NUMERICAL SIMULATION OF PINCH-OFF JET PROCESS BY THE LEVEL SET METHOD

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Abstract
Three-dimensional dynamic simulations of a liquid-liquid pinch-off jet are performed by using the level set method for tracking the interface between immiscible materials. The numerical scheme (Sussman *et al*. 1994, Chang *et al*. 1996) of the level set method is incorporated into a finite-volume package, CFDLIB, developed by Los Alamos National Laboratory (Kashiwa and Rauenzahn 1994; Kashiwa *et al*. 1994). The surface tension force is treated as a body force by adopting the continuum surface force (CSF) model by Brackbill *et al.* (1992). The numerical results are compared, both qualitatively and quantitatively, with the experimental data provided by Longmire *et al*. (2001). The possibility of using the level set method in multiphase problems with interface breakup/coalescence is explored by simulating such relative low speed, low density-ratio two phase flow. This work is a part of our effort to study the pressure-atomised fuel injection/spray that provides the fine atomisation needed for rapid mixing of liquid and gas phases during practical combustion processes.

Introduction

The pressure-atomized fuel injection/spray provides the fine atomization needed for rapid mixing of liquid and gas phases during practical combustion processes. We are interested in the behavior of liquid jet and the mechanism of jet breakup at the very initial stage right after the injector exit, i.e. the dense spray region. In order to simulate directly the two-phase flow field, one has to track the interface with reasonable accuracy. As is well known, there are several methods for expressing the moving interface between two fluids, such as the VOF method, the level-set method, and the front-tracking method. The VOF and level-set methods are categorized as a front capturing method, which tracks the movement of volume and finds the interface in an indirect way. One of the advantages of the front capturing methods is that collision and breakup of interfaces are easily treated. In this study, we have been using the level set method (Chang *et al*. 1996) as the interface tracking methodology. This method has been used by other investigators successfully for solving many multiphase problems, which contain interfaces between immiscible materials of different physical properties. Most of these studies deal with low speed, low density-ratio between the material and have been limited to two-dimensional problems. In fuel injection for engine combustion, however, the flow velocity is very high (the flow Reynolds number at the exit is in the order of 10^4) and the density ratio is typically in the order of 10^3. In order to test and validate the numerical schemes and the computer code before going to the more complicated situations, we need reliable experimental studies on some problems, which are physically similar to the fuel injection. Such work should provide not only the qualitative but also quantitative data for the purpose of comparison and assessment of the numerical results.

Round liquid jets flowing into a second immiscible liquid have been investigated by many researchers (Cohen *et al*. 1999; Wilkes *et al*. 1999; Zhang *et al*. 1999) due to their fundamental simplicity as well as their importance in a number of industrial systems. The experiments demonstrated that for low flow rates (and Reynolds number), droplets form at and detach from the jet outlet. As the flow rate is increased, the injected fluid forms a jet that develops axis-symmetric instabilities and pinches off at a finite length. Above a Reynolds number associated with the maximum length, three-dimensional instabilities and eventually direct atomisation occur. The Reynolds number range corresponding to each flow mode depends significantly on the other system parameters, including the fluid properties. One of the most recent experimental studies was done by Longmire *et al*. (2001). They employed a PIV technique to measure the velocity and vorticity distribution and thus provide some quantitative data beside the qualitative images of flow pattern. In the present paper, three-dimensional simulations of round liquid jets flowing into a second immiscible liquid have been performed with the same parameters and conditions as used by Longmire *et al*. (2001). Detailed comparisons are made, both qualitatively and quantitatively, to serve the purpose of assessment and validation of the level set method.

Numerical methods

We consider the fluid motion of a liquid injected into a space, which is initially filled with another liquid of different density and viscosity. Two liquids are immiscible to each other. The interface between the two liquids remains throughout the motion and a surface tension exists at the interface. The flow motion is governed by the

Navier-Stokes equation,

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla p+\nabla\cdot\left(2\mu\mathbf{S}\right)+\sigma\kappa\left(\theta\right)\nabla\theta\delta\left(\theta\right)+\rho\mathbf{g} \tag{1}$$

No turbulence model is included since the flow is laminar. The third term on the right-hand-side of the above equation represents a model, called the continuum surface force (CSF) model proposed by Brackbill *et al.* (1992), for approximating the surface tension forces. In this model, the effect of surface tension can be expressed in terms of a singular source function which is defined by an indicative function, here the level set function, θ.

The level set method (Chang *et al.* 1996) is used to capture the interface between two fluids of different densities and viscosities. A level set function, say, θ, is a distance function about the interface. It has positive values outside the interface and negative inside the interface. At the interface θ keeps a value of zero. The magnitude of the level set function at any location represents the distance from this location to the interface. The level set function, like any passive scalar variables, moves with the fluid, and it follows,

$$\frac{\partial\theta}{\partial t}+\mathbf{u}\cdot\nabla\theta=0 \tag{2}$$

which moves the zero level of θ exactly as the actual interface. The density and viscosity are calculated through out the computational domain depending on the value of θ, by

$$\rho=\begin{cases}\rho_i\\(\rho_i+\rho_o)/2\\\rho_o\end{cases}\quad\text{and}\quad\mu=\begin{cases}\mu_i\\(\mu_i+\mu_o)/2\\\mu_o\end{cases}\quad\text{for}\quad\begin{matrix}\theta<0\\\theta=0\\\theta>0\end{matrix} \tag{3}$$

The nature of the level set method being a signed normal distance function from the interface has essentially to be kept all the time throughout the simulation. A procedure of re-initialization and re-normalization (Sussman *et al.* 1994) is therefore performed at every time step during simulation to pertain such property. This is achieved by solving the following equation to a steady state,

$$\frac{\partial\theta}{\partial t}=\operatorname{sgn}\left(\theta_0\right)\left(1-\left|\nabla\theta\right|\right) \tag{4}$$

with an initial condition,

$$\theta\left(\mathbf{x},0\right)=\theta_0\left(\mathbf{x}\right) \tag{5}$$

where $\theta_0(\mathbf{x})$ is the level function before the renormalization. By the above procedure we build a distance function $\theta(\mathbf{x})$ whose zero set is the same as $\theta_0(\mathbf{x})$. We have implemented the level set method into CFDLIB (Kashiwa and Rauenzahn 1994; Kashiwa *et al.* 1994). for the numerical investigation of fuel injection.

Results

The cases examined represent a parameter set where the fluid properties as well as inertia, gravity, and surface tension are all significant. The flow conditions can be characterized by a set of dimensionless parameters. The values of these parameters for the two cases are given in Fig. 1(a).

Under the chosen flow conditions, gravitational effects cause the jet to accelerate and contract immediately after exiting the nozzle. In the absence of forcing, the above conditions yield a smooth jet column that travels all the way to the downstream fluid interface without developing any significant waves or instabilities. When the flow is forced with a sinusoidal velocity perturbation, however, instabilities are enhanced, and pinch-off occurs within the layer of surrounding fluid. The location of pinch-off can be controlled by adjusting the forcing amplitude such that increasing the amplitude moves the pinch-off location upstream. For the cases described below, the flow was forced at the laser pulsing frequency of 10 *Hz* yielding a Strouhal number, *St*, of about 4. Under the chosen conditions, one drop formed during each forcing cycle at a location approximately 7 diameters downstream of the nozzle exit. In the numerical simulation, this condition is achieved by setting a time-dependent velocity condition at the nozzle exit. The amplitude of the velocity fluctuation is adjusted such that a droplet is pinched off at the same downstream location as in the experiments.

Figure 1(b) shows a typical instantaneous jet shape in the computational domain. Figures 2 and 3 show one cycle of jet disintegration for the case I, and II, respectively. One can see that, for both cases, the numerical

results match with the experimental ones reasonably well. Notice that, for the case II, due to the lower viscosity, the ambient fluid offers less resistance to the inner jet fluid, and hence the jet accelerates to a higher velocity and develops a more complex shape of the evolving jet tip than in the case I. Specifically, the jet tip develops a small spherical structure before the wide part of a wave approaches and the `cusp' initiates near the upstream end of the drop. The simulation did catch such details. However, one can also observe that the drop recovers the spherical shape more slowly than in the reality. One can also notice that a "tear drop" shaped droplet is formed immediately after the disintegration in each simulation whereas the experimental results show that a more round shaped droplet is formed there. The disagreement is mainly due to the CSF model in which the surface tension force is imposed on layer of finite thickness. A remedy would require high mesh resolution.

The quantitative comparisons are made on the axial velocity component. Figure 4 shows the vertical velocity along the jet axis at different time instant during one disintegration cycle for case I. Figure 5 shows the radial profiles of the vertical velocity at a fixed axial location at different time instants during one disintegration cycle for case I. Figures 6 shows the similar comparisons for case II. One can see that better agreement is achieved in the case of higher ambient viscosity, i.e. case I (μ_i/μ_o=0.17). In general, the wavelengths of the velocity variation match with the experiments. The discrepancies are most likely caused by the finite thickness of jet and drop surface, which results in an inner fluid of lighter weight than the real one. Again a mesh system with finer resolution would definitely help to improve the situation.

Conclusion

The comparisons between the numerical results and the experimental data show that the simulation based on the level set method is able to handle the three-dimension problems with interface breakup quite well. The quantitative predictions of velocities are also satisfactory. The results demonstrated that the viscosity ratio had a significant effect on the evolving jet flow, the pinch-off process, and the resulting drop shapes. In general, the simulation gives more accurate prediction in the case of high-viscosity-ratio than in the case of low-viscosity-ratio. The liquid/liquid pinch-off differs from the liquid/air injection in the sense that it is basically a creeping flow with low density-ratio while the later is a two-phase flow of high density-ratio and is turbulent in most situations. The dominant factors in the former are surface tension and viscous force while they are the surface tension and inertia force including turbulent forces in the later. It would, therefore, be reasonable to expect some difficulties when one simulate the high-speed fuel injection problems.

Nomenclature

D	diameter of jet
U_e	velocity of inner liquid at jet exit
W	axial component of fluid velocity
$\mathbf{S}$	fluid strain rate, a tensor variable
$\mathbf{u}$	fluid velocity vector
f	frequency of jet forcing
g	gravitational constant
$\delta()$	Dirac delta function
κ	curvature of interface
ρ	density
μ	dynamic viscosity
σ	surface tension coefficient
Δ	density difference, ρ_I-ρ_o.

Subscripts

i	inside jet
o	outside jet

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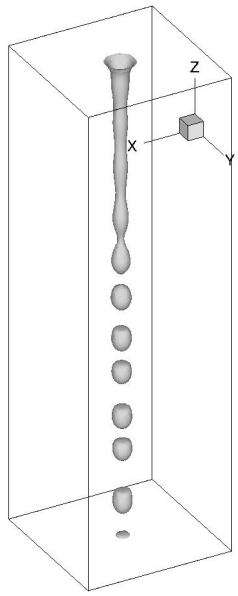
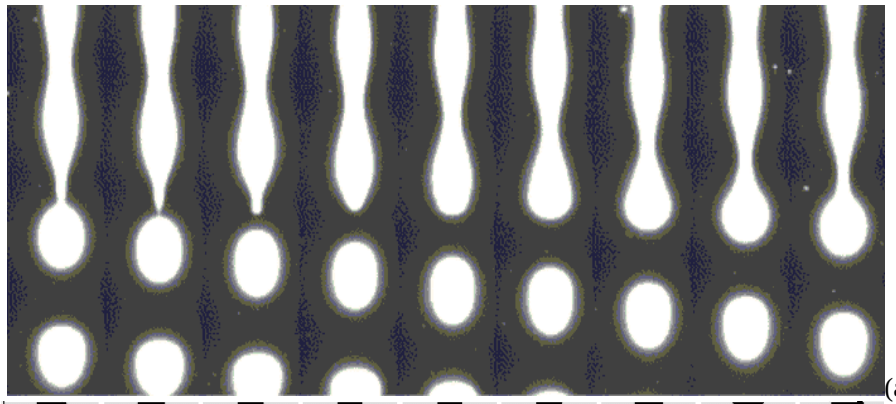
Parameters	Case I	Case II
$Re=\rho_l U_e D/\mu_i$	34	35.2
$Fr=U_e(\rho_i/gD\Delta\square)^{1/2}$	0.2	0.21
$Bo=gD^2\Delta\square/\sigma$	6.1	6.1
ρ_i/ρ_o	1.19	1.18
μ_i/μ_o	0.17	1.72
$St= fD/U_e$	4	3.9

(a)

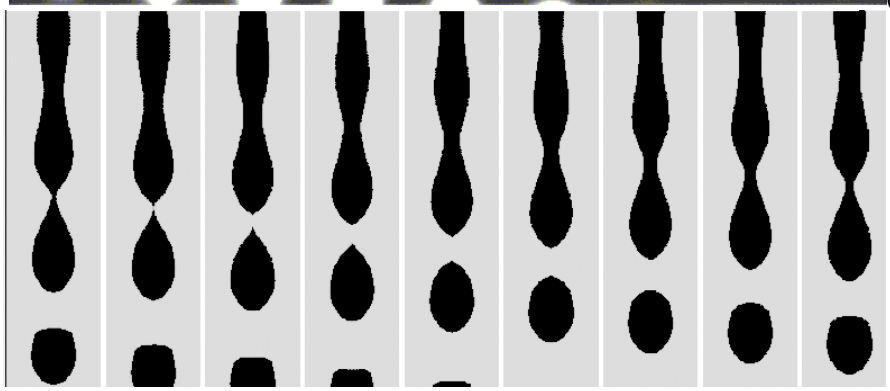


(b)

Figure 1. 3-D simulation of liquid-liquid pinch off. (a) Flow parameters. (b) Computational domain and mesh system: $\Delta x=\Delta y=\Delta z=0.0667$cm. The number of cells in x, y and z directions are 75, 75 and 250, respectively. The diameter of exit is 1 cm.



(a) Experiment



(b) Computation

Figure 2. One cycle of jet disintegration for case I (μ_i/μ_o=0.17, *Re*=34). The time interval between the images is 1/9 T, where T is the period of one disintegration cycle. The pinch off (t=0) is the second in each series. The images from computation show the iso-surface (3-D) of zero level set function ϕ=0.

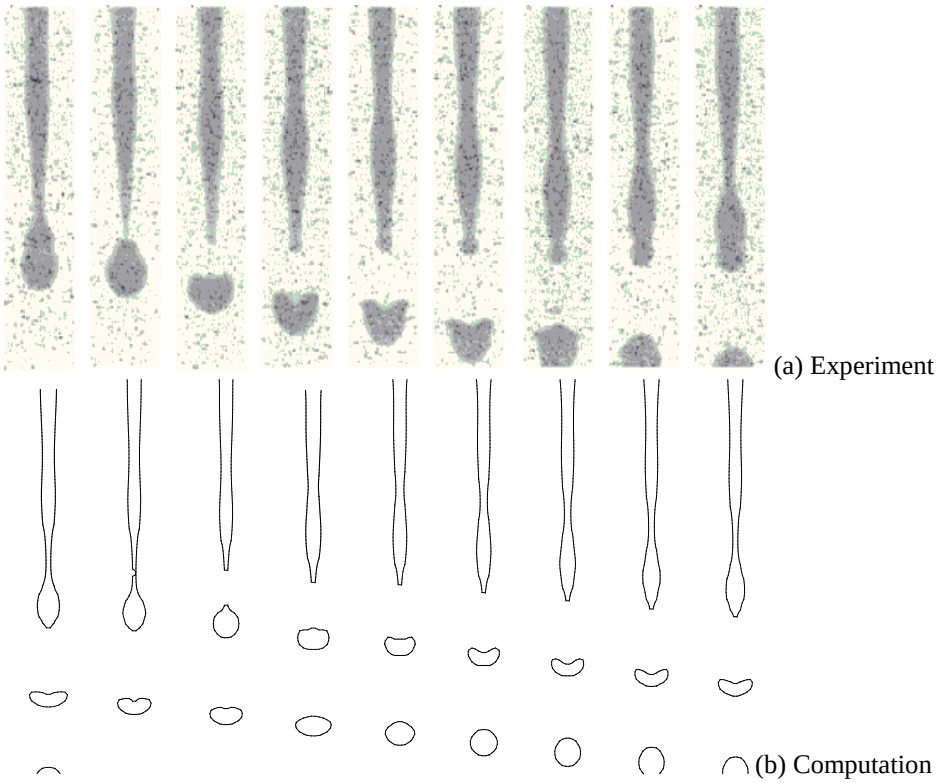


Figure 3. One cycle of jet disintegration for case **II** (μ_i/μ_o=1.72, *Re*=35.2). The time interval between the images is 1/9 T, where T is the period of one disintegration cycle. The pinch off (t=0) is the second in each series. The plots from computation show the contours (2-D) of zero level set function ϕ=0 projected onto an angular plane cutting through the axis of the jet.

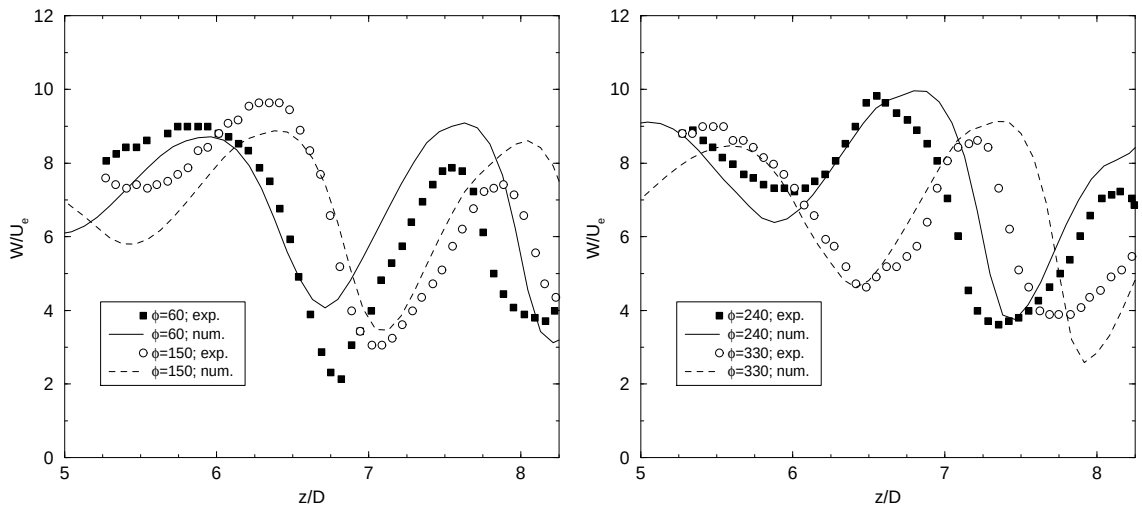


Figure 4. The axial velocity along the centerline of the jet for case **I** (μ_i/μ_o=0.17, *Re*=34), at different time during one cycle of disintegration. Symbols and lines represent the experimentally measured data and numerically calculated values, respectively. ϕ is the phase of one cycle of jet disintegration, defined as ϕ=360t/T. The phase corresponds to pinch is $\phi=0^0$. The letter, z, is the distance from jet exit.

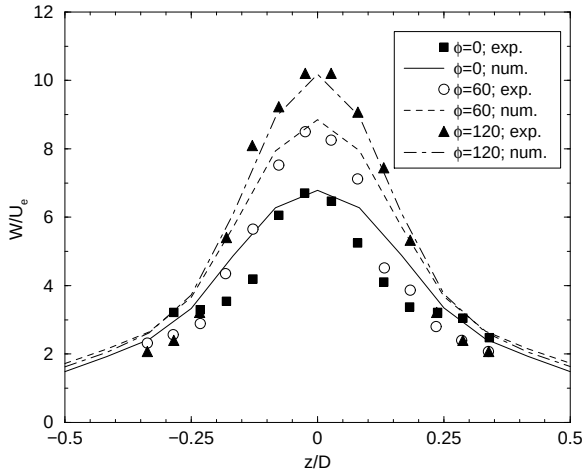
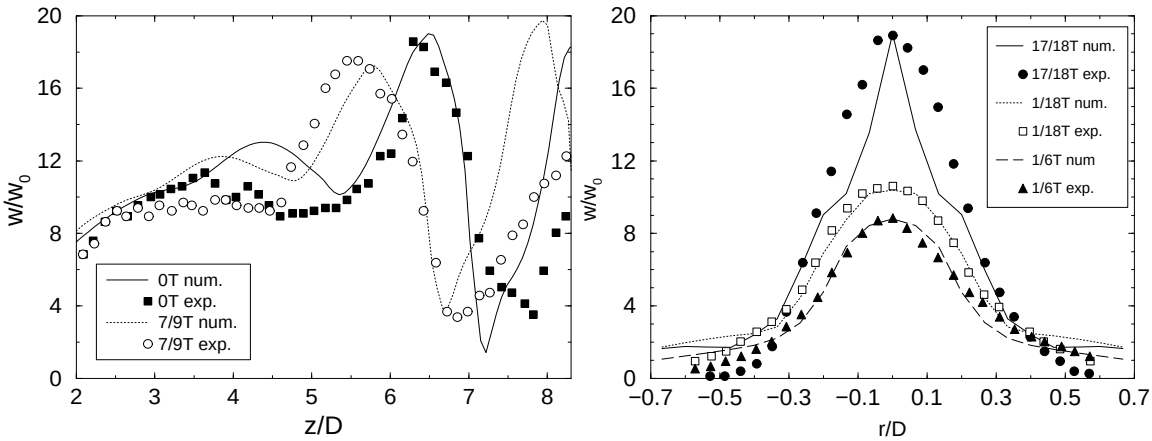


Figure 5 The radial profiles of axial velocity at an axial location (z/D=6.15) for case **I** (μ_i/μ_o=0.17, *Re*=34) at different time during one cycle of disintegration. Symbols and lines represent the experimentally measured data and numerically calculated values, respectively. ϕ is the phase of one cycle of jet disintegration, defined as ϕ=360t/T. The phase corresponds to pinch is $\phi=0^0$.



(a) (b)

Figure 6. The axial velocity profiles for case **II** (μ_i/μ_o=1.72, *Re*=35.2), at different time during one cycle of disintegration. Symbols and lines represent the experimentally measured data and numerically calculated values, respectively. The time instant corresponds to pinch is t=0T. T is the period of one cycle of jet disintegration. (a) The axial profiles along the centerline of the jet. The letter, z, is the distance from jet exit. (b) The radial profiles at a axial location (z/D=6). The letter, r, is the radial coordinate from the center of jet, r=0.