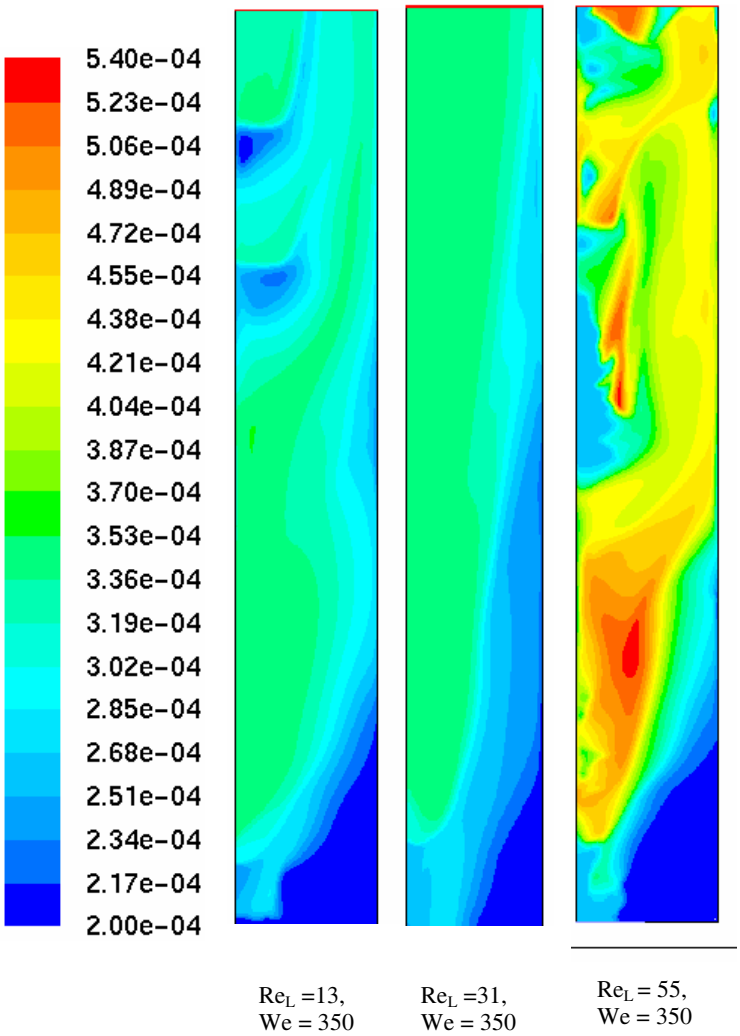

Fig. (6): Computed spatial distributions of Sauter mean diameter of secondary droplets formed by atomizing water by a high speed co-axial air stream.