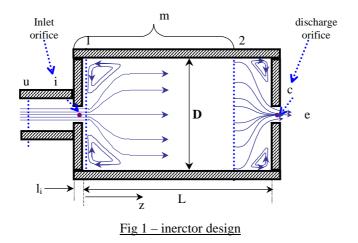
# SPRAY FORMATION BY BI-COMPONENT LIQUID FLASHING: A THEORETICAL APPROACH

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#### Abstract

Spray formation by bi-component liquid flashing through a special-design injector has been studied. The injector is composed of an inlet orifice (i), an expansion chamber (m), and a discharge orifice (c). In principle, in this method, a given mixture comprising of two different types of liquids; a solvent (s) and a propellant (p), flows through the inlet orifice into the expansion chamber (Fig 1), and leaves it through the discharge orifice. The propellant, which has a higher vapor pressure than the solvent, undergoes a rapid boiling process (flashing process), in which many tiny bubbles are created, and then grows along the expansion chamber. In this process, the specific volume of the mixture increases and therefore it's velocity increases. Nevertheless, due to the very low acceleration, no slip between the two phases inside the expansion is expected. Slip, however, is important inside the discharge orifice.



The various relevant processes, which include the pressure drop at the inlet orifice, nuclei formation, bubble growth inside the expansion chamber and the pressure drop and acceleration at the discharge orifice, have been analyzed by using a one-dimensional model approach. Whilst the one-dimensional assumption cannot fully justified, it enables simple analysis and yet provides realistic quantitative results.

In the present work, special attention has been drawn to the processes inside the expansion chamber. It was found that the optimal length of the expansion chamber strongly depends on the growth rate of the bubble and on the cross-section area ratio between the inlet orifice and the chamber. Slower growth rates demand longer expansion chambers in order to attain the optimal void fraction at the discharge orifice.

## Introduction

Spray formation of a bi-component liquid is widely used in household applications; odors, medical, and painting sprays, are few examples. This type of spraying method provides remarkably low SMD spray for a relatively low operating pressure (SMD<100  $\mu$ m for pressure difference of less than 200kPa, Zeigerson-Katz & Sher, [1,2]). As compared to spay formation by mechanical means, for the same operating pressure, spray formation by flash boiling is characterized by smaller mean diameter, more droplets homogeneous distribution, wider cone angle and shorter penetration depth. These characteristics are needed for many applications; smaller and more homogeneous spray are important in almost every application, shorter penetration depth is important in fuel injection systems, where droplet wall impingement should be avoided, and lower injection pressure is important

where safety is a major concern (Smallwood [3]). Spray formation by flash boiling provides the opportunity to generate the desired spray at low injection pressures.

Many experimental works have been focused on finding relationships between the injection properties and the final spray properties (Oza [4], Zeigerson-Katz & Sher, [1,2], Sher & Elata [5]). A detailed review can be found in Witlox & Bowen [6]. Senda et al. [7] presented a semi-empirical model to correlate the rate of generated nuclei to the operation conditions in a flash boiling spray. They measured the number density of bubble nuclei that are created at the inlet orifice by using microscopic photographs. Solomon et al. [8] investigated spray characteristics for flashing injectors of fuels containing dissolved air and superheated fuels.

The present analysis suggests a model, to describe quantitavely the different processes that occur in the different parts of the injection system (the inlet orifice, the expansion chamber and the discharge orifice). This model is used to estimate the pressure inside the expansion chamber, the flow regime in it, and the mean diameter of the spray's droplets, all in terms of the design parameters and the orifices and expansion chamber geometries.

## Model

#### Nucleation

A given mixture of two different types of liquids (a solvent, s, and a propellant, p) flows through an inlet orifice into an expansion chamber. The propellant, having a higher vapor pressure, undergoes a nucleation process. It is assumed that a mixed (homogeneous and heterogeneous) nucleation occurs at the inlet orifice. Riznic & Ishii [9] use the heterogeneity factor in their estimation of the bubble nuclei formation rate. Various heterogeneity factors have been developed (Alamgir & Lienhard [10], Deligiannis & Cleaver [11] and Elias & Chambré [12]) in order to modify the classical kinetic theory expression for homogenous nucleation to include the wall heterogenous nucleation. Schematics representation is shown in Fig 2. The rate of nuclei formation is:

 $\dot{n} = J_s \cdot \pi d_i l_i$ , where  $J_s$  – nucleation rate per unit area,  $d_i$  – diameter of inlet orifice,  $l_i$  – length of inlet orifice.

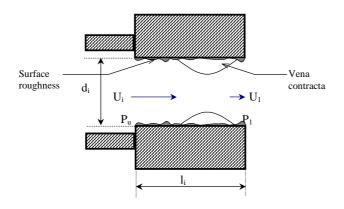


Fig 2 – Wall necleation at the inlet orifice (based on Fig 1 in He & Ruiz [13]).

#### General

The following definitions were used:

The void fraction,  $\varepsilon \equiv \frac{\dot{V}_v}{\dot{V}}$ ; the Cross-sectional fraction,  $\varepsilon_A \equiv \frac{A_v}{A}$ ; the mixture quality,  $x \equiv \frac{\dot{m}_v}{\dot{m}_p}$ ; the propellant mass fraction,  $\beta \equiv \frac{\dot{m}_p}{\dot{m}_p + \dot{m}_s}$ ; and the slip ratio,  $k \equiv \frac{U_v}{U_L}$ .

The following assumptions were made:

I – Steady state process.

**II** – The fluid to be dispersed is incompressible ( $\rho_s$ =const).

III – Nucleation takes place only at the entrance orifice droplet (Senda et al [7]).

- IV coalescence of bubbles inside the expansion chamber are neglected.
- V bubble growth rate is dominated by thermal diffusion.

VI – No slip velocity between the vapor and liquid of the propellant.

**VII**  $-D \gg d_i$ ,  $d_e$  and therefore the pressure inside the chamber is constant.

**VIII** – The temperature inside the expansion chamber is fairly constant  $(T_i = T_m)$ .

**IX** – The kinetic energy/pressure head ratio of the mixture upstream (u) is negligible.

For a steady state process, the mass conservation is:

$$\dot{m} = \dot{m}_s + \dot{m}_p + \dot{m}_v$$
;  $s \equiv \text{solvent (Liq.)}$ ;  $p \equiv \text{propellant (Liq.)}$ ;  $v \equiv \text{propellant (Vap.)}$ 

# Bubble Growth

Bubble growth is controlled by thermal diffusion (V):  $R_b(t) = R_{b,0} + 2C_R \sqrt{\alpha_p \cdot t}$ , where,

$$C_{R} = \sqrt{\frac{3}{\pi}} \cdot Ja \qquad ; Ja = \frac{\left[T_{i} - T_{sat}(P_{m})\right] \cdot \rho_{p} \cdot C_{P,p}}{\rho_{v} \cdot h_{fg}} \qquad ; T_{m} = T_{i}$$
  
Using Clausius-Clapeyron: 
$$Ja = \frac{T_{i}(P_{i} - P_{m}) \cdot \rho_{p} \cdot C_{P,p}}{\rho_{v^{2}} \cdot h_{fg}^{2}}$$

Flow inside the expansion chamber

For non-choked flow: 
$$U_i = \sqrt{2 \frac{(P_u - P_i)}{\rho_i}} = \sqrt{2 \frac{(P_u - P_m)}{\rho_i}}$$

At z =0 (inside the expansion chamber):  $U_1 = U_i \left(\frac{A_i}{A}\right) = U_i \left(\frac{d_i}{D}\right)^2$ 

(1) It follows that 
$$U_m = U_i \frac{A_i}{A} \cdot \frac{1}{1 - \varepsilon_A} = \frac{U_1}{1 - \varepsilon_A}$$

Since  $U_I$  is very low (for  $A_i/A \approx 10^{-2}$ ,  $U_I \approx 10^{-1}$  m/s), the velocity at the end of the chamber is low comparing with the velocity at the exit orifice, and hence the acceleration inside the chamber  $(a \approx \frac{\Delta U}{\Delta t} \approx \frac{U_2^2}{L} \approx \frac{10^{-2}}{10^{-2}} \approx 1)$  is

negligible comparing to the accelartion at the discharge orifice  $(a \approx \frac{\Delta U}{\Delta t} \approx \frac{U_2 \frac{A}{A_e}}{z_e - z_2} \approx \frac{10^1}{10^{-3}} \approx 10^4)$ . It is therefore assumed that there is no slip between the data in the second state of the data.

#### Velocity at the exit orifice

For non-choked flow:  $U_{Le} = \sqrt{2 \frac{(P_m - P_e)}{\rho_L}}$ . The mixture accelerates towards the second orifice, therefore

drag should be accounted for, and slip between the two phases is possible.

The vapor's velocity is calculated through the momentum equation:  $-(P_e - P_m) \cdot (A - A_{v,e}) - F_{D,L} = \dot{m}_{L,e} \cdot (U_{L,e} - U_m) \qquad ; U_m << U_{L,e}, \text{ where, } F_{D,L} \text{ is the net drag force}$ acting on the liquid:  $F_{D,L} = \frac{c_D \cdot \rho_L}{2} |U_{L,e} - U_{v,e}| \cdot (U_{L,e} - U_{v,e}) \cdot A_{v,e}$ The slip ratio is therefore:  $k = 1 + \sqrt{\frac{(1 - \varepsilon_A)}{c_D}}$ .

# Pressure Inside the expansion Chamber

The pressure inside the chamber depends both on the inlet and exit conditions.  $P_m$  is estimated from the mass conservation

$$\begin{split} C_{D,i} \cdot A_i \cdot \rho_i \sqrt{2 \frac{(P_u - P_m)}{\rho_i}} &= C_{D,e} \cdot A_e \cdot \sqrt{2 \frac{(P_m - P_e)}{\rho_L}} \left[ \rho_v \cdot \varepsilon_A \cdot k + \rho_L \cdot (1 - \varepsilon_A) \right] \\ \text{and therefore: } P_m &= \frac{P_u + P_e \cdot \left[ \frac{\rho_v}{\rho_L} \varepsilon_A \cdot k + (1 - \varepsilon_A) \right]^2 \left( \frac{A_e C_{D,e}}{A_i C_{D,i}} \right)^2}{1 + \left[ \frac{\rho_v}{\rho_L} \varepsilon_A \cdot k + (1 - \varepsilon_A) \right]^2 \left( \frac{A_e C_{D,e}}{A_i C_{D,i}} \right)^2}, \text{ and } \\ P_u - P_m &= \frac{P_u - P_e}{1 + \left[ \frac{\rho_v}{\rho_L} \varepsilon_A \cdot k + (1 - \varepsilon_A) \right]^2 \left( \frac{A_e C_{D,e}}{A_i C_{D,i}} \right)^2} \\ \text{It follows that } U_i &= \sqrt{\frac{2}{\rho_i} \cdot \frac{(P_u - P_e)}{1 + \left[ \frac{\rho_v}{\rho_L} \varepsilon_A \cdot k + (1 - \varepsilon_A) \right]^2 \left( \frac{A_e C_{D,e}}{A_i C_{D,i}} \right)^2}. \end{split}$$

Optimal chamber's length and void fraction

Expressing the void fraction in terms of the total nuclei number and bubble volume yields:

$$\varepsilon = \frac{\dot{n} \cdot V_{bub}}{\dot{V}} = \frac{\dot{V}_v}{\dot{V}_v + \dot{V}_L} = \frac{\dot{V}_v}{\dot{V}_v + \frac{\dot{m}_i}{\rho_i} \left(1 - \frac{\dot{m}_v}{\dot{m}_i}\right)} \quad \longrightarrow \quad \dot{n} = \frac{\varepsilon}{1 - \varepsilon} \cdot \frac{U_i A_i}{\frac{4}{3} \pi R_b^3},$$

$$\varepsilon_A = \frac{n \cdot A_{bub}}{A} = \frac{\dot{n}}{A} \frac{l_i}{U_i} \cdot \pi \left(2 C_R \sqrt{\alpha_p}\right)^2 \cdot t, \text{ and } U_m = \frac{U_i \frac{A_i}{A}}{1 - \frac{\dot{n}}{A} \frac{l_i}{U_i}} \cdot \pi \left(2 C_R \sqrt{\alpha_p}\right)^2 \cdot t$$
Integrating along the chamber:  $U_m = \frac{dz}{dt}$ , and  $L_{opt} = \frac{U_i \cdot A_i}{n \frac{l_i}{U_i} \pi \left(2 C_R \sqrt{\alpha_p}\right)^2} Ln(1 - \varepsilon_A)$ 

#### Number and mean diameter of droplets

At the end of the chamber, 6 droplets surround each bubble and every 3 bubbles create 1 droplet (Senda et al.

)

[7]): 
$$n_d = 2J_S \cdot (\pi d_i \cdot l_i) \cdot \frac{l_i}{U_i}$$
;  $n_d$  – number of droplets, and  $\frac{V_{Liq,L_m}}{V_{tot,L_m}} = (1-\varepsilon) = 2n \cdot \frac{4}{3}\pi \overline{d_d}^3$ 

Each droplet is most likely to be disintegrated into m additional droplets due to hydrodynamic effects and possible further flashing. The final mean diameter is therfore:  $m \cdot \overline{d_d} = 3\sqrt[3]{\frac{(1-\varepsilon)}{2n}}\frac{3}{4\pi} \cdot V$ 

## **Discussion and conclusions**

A simple 1D model for spray formation by flashing of a given binary mixture has been developed. It is postulated that in a well-designed expansion chamber, a pre-specified void fraction has to be attained at the end of the expansion chamber. The latter is designed to yield this void fraction, subjected to the thermodynamic conditions of the entering mixture and orifices geometries.

For a given mixture properties and pre-specified packaging (from which the void fraction is derived), increasing of bubble nucleation rate, either by altering the inlet orifice surface roughness or superheating degree, results in a decreasing of the maximum bubble radius. Smaller bubbles need shorter residence time, and thus reducing the optimal length for a given cross-section ratio ( $A_i/A=const$ ).

For a given mixture, bubble nucleation rate and cross-section ratio, decreasing of the optimal length will shorten the residence time of the bubbles in the expansion chamber, and thus reducing the maximal bubble radius. Consequently, the void fraction cannot reach its maximal packaging as required. This, in turn, decreases the efficiency of the disintegrating process, thus increases the mean diameter of the spray droplets.

#### References

- Katz-Zeigerson, M. and Sher, E., (1998), "Spray Formation by Flashing of a Binary Mixture: a Parametric 1. Study", Atomization and Sprays, Vol. 8, pp. 255-266
- 2. Katz-Zeigerson, M. and Sher, E., (1996), "Spray Formation by Flashing of a Binary Mixturean Energy Balance Approach", Atomization and Sprays, Vol. 6, pp. 447-459
- Smallwood, G.J and Gulder, O.L., (2000), "Views On The Structure of Transient Diesel Sprays", 3. Atomization and Sprays, Vol. 10, 355-386

- 4. Oza, R.D., (1984), "On Mechanism of Flashing Injection of Initially Subcooled Fuels", J. of Fluid Mech., Vol.106, pp. 105-109
- 5. Sher, E. and Elata, C., (1977), "Spray Formation From Pressure Cans by Flashing", I&EC, Process Design and Development, Vol. 16, pp. 237-242
- 6. Witlox, H.W.M. and Bowen, P.J., (2002), "Flashing Liquid Jets and Two-Phase Dispersion. A Review", Health & Safety Executive, Contract research report 403/2002
- Senda, J., Hojyo, Y. and Fujimoto, H., (1994), "Modeling on Atomization and Vaporization Process in Flash Boiling Spray", JSME Review, Vol. 15, pp. 291-296
- 8. Solomon, A.P., Chen, L-D. and Faeth, G.M., (1982), "Investigation of Spray Characteristics for Flashing Injection of Fuels Containing Dissolved Air and Superheated Fuels", NASA Contractor Report 3563
- 9. Riznic, J.R. and Ishii, M., (1989), "Bubble Number Density and Vapor Generation in Flashing Flow", J. Heat Mass Trans., Vol. 32 (10), pp. 1821-1833
- Alamgir, Md. and Lienhard, J.H., (1981), "Correlation of Pressure Undershoot of Pressure During Hot Water Depressurization", J. Heat Trans., Vol. 103, 52-56
- 11. Deligiannis, P. And Cleaver, J.W., (1990), "The Role of Nucleation in the Initial Phases of a Rapid Depressurization of a Subcooled Liquid", Int. J. Multiphase Flow, Vol.16 (6), 975-984
- 12. Elias, E. and Chambré, P.L., (1993), "Flashing Inception in Water During Rapid Decompression", J. Heat Trans., Vol. 115, 213-238
- 13. He, L. and Ruiz F., (1995), "Effect of Cavitation on Flow and Turbulence in Plain Orifices for High-Speed Atomization", Atomization and Sprays, Vol. 5, pp. 569-584