# LAMELLA DISINTEGRATION AT SHEET FORMING NOZZLES, ESTIMATES FOR DROP SIZES

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## Abstract

Breakup of attenuating lamella is most often initiated by aerodynamic waves. At relatively low Weber numbers instability due to long waves is dominant. At higher Weber numbers short waves occur according to *Senecal et al.* with a wave growth rate independent of the sheet thickness. The authors presented their results as a CPU model for fast moving sheets to predict mean droplet sizes. Here, their equations are transformed into a non dimensional  $\pi$ -number relationship allowing for a parametric study valid in the general case. Drop sizes were measured with PDA and compared with theoretical predictions. Despite of fairly satisfactory correlations between experiment and theory questions about the breakup process still exist.

#### Introduction

Amplitudes of aerodynamic waves show exponential growth along streamlines. The pressure drop in the gas environment at convex contours of the waves increases at stronger deformations like at airfoils. After a certain running length the lamella will break. One can observe fragmentation at sufficient amplitudes and the formation of threads from the fragments due to rim contraction of these fragments. Finally break up of the threads into droplets takes place due to Rayleigh instability.

Amplitude, Wavelength and growth rates of these sinusoidal waves have been examined by a linear stability theory. First results have been reported by Squire [1] for lamella of constant thickness und Dombrowski et al. [2] for attenuating lamella. Viscous lamella have been treated in [3]. Long waves of liquids with negligible viscosity have a wave length of  $\lambda = 4\pi\sigma/\rho_g v^2$ . In this regime it is assumed that two threads are formed from one wave during disruption of the lamella fragments.

In a new analysis reported by Senecal et. al. [4], it is was found that short waves appear at high lamella velocities, have higher growth rates and a higher probability to form droplets. The onset of this regime can be more closely described for non viscous liquids as

$$We^2 \kappa \rho *^{5/2} > 18,2.$$
 (1)

In Eq. (1) the Weber number is defined as  $We = v^2 d\rho/\sigma$ , v is the velocity of the lamella, d is the nozzle diameter. The sheet number is defined for a hyperbolic layer decay as in [5] by  $\kappa = 4\delta x/\pi d^2$  and x is the running length along a streamline,  $\delta$  is the lamella thickness at x,  $\rho * = (\rho_{\alpha}/\rho)$  is the ratio of densities from gas to liquid.

## **Growth Rate of Short Waves**

The dispersion relation in [4] can also be presented in a non dimensional form:

$$\frac{\omega \sigma}{v^3 \rho} = -\frac{2}{W e_k \operatorname{Re}_k} + \frac{1}{W e_k} \sqrt{\frac{4}{\operatorname{Re}_k^2} + \rho^* - \frac{1}{W e_k}}$$
(2)

Defining the following numbers  $Re_k = We_k/Ca$  and  $We_{kg} = We_k (\rho_g/\rho)$ , and by means of the Capillary number  $Ca = v\eta/\sigma$  the growth rate is

$$\Omega = -\frac{2\rho * Ca^2}{We_{kg}^2} + \frac{1}{We_{kg}} \sqrt{\frac{4Ca^2 \rho *^2}{We_{kg}^2} + \rho * -\frac{\rho *}{We_{kg}}}$$
(3)

When  $We_k = \rho v^2 / \sigma k$  is a wave Weber number containing the wave number  $k = 2\pi / \lambda$ . We<sub>k</sub> may be defined either with the liquid  $\rho$  or gas density  $\rho_g$  as We<sub>kg</sub>.

Fig. 1 shows the non dimensional growth rate of short waves depending on the wave Weber number and the capillary number , calculated by Eq. (3).



Fig. 1. Non dimensional growth rate of short waves depending on the wave Weber number, the Capillary number and the density ratio, calc. by Eq.(3). The largest growth rate is found at highest gas densities. The values of  $\rho^*$  are only indicated at the curves for Ca = 0.

The wavelength of short waves for the non-viscous case is given by  $\lambda = 3 \pi \sigma / \rho_g v^2$ . In this regime it does not depend on the sheet thickness  $\delta$ .



Fig. 2. Maximum Growth rate of short waves depending on the density ratio and Capillary number

**Fig. 3.** Gas wave Weber number at maximum growth rates.

The non-dimensional maximum growth rate and the according wave Weber number and their dependence on the density ratio and capillary number can be calculated by an infinitesimal calculation for the general case. Results are given in Figs. 2 and 3. The maximum growth rate decreases with increasing viscosity or capillary number resp. Simultaneously the wave Weber number belonging to this maximum increases, i.e. the wave length becomes larger.

#### **Drop Sizes**

It is assumed that only one ligament is formed per wave in the short wave regime. It is also assumed that only waves with maximum growth rate lead to breakup. Now the ligament diameters and the according drop diameters D can be calculated. In literature, for break up the amplitude term is set to  $ln(A_Z/A_0) = 12$ . Even though the size of the initial disturbance  $A_0$  is unknown this break up criterion is needed for determination of the sheet thickness at the point of break up.

In describing the geometry of attenuating sheets, the sheet number  $\kappa = C_D/[2\pi \varphi \sin(\theta/2)]$  at swirl nozzles is obtained from the discharge coefficient  $C_D$  and the spray cone angle  $\theta$  as in [5]. The sheet number is determined by the geometry of the nozzle and the Reynolds number of the flow. Depending on the spray angle, e.g. at swirl nozzles, the lamella numbers lies in the range  $0,05 < \kappa < 0,2$ , at fan jet nozzles it lies in the range of  $0,3 < \kappa < 0,8$ . Geometrically similar nozzles produce lamella with the same sheet numbers independent on their size.

 $\kappa$  allows for a uniform presentation of the dispersion equations for long and short waves as well as a correlation of the drop sizes with the nozzle diameter. For long waves the well known related drop size for low viscous liquids is

$$D/d = 1.67 \kappa^{1/3} W e^{-1/3} \rho *^{-1/6} .$$
(4)

When short waves lead to break up at higher Weber numbers, more accurately when Eq. (1) holds, and when the viscosity is low, the drop size is given as

$$D/d = 1,03 \kappa^{1/2} \rho *^{1/4}.$$
 (5)

It is interesting to see that from a certain Weber number on the drop size should no longer depend on the sheet velocity or Weber number. The same is true for the pressure dependence of the drop sizes. Theoretical results for short and long wave break up are shown in Fig. 4. According to this plot at a given sheet number and density ratio the drop sizes should decrease first with increasing pressure or sheet velocity but then are predicted to be constant.



**Fig. 4.** Related drop diameters from breakup of attenuating sheets due to long and short aerodynamic waves depending on the nozzle Weber number. At a certain limiting Weber number the drop size does not decrease further with increasing Weber numbers.

The plateau like course of the drop sizes has sometimes also been observed in experiments, e.g. at [6]. Even so the drop size is expected to remain constant from a certain Weber number on, one has to consider other flow regimes, i.e. turbulent breakup at even higher Weber numbers or higher sheet Reynolds numbers. Then the assumed break up regime does not hold further. Further calculations indicated very little influence of the liquid viscosity on the drop size in the short wave regime at  $\rho^* = \rho_g / \rho = 1,2.10^{-3}$ , see e.g. [7].

At typical lamella forming nozzles such as swirl nozzles, the Weber number is not a well known parameter in technical applications. In contrary the pressure, the flow rate and the spray cone angle can be obtained without great difficulties. A comparison between calculated and measured drop sizes therefore is presented by means of the Laplace number  $\Delta p^* = \Delta p d/\sigma$  instead of the Weber number. To perform the calculation with the long and short wave model it is necessary to know the course of the velocity coefficient  $\varphi = v/u$ , and of the sheet number  $\kappa$  on the Reynolds number. This may be taken from measurements at geometrically similar larger nozzles, e.g. from [8] obtained with the e-PIV<sup>1</sup> method. In order to stay with just one characteristic geometrical dimension, the Reynolds number is defined solely by well known parameters, i.e., the nozzle diameter, the pressure and the liquid properties. The flow within the nozzle is known to be practically independent on the gas density and the capillary pressure. Fig. 5. shows the discharge coefficient, the velocity coefficient and the sheet number vs. the pressure Reynolds number at a swirl nozzle. The geometry was optimized according to [9] with a short orifice length of l = 0.2 mm. The nozzle Diameter was d = 1,03 mm, the cone angle of the spin chamber was 60°, there were two tangential inlet slots with a width of 0.5 mm and a height of 0.8 mm.

For a swirl nozzle with the given geometry, the sheet number reaches  $\kappa = 0.07$  at  $Re_p > 50.000$ , below that limit there is a strong dependence on  $Re_p$ . The discharge coefficient was obtained at the 1 mm nozzle within the pressure range of  $3 < \Delta p < 25$  bar with water and glycerol water mixtures. It hardly exceeds  $\varphi = 0.8$  even for high Reynolds numbers,  $Re_p > 50.000$  resp.



**Fig. 5.** Sheet number, velocity coefficient and discharge coefficient at a swirl nozzle with a geometry as described in the text. The Reynolds number is defined with easy accessible parameters.

Based on the given liquid properties the drop sizes were calculated for the two breakup regimes. Measurements of drop sizes in the spray were performed with a TSI PDA. For water with Oh = 0,0038, the droplet data appear to fit more closely to the short wave theory even at low pressures. In the case of a water glycerol mixture with  $\mu = 3,48$  m Pas and Oh = 0,0136 the droplet data obtained with the PDA indicate a satisfactory fit to the calculated long wave results as predicted by the theory.

Besides turbulence there are other remarkable theories, explaining waves on the sheet e.g. as a propagation of oscillations generated at the air core as gravity or centrifugal waves [10]. Further experimental examinations of the wave pattern on the sheets are needed to give a better insight into the break up process.

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**Fig. 6.** Related drop sizes at a swirl nozzle with d = 1,03 mm orifice diameter. For water, Oh = 0,0038, drop sizes are more accurately described by the short wave computations even in the case of low pressures. For a water glycerol mixture, Oh = 0,0136, within the low pressure range, the drop sizes are predicted better by the long wave theory as expected.

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## Nomenclature

d	nozzle diameter	$\Delta p^* = \Delta p d / \sigma$	Laplace number
D	mean drop diameter	$\kappa = C_D / [2\pi \varphi \sin(\theta/2)]$	sheet number
$k = 2\pi/\lambda$	wave number	$Ca = v\mu/\sigma$	capillary number
V	volumetric flow rate	$C_{\rm D} = 4V/\pi d^2 u$	discharge coefficient
v	sheet velocity	$We_k = v^2 \rho / k \sigma$	wave Weber number
$u = \left(2\Delta p/\rho\right)^{1/2}$	Bernoulli velocity	$We_{kg} = v^2 \rho_{\Box} / k\sigma$	gas wave Weber number
x	running length	$Re_p = d(\Delta p\rho)^{1/2}/\mu$	pressure Reynolds number
$\varphi = \tilde{v}/u$	velocity coefficient		
δ	sheet thickness	$\Omega = \omega \sigma / v^3 \rho_g$	non dim. growth rate (1)
λ	wave length	$\Omega * = (\omega \sigma / v^3 \rho_g) / \rho^{*^{1/2}}$	non dim. growth rate (2)
ρ	liquid density	$Oh = \mu / (\rho d\sigma)^{1/2}$	nozzle Ohnesorge number
$ ho_{g}$	gas density	$We = v^2 d\rho/\sigma$	nozzle Weber number
σ	surface tension	$\rho^* = \rho_g / \rho$	density ratio
μ	liquid viscosity	-	
$\theta$	spray cone angle		
ω	growth rate of amplitudes		

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