FLOW ON A WALL SURFACE DUE TO SPRAY IMPACT

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Abstract
A spray impacting onto a wall creates on it a moving, oscillating liquid film. In the present paper the parameters of the single drop impact and those of the impinging spray influencing the dynamics of this film are determined. Next, the statistically averaged fluctuations in the motion of the film at short times after drop impact are considered, such that the dynamic pressure applied by the spray can no longer be considered as continuous. This motion causes the formation of finger-like jets, as observed in experiments of a diesel spray impacting onto a rigid wall.

Introduction

The usual approach to the modelling of spray impact treats the phenomenon as a simple superposition of single drop impact events [1]. Such models result from either experimental [2, 3] or theoretical [4] studies of the impact of a single drop onto a dry wall, onto a uniform, undisturbed liquid film or into a deep pool [5]. One important result of these studies is the splashing threshold in terms of the impact parameters, which may differ, depending on the mode of splash. One empirical condition found for a splash after the crown-like uprising of jets is when the thickness of the film and the drop diameter are of the same order of magnitude [6]: $We^{4/5}Re^{2/5} \geq 2800$, whereas the condition for a drop ejected from the central jet formed after the crater created by the impacted drop recedes is [5]: $We \geq 48.3\ Fr^{0.247}$, where $We = \rho DU^2/\sigma$ is the Weber number, $Re = DU/\nu$ is the Reynolds number and $Fr = U^2/(gD)$ is the Froude number; ρ, σ, ν being the density, surface tension and the kinematic viscosity of the liquid, D and U being the drop initial diameter and the impact velocity.

However, the spray impact phenomenon is much more complicated. In [6] it was shown that this conventional approach is not universal in the description of the spray impact and that in the case of relatively dense sprays, the interaction of crowns and the oscillations of the liquid-wall film must be taken into account. In Fig. 1 the impact of a diesel spray onto an inclined, cylindrical target is shown. In this picture the spray impact results in the creation of numerous uprising finger-like jets on the wall. In only one of 300 such pictures is a crown present. Measurements performed using the phase Doppler technique indicate that secondary droplets are ejected from the wall. The average drop diameter in the impinging spray was $D \sim 10\ \mu$m, the impact velocity $U \sim 40$ m/s, and the volume flux density approximately $\dot{q} \sim 0.1$ m/s. These parameters are far below either of the above-mentioned splash conditions. Therefore, the motion of the film and its oscillations appear to influence significantly the drop impact process and apparently can enhance the splash.

The main subject of the present work is the theoretical description of the behavior of the liquid film created on a wall by a spray at very short times after impact of a single drop and the creation of finger-like jets.

Single drop impact

Consider first the normal impact of a single drop with the impact velocity U and the initial diameter D, onto a thin liquid stationary film of the thickness h_f. If the impact velocity is relatively high, the impact produces a crown-like jet. Following the theory of [4], the base of the crown is of radius $R_{cr}(t)$, which is described as the propagation of a kinematic instability in the film, dividing it into two parts: an outer undisturbed film and an inner part of thickness $h(r,t)$, consisting of radially moving liquid with the velocity $\boldsymbol{V}(r,t)$.

Let us introduce two parameters of the single drop impact important for the characterization of a polydisperse spray impact:

$$S = \int_0^{T_m} \pi R_{cr}^2(t)\,\mathrm{d}t, \tag{1}$$

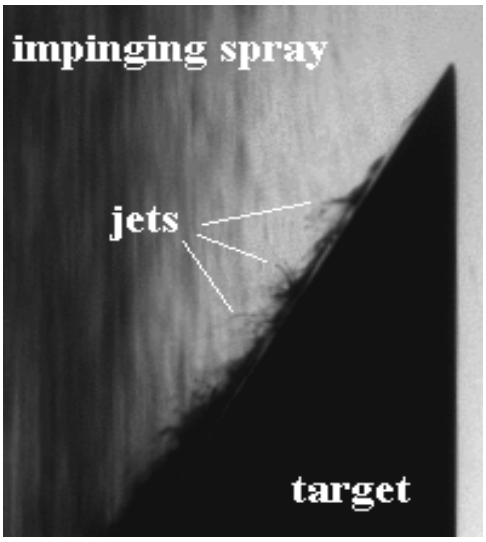

Figure 1. Image of the diesel spray impacting onto an inclined target. The uprising finger-like jets can be clearly seen.

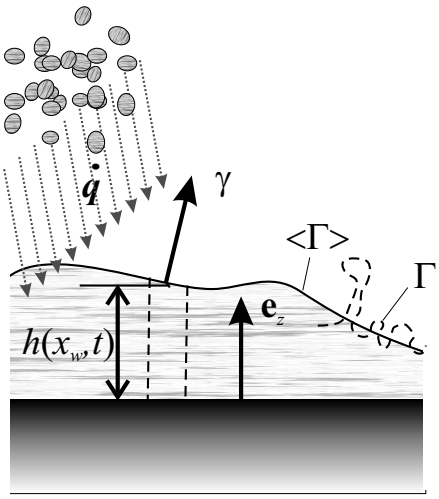

Figure 2. Schematic representation of the film created on the wall.

$$\boldsymbol{Y} = \rho \int_0^{T_m} \int_0^{R_{cr}} \int_0^{2\pi} r\, h\, (\boldsymbol{V} \otimes \boldsymbol{V})\, \mathrm{d}\theta \mathrm{d}r \mathrm{d}t, \tag{2}$$

where symbol $\otimes$ denotes the usual tensor product. The first parameter is associated with the presence of the crown in a space/time volume and the second one with the axial momentum of the flow in the film produced by the drop impact. The upper bound in the time integration, T_m, is the maximum time of the crown propagation corresponding to the instant when the free rim bounding the crown falls down onto the film. This time is usually much larger than the characteristic time of drop impact.

The integrals in expressions (1) and (2) can be evaluated using the remote asymptotic solution of [4]. Approximating the results for $T_m \gg D/U$ yields the expressions for the parameter $S(D, U, h_f)$ and the tensor $\boldsymbol{Y}(D, U, h_f)$ in the following form

$$S = \pi D^{3/2} U\, T_m^2\, h_f^{-1}/6, \qquad \boldsymbol{Y} = Y\, \boldsymbol{I}, \tag{3}$$

where

$$Y = \pi \rho\, D^4\, U/36\, \left[1 + \sqrt{2/3}\, D^{1/2}\, h_f^{-1/2}\right], \tag{4}$$

and $\boldsymbol{I}$ is the two dimensional identity tensor in the plane parallel to the wall.

Characterization of a spray

Consider first the spray as continuous in space and denote with $c_n(\boldsymbol{x}, t)$ the number concentration of the spray at point $\boldsymbol{x}$, and $f(\boldsymbol{x}, t, \boldsymbol{v}, D)$ as the local probability density function. Let us introduce for convenience a functional $\Im$ defined as

$$\Im\{a\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} a(\boldsymbol{x}, t, \boldsymbol{v}, D)\, \mathrm{d}D \mathrm{d}v_x \mathrm{d}v_y \mathrm{d}v_z, \tag{5}$$

where $a(\boldsymbol{x}, t, \boldsymbol{v}, D)$ is an arbitrary function. Here D is the drop diameter (for simplicity the drops in the spray are assumed spherical) and $\boldsymbol{v}$ is the drop velocity vector with the components v_x, v_y and v_z. The normalization condition of the probability density function f can be thus written in the form $\Im\{f\} = 1$.

These two functions, c_n and f, completely determine the propagation of the liquid phase of the spray. For example, the local number flux density vector, $\dot{\boldsymbol{n}}$, and the local volume flux density vector, $\dot{\boldsymbol{q}}$ are determined as

$$\dot{\boldsymbol{n}}(\boldsymbol{x}, t) = c_n\, \Im\{\boldsymbol{v}\, f\}, \tag{6}$$

$$\dot{\boldsymbol{q}}(\boldsymbol{x}, t) = \pi\, c_n/6\, \Im\left\{\boldsymbol{v}\, D^3\, f\right\}. \tag{7}$$

Next, determine also the local momentum flux density tensor as

$$\dot{\boldsymbol{P}}(\boldsymbol{x},t) = \rho\pi\ c_n/6\ \Im\left\{\boldsymbol{v}\otimes\boldsymbol{v}\ D^3\ f\right\}. \tag{8}$$

These quantities, characterizing the spray transport, will be used here for the description of the hydrodynamics of the film created by the spray on the wall.

Time averaged motion of a liquid film

Consider a liquid film formed due to the impact of a spray onto a rigid wall (Fig. 2). Denote $\boldsymbol{e}_z$ as the unit vector normal to the wall and $\boldsymbol{x}_w$ as the radius vector of a point at the wall. Define also the gradient operator $\boldsymbol{\nabla} \equiv \partial/\partial\boldsymbol{x}_w$ in the plane parallel to the wall.

The liquid film is bounded on the one side by the wall and from the other side by the free surface Γ exposed to the spray impact. The boundary Γ includes the jets, capillary waves, crowns and craters produced by drop impacts. We consider here the time averaged film surface $\langle\Gamma\rangle$ bounding a time averaged film of the thickness $h(\boldsymbol{x}_w,t)$. The local unit vector normal to $\langle\Gamma\rangle$ is designated $\boldsymbol{\gamma} = \boldsymbol{\gamma}(\boldsymbol{x}_w,t)$.

One of the aims of this contribution is to indicate the influence of the parameters of the single drop impacts on the motion of the film. This influence is described here using the simplest example: normal impact of a spray onto a plane wall. Normal impact of the spray means that the values of $\dot{\boldsymbol{q}}\cdot\boldsymbol{\nabla}h$ and $(\dot{\boldsymbol{P}}\cdot\boldsymbol{e}_z)\times\boldsymbol{e}_z$ vanish at $\boldsymbol{x}\in\langle\Gamma\rangle$.

Next, we assume $|\boldsymbol{\nabla}h| \ll 1$, such that the quasi-two-dimensional description of the film can be applied. The depth-averaged velocity component of the liquid, $\boldsymbol{u}$, parallel to the wall, can be represented in the form $\boldsymbol{u} = \langle\boldsymbol{u}\rangle + \boldsymbol{u}'$, where $\langle\boldsymbol{u}\rangle$ is the time averaged part of the velocity and $\boldsymbol{u}'$ is its oscillating part.

Assuming that the rate of change of the film thickness, $\partial h/\partial t$, is much smaller than the characteristic velocity of the drops in the spray, the continuity of the film can then be written as

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\langle\boldsymbol{u}\rangle) + \dot{\boldsymbol{q}}\cdot\boldsymbol{e}_z = 0. \tag{9}$$

Then, consider for simplicity the spray of a low viscous, low surface tension liquid impacting onto the wall with a high velocity, such as the diesel spray shown in Fig. 1. We neglect the viscous and the surface tension effects, neglect the gradient of the forces produced by the gas phase, and write the momentum equation representing the balance of the inertial forces in the film in the plane parallel to the wall:

$$\frac{\partial(h\boldsymbol{u})}{\partial t} + \nabla\cdot(h\ \langle\boldsymbol{u}\rangle\otimes\langle\boldsymbol{u}\rangle) + \nabla\cdot h\langle\boldsymbol{u}'\otimes\boldsymbol{u}'\rangle = -\frac{h}{\rho}\,\boldsymbol{\nabla}p \tag{10}$$

The right-hand-side of (10) is the force associated with the the distribution of the dynamic pressure p averaged through the film thickness, which is given in the form

$$p = -\boldsymbol{e}_z\cdot(\dot{\boldsymbol{P}}\cdot\boldsymbol{e}_z), \qquad \text{at } \boldsymbol{x}\in\langle\Gamma\rangle, \tag{11}$$

where the momentum flux density tensor $\dot{\boldsymbol{P}}$ of the impacting spray is defined in Eq. (8).

The first term in the left-hand-side of (10) is the inertia of the liquid in the film associated with the acceleration and the second term is the inertial term associated with the time averaged velocity.

The third term is the divergence of the momentum associated with the oscillating part of the flow in the film. It is estimated with the help of (3) and (4) as the sum of the momentum tensors of impacting drops:

$$\rho\,\nabla\cdot h\langle\boldsymbol{u}'\otimes\boldsymbol{u}'\rangle = \boldsymbol{\nabla}(C_s\ c_n\ \Im\{-Y\ \boldsymbol{v}\cdot\boldsymbol{e}_z\ f\}), \tag{12}$$

at $\boldsymbol{x}\in\langle\Gamma\rangle$, where C_s is a correction factor accounting for the probability of intersection of the crowns on the film surface. These crowns in the absence of the interactions should be the circles of the radii R_{cr}. The specific cumulative area of these circles can be estimated as $A_{cr} = c_n\Im\{-S\ \boldsymbol{v}\cdot\boldsymbol{e}_z\ f\}$, at $\boldsymbol{x}\in\langle\Gamma\rangle$, (with S being defined in (1)). Assuming the random distribution of these circles in space and time, the probability that given point at the film surface does not belong to any crown can be determined with the help of the Poisson's distribution [7] as $e^{-A_{cr}}$. Therefore, the correction factor C_s, which is the ratio of the specific area occupied by crowns to the specific cumulative area A_{cr} of the circles, is given as

$$C_s = (1 - e^{-A_{cr}})/A_{cr}. \tag{13}$$

The value of C_s is near unity only for very dilute sprays.

Note, also, that the ratio of the term (12), associated with the fluctuations of the film velocity, to the right-hand-side of the equation (10), the term associated with the dynamic pressure, is estimated with the help of Eq. (4) to be of order of D/h. Here D is the average drop diameter. Therefore, the momentum of the radial flow produced in the film by the single drop impacts can be neglected in equation (10) only when $D \ll h$, which is not always the case.

Short time analysis

In the approach considering the spray as a continuous media, the minimal characteristic length scale, l_{cont}, should be much larger than $c_n^{-1/3}$. However the number concentration can not be used to describe the length scale of spray impact. Consider an element of area, dA, of the wall surface . The dynamic pressure produced by the spray can be considered as continuous if the observation time is much larger than $t_{cont} = dA^{-1}\dot{n}_z^{-1}$, where $\dot{n}_z$ is the number flux density of the impacting drops. During the times much shorter than the characteristic time t_{cont} the dynamic pressure can no longer be considered as a continuous function but as an ensemble of single drop impacts.

We describe the fluctuations of the dynamic pressure considering the most singular event of the spray/wall interaction - the impact of a single drop D_0: – an arbitrary one among drops of the spray impacting normal to the wall. The average number of drops (including drop D_0) impacting onto a circle of radius r during the time interval t after impact is $\lambda = \pi\, r^2\, t\, \dot{n}_z$, where the constant number flux density into the film is defined as $\dot{n}_z = -\dot{\boldsymbol{n}} \cdot \boldsymbol{e}_z$. What is the distribution of drops around our drop D_0? Intuitively, when we will remove one drop from the uniformly distributed drops, we will obtain a "hole" in the neighborhood of the location of this removed drop. It is obvious that if $\lambda \gg 1$ the average number of drops around the drop D_0 is $\lambda_1 \approx \lambda - 1$. Below is the exact analysis of the drop distribution around the drop D_0 for any λ.

Assuming that the drops of the spray are distributed randomly in space and in time, the probability $\mathcal{P}(k, \lambda)$ that exactly k drops impact onto the considered circle can be described by the Poisson distribution [7]:

$$\mathcal{P}(k, \lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{14}$$

In order to determine the distribution $\mathcal{P}_1$ of drops around the given drop D_0, the case $k = 0$ with the probability $\mathcal{P}(0, \lambda) = e^{-\lambda}$ should be excluded from the set of possible events. The minimum possible k number is 1 (because the drop D_0 has already impacted onto the considered area). Thus, the distribution $\mathcal{P}_1$ can be given with the help of (14) by

$$\mathcal{P}_1(k_1, \lambda) = \frac{e^{-\lambda}\,\lambda^{k_1+1}}{(1 - e^{-\lambda})(k_1 + 1)!}, \tag{15}$$

where $k_1 = k - 1$ is the number of drops impacted into the circle except the drop D_0. The term $(1 - e^{-\lambda})$, which is the probability that at least one drop including D_0 will impact onto the circle, appears in the denominator of the expression for $\mathcal{P}_1$ as a normalization parameter.

The average number of drops impacting onto the circle around the given point D_0 is

$$\lambda_1(r, t) = \sum_{k_1=0}^{\infty} k_1\, \mathcal{P}_1 = \lambda\, e^{\lambda}\xi^{-1} - 1, \tag{16}$$

where $\xi = e^{\lambda} - 1$. This number depends on the radius of the considered area around the drop D_0 and the time.

From the other hand, the number of drops λ_1 can be determined using the statistically averaged number flux density $\dot{n}_{1z}(r, t)$ of the drops around the drop D_0:

$$\lambda_1 = \int_0^t \int_0^r 2\pi r \dot{n}_{1z}(r, t) \mathrm{d}r \mathrm{d}t \tag{17}$$

The expressions (16) and (17) are used to determine $\dot{n}_{1z}(r, t)$ in the form $\dot{n}_{1z}(r, t) = G(r, t)\, \dot{n}_z$ where

$$G(r, t) = \frac{1}{2\pi\, r\, \dot{n}_z} \frac{\partial^2 \lambda_1}{\partial t \partial r} = e^{\lambda} \left[\xi^{-1} - 3\lambda\xi^{-2} + (2 + \xi)\lambda^2\xi^{-3}\right]. \tag{18}$$

The function G expresses the ratio of the statistically averaged number flux $\dot{n}_{1z}$ of drops around the given drop D_0 to the number constant flux $\dot{n}_z$. This function approaches unity in the limit $\lambda \to \infty$. This means that the average distribution of the drops far from D_0 is not influenced by the drop impact. At small λ, in the neighborhood of the drop D_0, the function G is not longer unity and can be approximated as

$$G \approx 1/2 + \lambda/3. \tag{19}$$

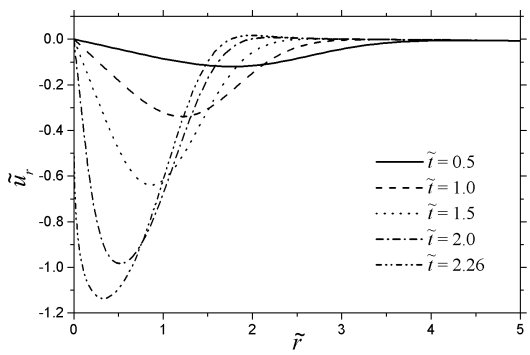

Figure 3. The dimensionless, statistically averaged radial velocity $\tilde{u}_r$ in the film.

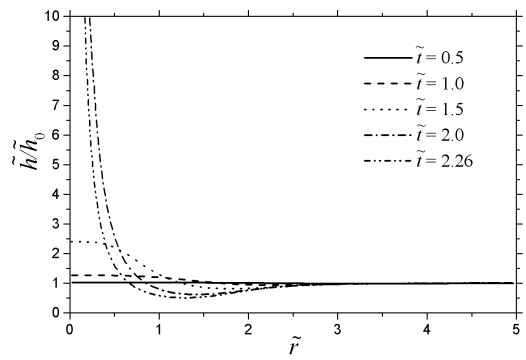

Figure 4. The dimensionless, statistically averaged thickness $\tilde{h}$ of the film.

Variation of G in the radial direction means that the statistically averaged dynamic pressure p applied by surrounding drops also varies, causing flow fluctuations in the film. This statistically averaged flow will be analyzed below.

Using the length scale Λ and the time scale T in the form

$$\Lambda = \left[\frac{p}{\dot{n}_z^2\,\pi^2\rho}\right]^{1/6}, \qquad T = \left[\frac{\rho}{p\,\dot{n}_z\pi}\right]^{1/3}, \tag{20}$$

the equation for the statistically averaged mass balance and the momentum in the radial direction can be obtained using the equations (9) and (10) in dimensionless form

$$\frac{\partial\tilde{h}}{\partial\tilde{t}} + \frac{1}{\tilde{r}}\,\frac{\partial(\tilde{r}\,\tilde{h}\,\tilde{u}_r)}{\partial\tilde{r}} = 0, \tag{21}$$

$$\frac{\partial(\tilde{r}\tilde{h}\tilde{u}_r)}{\partial\tilde{t}} + \frac{\partial(\tilde{r}\,\tilde{h}\,\tilde{u}_r^2)}{\partial\tilde{r}} = -\,\tilde{r}\,\tilde{h}\,\frac{\partial G}{\partial\tilde{r}}, \tag{22}$$

where $\tilde{u}_r$ is the dimensionless, statistically averaged radial velocity of fluctuations in the film. Note, that the volume flux of the spray and the gradient of the dynamic pressure determine the temporary averaged flow in the film, wich is expressed in the governing equations (9) and (10). However equations (21) and (22) describe the motion only the fluctuations in the velocity of the film due to the fact that the dynamic pressure is not continuous function.

The parameter λ can be written using the new dimensionless variables as $\lambda = \tilde{r}^2\,\tilde{t}$.

Let us find the solution of the system of equations (21)-(22) subject to the initial conditions:

$$\tilde{h} = \tilde{h}_0 = const, \qquad \tilde{u}_r = 0, \qquad \text{at } \tilde{t} = 0,$$

corresponding to the initially stationary uniform film.

The asymptotical solution at small radii r can be obtained by substitution of the approximate value of G expressed in (19) into equation (22). This solution reads:

$$\tilde{u}_r = -\tilde{r}\tilde{t}^2/3\;\frac{{}_0F_1\left(;5/3;-2\tilde{t}^3/27\right)}{{}_0F_1\left(;2/3;-2\tilde{t}^3/27\right)}, \qquad \tilde{h} = \tilde{h}_0\;{}_0F_1\left(;2/3;-2\tilde{t}^3/27\right)^{-2},$$

where ${}_0F_1(;a;b)$ is the hypergeometric function. It can be shown that this solution becomes singular near the axis $\tilde{r} = 0$ at the time instant $\tilde{t} \approx 2.274$. This instant corresponds to the shock causing the creation of the uprising central jet. This phenomena was observed in the experiments with the diesel spray impact (see Fig. 1).

The numerical solutions of the system of the differential equations (21) and (22) obtained using the expression (18) for the function G is shown in Figs. 3 and 4. The velocity distribution $\tilde{u}_r$ in the film is shown in Fig. 3. As the time approaches the critical value $\tilde{t} = 2.274$ the velocity gradient at the center ($\tilde{r} = 0$) goes to minus infinity, whereas the film thickness grows there infinitely (see Fig. 4). Note however,

that at this time interval the inertial effects in the direction normal to the wall, as well as the surface tension effects become significant and therefore the above model is no longer valid. In order to estimate the height of the jets, the vertical motion of the jets must be modelled separately.

On the other hand, the diameter of the uprising jet shown in Fig. 4 is of order of the length scale Λ. For the spray impact shown in Fig. 1 some parameter estimates were possible using results from phase Doppler measurements. The length scale is of order $\Lambda \sim 15$ μm, and the diameter of the observed jets is approximately 30 μm, which is the same order as Λ. Moreover, the characteristic time of the drop deformation is $D/U \sim 0.4$ μs and the time of the crown propagation is approximately 10 $D/U \sim 4$ μs. The estimated time scale $T \sim 7$ μs, is therefore of the order of the time of crown propagation, thus the effect analyzed above should influence the drop impact. Next, the characteristic velocity of these short-time fluctuations in the film is $\Lambda/T \approx 2$ m/s.

Conclusions

In the present paper the hydrodynamics of a liquid film produced by an impinging spray is considered. The spray is described as a continuum media, exhibiting specific properties, such as number concentration of particles and their probability density function. Also, the definitions of the local flux density vectors and tensors characterizing the spray are introduced.

The equations of motion of the film on the wall account, among others, for the momentum of the fluctuating part of the velocity of the liquid in this film, the volume flux into the film, and the pressure produced by the impacting spray.

The paper explains the emergence of the liquid jets during the impingement of very intense spay by fluctuations of the pressure. The short times are considered during which the pressure produced by the impacting drops can no longer be considered as continuous over the wall surface. Moreover, the characteristic time and length of these fluctuations are determined.

Acknowledgements

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