DEVELOPMENT OF A SPRAY WALL IMPACTION MODEL WITHOUT DISCRETISATION INTO DROPLET SIZE CLASSES

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Abstract
The Eulerian-Eulerian approach proposed by Beck [1] is used here to study the phenomena of a spray impinging on a wall. The basic fluid dynamics of an impinging spray can be simulated with this approach once the appropriate wall boundary conditions have been imposed. However, near the wall, the SMD and the velocity component normal to the wall are not simulated appropriately. Hence a compatible wall impaction model has been developed to take into account the different outcomes that a droplet can undergo after impaction on a wall (deposition, rebound and splash), and to correct the size and velocity distributions in the near wall region. This paper considers the theory of the spray wall impaction model and presents some preliminary results.

Introduction

The method of modelling sprays using the moments of the droplet number size distribution was first introduced by Beck and Watkins [2]. This method does not require the discretisation of droplets into size classes to capture the polydisperse nature of a spray flow. Instead, it solves both the liquid and the gaseous phases in an Eulerian manner [3]. In this way, the liquid phase is considered as a coherent whole and its properties are written in terms of the first four moments of its number size distribution function. The third and fourth moments are calculated using transport equations, and the other two are approximated from a presumed distribution function. Because the size distribution is different along the spray, the size distribution function is truncated in order to fit the SMR obtained with the transport equations of the third and fourth moments.

This model is capable of predicting a basic wall spray, once the appropriate wall boundary conditions have been imposed. However, this does not take into account important effects that occur in a real spray, namely the deposition, rebound or break-up of droplets and the associated effects on the drop velocities. The modelling scheme of Beck and Watkins gives much information about the size distribution of the droplets in each computational cell. Hence it is possible to build a statistical wall impaction model capable of predicting the amount of liquid, within each near wall computational cell, that is involved in each one of the impaction regimes (i.e. deposition, rebound and splash). The effects on the number size distribution moments of the liquid can then be calculated, along with the associated velocity components. Thus much information about the spray after impaction is available, at least matching that obtained by Lagrangian spray wall impaction models used in DDM schemes.

Mathematical Model

The present impaction model uses the droplet number size distribution proposed by Beck [3] to predict the amount of liquid within a computational cell that lies in each of the impaction regimes. The distribution proposed by Beck is:

$$n(r) = \frac{16r}{r_{32}^2} e^{\frac{-4r}{r_{32}}} \tag{1}$$

where r is the drop radius, and r_{32} is the local Sauter Mean Radius (SMR) of the spray. This distribution may be truncated at either end as discussed above.

It is necessary to choose a dimensionless parameter, a function of the size and velocity of the impinging droplets, which will provide a criterion for the outcome of an impinging droplet. Mundo et. al. [4] proposed the following number:

$$k = Oh \cdot Re^{1.25} = \left(\frac{\rho^3 d^3 u^5}{\sigma^2 \mu}\right)^{0.25} \tag{2}$$

This number is very convenient because it has successfully predicted the splashing limit and also there is some information about the outgoing droplet size distribution based on the k number of the impinging droplet. It is important to note that, depending on the type of experimental data available, the model could be built using other dimensionless numbers like the Weber and Laplace numbers.

In this scheme, all properties of the liquid (density, viscosity, surface tension and velocity) within a computational cell are the same for all the droplets. Hence, from equations (1) and (2), it is possible to produce a '*k density distribution*', *g(k)*:

$$g(k) = Ak^{5/3}e^{-Bk^{4/3}} \tag{3}$$

where

$$A = \frac{16}{3r_{32}^2\rho^2}\left(\frac{\sigma^2\mu}{u^5}\right)^{\frac{2}{3}} \text{ and } B = \frac{2}{r_{32}\rho}\left(\frac{\sigma^2\mu}{u^5}\right)^{\frac{1}{3}} \tag{4}$$

This is done by writing r as a function of k (and the rest of the parameters) in equation (2) and making a change of variable in equation (1) from r to k.

Having this distribution one can calculate the fraction of droplets, wf_j that are included in each of the impinging regimes as follows:

Deposition $$wf_D = \int_0^{k_D} g(k)dk \tag{5}$$

Rebound $$wf_R = \int_{k_D}^{k_S} g(k)dk \tag{6}$$

Splash $$wf_S = 1 - wf_D - wf_R \tag{7}$$

The integration limits k_D and k_S refer to liquid characteristic transition numbers of deposition and splash respectively. Mundo et. al. [4] report that the splashing limit is k_S=57.7. But there is no information about the deposition limit, hence, in the present work, it has been assumed as k_D=15.

At the present stage of the modelling, the deposited volume fraction is being 'removed' from the total volume of liquid in the cell. In the future it will be used to predict a wet film. This amount of liquid is represented in the moment equations by the sink terms SQ_{iD}:

$$SQ_{iD} = wf_D Q_i \tag{8}$$

where the subscript i denotes the moment number (i = 0, 1, 2, 3), and the Q_i are the number size distribution moments.

For the rebound fraction all moments of the spray remain constant, since there are no changes to the drop sizes; only the velocity component of the liquid normal to the wall in the cell alters. Lee and Ryou [5] successfully predicted this velocity with the correlations proposed by Matsumoto and Saito [6] and using the coefficient of restitution, e, reported by Bai and Gosman [7]:

$$u_a = -e \cdot v \tag{9}$$

$$e = 0.993 - 1.76\theta + 1.5\theta^2 - 0.49\theta^3 \tag{10}$$

Here u_a is the velocity component normal to the wall *after* the impact and v is the relative velocity between the fourth moment or volume averaged velocity of the impinging droplet and the wall velocity; it is defined as $v=U_3-U_{wall}$. Also θ is the angle of impaction formed between the wall and the incoming velocity vector.

Both outgoing moment-averaged velocities required in the model (i.e. U_3 and U_2) are considered to be the same and equal to u_a. The liquid velocity components parallel to the wall are calculated by the liquid phase continuity equations once the appropriate wall boundary conditions have been set.

The splashing regime is modelled through source terms for each moment Q_i. In fact, only the source terms for Q_0, Q_1 and Q_2 have to be calculated since the volume of liquid, represented by Q_3, is already known through equation (7) after the splash.

In order to calculate the source terms SQ_{iS} a droplet size distribution of the outgoing droplets is necessary. Mundo et. al. [4] report a set of data for a dimensionless droplet size distribution of secondary droplets for

different k numbers, for both smooth and rough surfaces. The present model has been developed using the dataset obtained with a smooth surface.

This experimental data was fitted into a function $m(k,r^*)$ to generate a surface that interpolates between the lines reported by the experiments. The function is as follows:

$$m(k,r^*) = \frac{r^*}{a} e^{-\left(\frac{r^*}{b}\right)^{\alpha}} \tag{11}$$

where

$$a = 1.96602\times10^{-4}k^2 - 9.1349\times10^{-2}k + 10.419257$$

$$b = -5.3313\times10^{-5}k^2 + 1.53062\times10^{-2}k - 0.936192$$

$$\alpha = -1.05522\times10^{-3}k^2 + 0.34599k - 26.130703$$

$$r^* = \frac{r_{out}}{r_{in}} \tag{12}$$

This function is illustrated in Figure 1 and comparisons between it and the experimental data are shown in Figure 2. Clearly the fit is very good except for the data for k = 162.

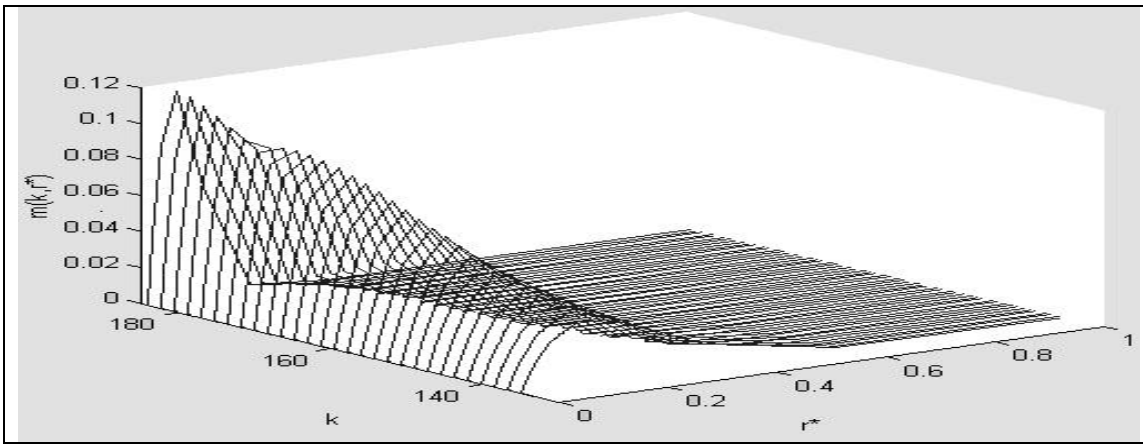

Figure 1. Surface of the size density function of secondary droplets $m(k,r^*)$.

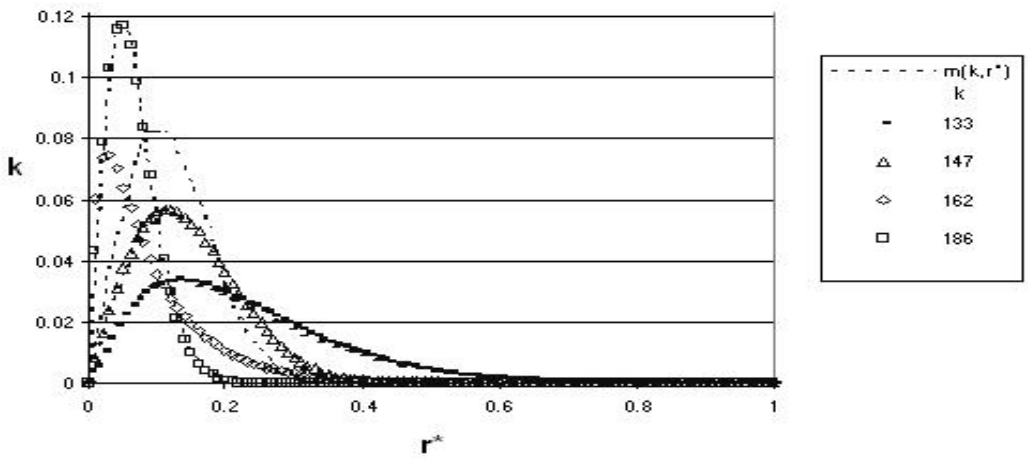

Figure 2. Comparisons between $m(k,r^*)$ and the experimental data of Mundo et al [4].

To obtain the moments of the secondary droplets one needs an expression involving the density function $m(k,r^*)$, which would give a number distribution. This expression, $d(k,v)$, is a function of k and the relative velocity of the liquid phase in the computational cell and the wall. The deduction of this expression is based on the fact that the volume of splashing liquid is equal to the volume of liquid after the splashing takes place. Suppose that one has a incoming splashing droplet with $k=k_l$, $v=v_l$ and $r=r_{in}$, then it can be shown that,

$$d(k_l, v_l) r_{in}^4 f_3^* = r_{in}^3 \tag{13}$$

where

$$f_i^* = \int_0^{r_{in}} \frac{r}{a r_{in}} e^{-\left(\frac{r}{b r_{in}}\right)^{\alpha}} \left(\frac{r}{r_{in}}\right)^i \frac{dr}{r_{in}} \tag{14}$$

From equations (13) and (14) one can get,

$$d(k_l, v_l) = \frac{1}{r_{in} \int_0^1 m(k_l, r^*) r^{*3} dr^*} \tag{15}$$

For given values of k and v the value of r_{in} for each drop can be calculated from equation (2). This is then inserted into equation (15). Values of $d(k,v)$ can then be obtained. These are shown in Figure 3 for different pairs of k and v values. The equation obtained to fit this data is:

$$d(k, v) = (0.5157v^2 + 37.42v + 870.56)k^{3.32} \tag{16}$$

and this is also shown in Figure 3.

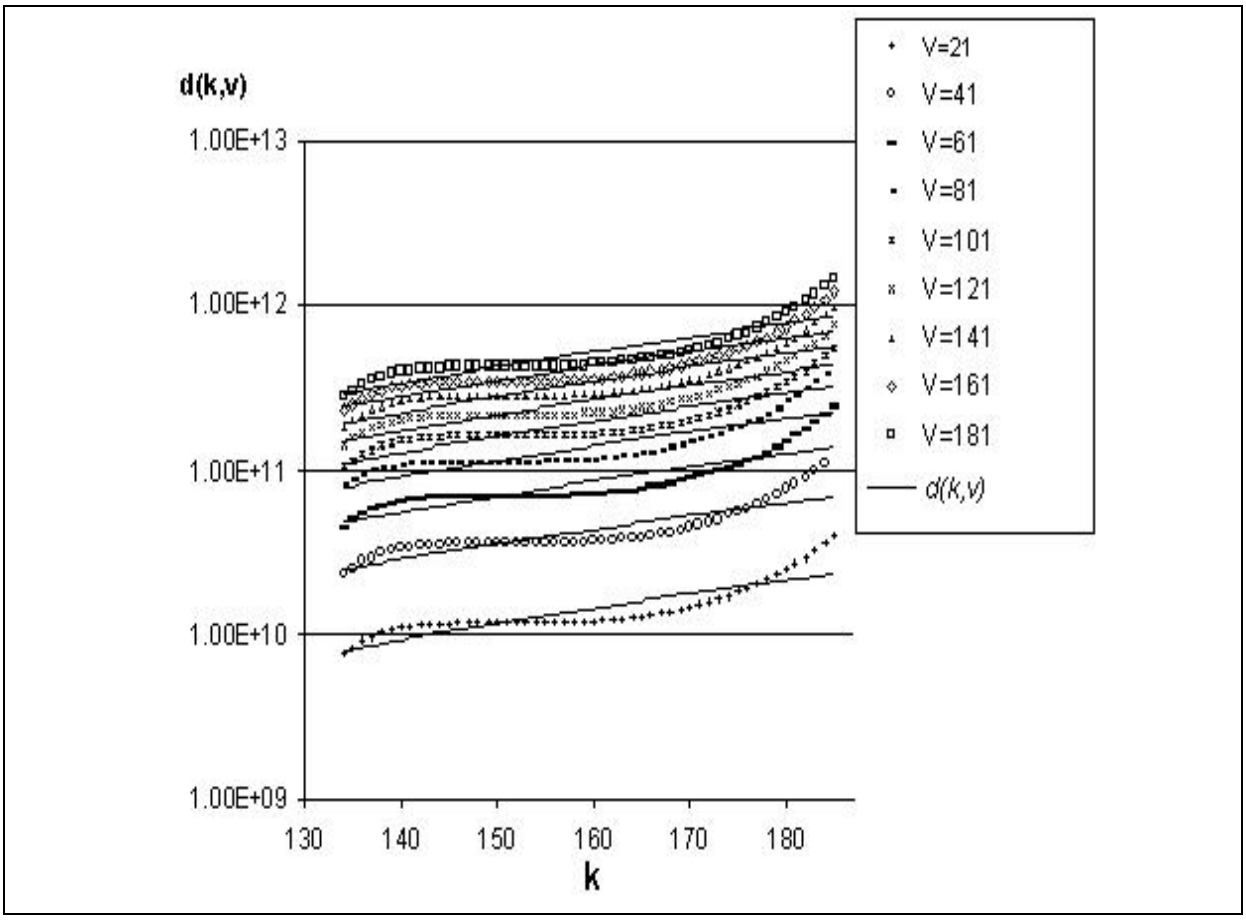

Figure 3. Comparisons between the calculation of $d(k,v)$ from equations (15) and (16).

The general form to calculate the source term for the ith dimensionless moment of the secondary droplets is:

$$SQ_{iS}^{*} = \int_{k_S}^{\infty}\int_{0}^{1} d(k,v)g(k)m(k,r^{*})r^{*i}dr^{*}dk \tag{17}$$

From dimensional considerations the dimensional source term is:

$$SQ_{iS} = r_{in}^{i-1} SQ_{iS}^{*} \tag{18}$$

The final moments within a computational cell are calculated as:

$$Q_i = SQ_{iS} + Q_i^{old}(wf_R) \qquad \text{i = (0, 1, 2)}$$
$$Q_3 = Q_3^{old} - SQ_{3D} \tag{19}$$

According to the experimental data of Mundo et. al. [4] the mean velocity of the secondary droplets is approximately -0.25 of the incoming velocity. Hence, the present model calculates the velocity of the secondary droplets as follows:

$$u_{3S} = u_{2S} = -\frac{u_3}{4} \tag{20}$$

The final moment-averaged velocities of the droplets within a computational cell are calculated as a weighted average velocity between the rebound and the splashing velocities.

$$u_2 = u_3 = Cu_a + (1-C)u_{3S} \tag{21}$$

where

$$C = \frac{wf_R}{wf_R + wf_S} \tag{22}$$

Results

Figures 4(a), 4(b) and 5 show how the model works. The input is a set of data that varies in order to obtain the different possible outcomes in an impinging spray. However, it should be noted that the functional forms for a, b and α given in equation (12) are not appropriate for values of k outside the range of data used to develop these equations (k = 133 to 186), because the values of b and α become negative. For values of k outside this range the values of $m(k,r^{*})$ are set to the appropriate end-of-range value.

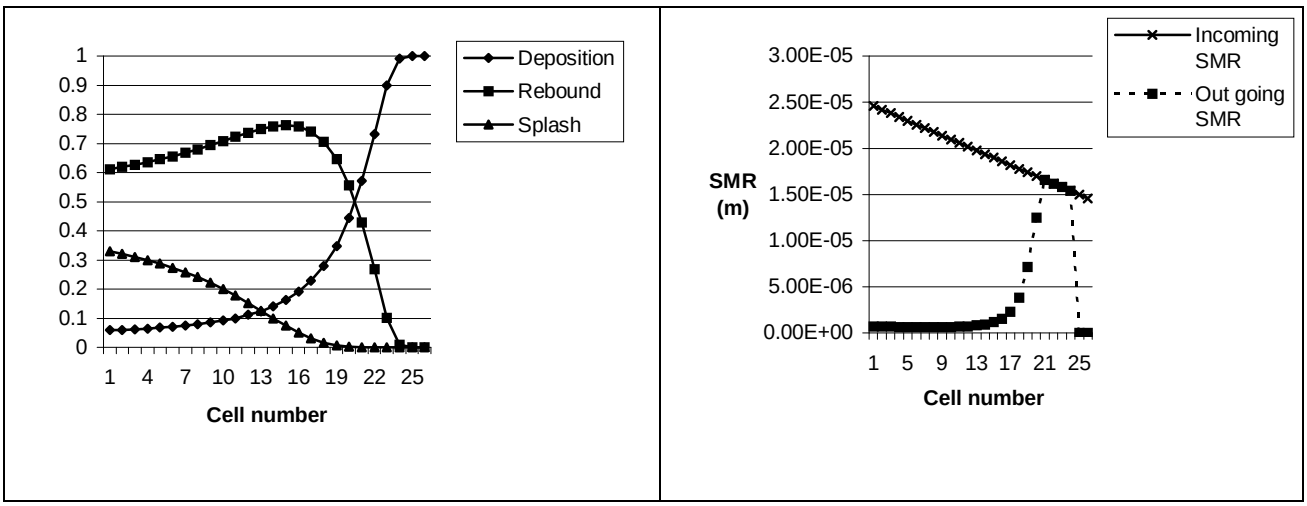

Figure4. (a) Fraction of droplets undergoing deposition, bouncing or splash. (b) SMR of the liquid phase.

It can be seen that near the axis of symmetry (denoted by low values of cell number) mainly splashing and rebound takes place and the SMR decreases substantially. Further from the axis, mainly rebound and deposition take place and the outgoing SMR starts to increase in value. When no splashing occurs, the outgoing SMR matches the incoming one because no break-up occurs. Even further away from the axis only deposition takes place and the SMR drops down to zero because there are no longer any droplets in the cell.

The velocity outcome shows a trend of smaller outgoing velocity with smaller incoming velocity, as would be expected. A fairly flat trend can be observed in the region where the splashing regime is more important. As the number of splashing droplets diminishes the outgoing velocity tends to zero more rapidly.

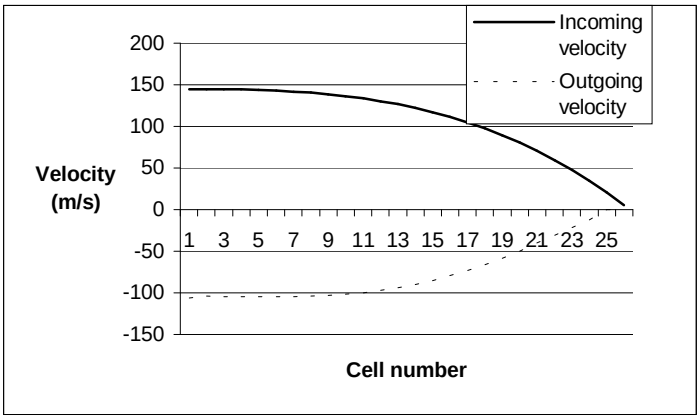

Figure 5. Volume-averaged velocity of the liquid phase versus distance from the axis of symmetry.

Conclusions

The new spray wall impingement model presented here is capable of predicting a new size distribution of the spray in the near wall region. At the present stage, because $m(k,r^*)$ is a Poisson distribution of variable exponent, α, the integration is being done numerically. This makes the model computational expensive; hence further work will be focused on obtaining a function that can be integrated analytically but that is still capable of delivering good results. Nevertheless, the model has proved to be efficient in predicting the different fractions of liquid volume coming from each impingement regime, and the SMRs and normal velocities obtained after impingement showed trends that appear sensible.

It is also necessary to obtain more statistical data about the size distribution of secondary droplets to make $m(k,r^*)$ a more robust function capable of being widely used. Another assumption of the present work is the value used for k_D (15.0), which is currently just guessed in order to test the performance of the model.

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5 **Lee, S.H. and Ryou, H.S.**, *'Development of a new spray/wall interaction model'*, Int. J. Multiphase Flow 26, 1209-1234, 2000.

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