# INFLUENCE OF PARTICLE-PARTICLE COLLISIONS ON A ROUND JET LADEN WITH SOLIDS 

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#### Abstract

This paper explores the influence of inter-particle collisions in the particle phase variables in the configuration of a free turbulent round jet laden with solids, i.e., considering the so-called four-way coupling. As a result, and due to the absence of walls confining the flow, the effects of particle-particle interactions start to be relevant for larger mass loading ratios as compared with confined flows, such as those occurring in pipes or channels. However, the particle phase fields are modified similarly to what happens in the previous confined flows: the profile of mean axial velocity flattens and the turbulence tends to be more isotropic when inter-particle collisions are taken into account.


## Introduction

Gas-solid and gas-droplet flows are frequently found in industrial and chemical process technology. Examples are pneumatic conveying, fluidised beds, vertical risers, mixing devices, spray systems and others. As a result of the complex micro-physical phenomena affecting the particle motion, such as turbulent dispersion, wall collisions, inter-particle collisions, and flow modulation by the particles, a reliable numerical prediction is rather sophisticated. In such flows the particle (hereafter we will refer to droplets or particles with the generic name particle) behaviour may be considerably affected by inter-particle collisions if the mass loading is high or regions of high concentration develop as a result of inertial effects. Quite a number of theoretical studies on the collision rate of particles and droplets in turbulent flows have been published in the past. A detail review was given, for instance, by Williams and Crane (1983) [1] and Pearson et al. (1984) [2].

A number of experiments were performed in the past aiming at a detailed analysis of particulate confined flows in pipes and channels $[3,4,5,6]$. Numerical calculations based on the Euler/Lagrange approach were performed in [6]. By accounting for wall roughness and inter-particle collisions good agreement with the measurements was achieved for the stainless steel and the glass pipe. Also, calculations by Lun and Liu (1997) [7] and Lun (2000) [8] were based on the Euler/Lagrange approach considering inter-particle collisions and wall collisions without roughness. Initially in [7], a horizontal channel according to Tsuji et al. (1987) [9] was considered. The results clearly revealed the importance of restitution ratio and friction coefficient in modelling inter-particle collisions. The second study [8] concentrated on modelling turbulence modulation for larger particles where wakes may result in a considerable enhancement of turbulence. On the basis of the experimental data of Wu and Faeth (1994) [10], an empirical wake model was proposed. However, the agreement of the calculations for a vertical pipe [4] was found to be not very good.

One of the first studies where a full Reynolds-stress model with two-way coupling based on the standard terms was used for the prediction of gas-particle flows in a channel was introduced by Kohnen and Sommerfeld (1997) [11]. Additionally, a two-time-scale $k-\varepsilon$ turbulence model was used and the predictions were compared with the data of [5]. Recently, Berlemont and Achim (2001) [12] analysed the effect of four-way coupling (i.e., inter-particle collisions) on the prediction of a vertical gas-solid flow [4] using the Euler/Lagrange approach with a Reynolds-stress turbulence model. It was demonstrated that the gas-phase fluctuating velocity components were considerably reduced due to the effect of inter-particle collisions. As an extension of the experimental studies previously performed in different pipes, Laín et al. (2002) [13] presented a detailed analysis of a gas-solid flow in an horizontal channel by varying parameters such as conveying velocity, particle mass loading, particle mean diameter, and size distribution. These data were used for the validation and improvement of the Euler/Lagrange approach using recently developed models on wall collisions [14] and inter-particle collisions [15].

In this paper the effect of inter-particle collisions is studied numerically in a turbulent round free jet. The main difference with the pipe or channel configurations is the absence of particle-wall interactions which promotes

| $\phi$ | $\Gamma_{i k}$ | $S_{\phi}$ |  |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 0 |  |
| $U_{j}$ | $\mu \delta_{i k}$ | $-P_{, j}+\left(\Gamma_{j k} U_{i, k}\right)_{, i}-\rho R_{i j, i}+\rho g_{j}$ |  |
| $R_{j l}$ | $c_{S} \rho R_{i k} k / \varepsilon$ | $\mathcal{P}_{j l}-\varepsilon_{j l}+\Pi_{j l}$ |  |
| $\varepsilon$ | $c_{\varepsilon} \rho R_{i k} k / \varepsilon$ | $c_{\varepsilon 1} \mathcal{P}_{k k} \varepsilon / k-\rho c_{\varepsilon 2} \varepsilon^{2} / k$ |  |
| $\mathcal{P}_{j l}=-\rho\left(R_{j k} U_{l, k}+R_{l k} U_{j, k}\right) ; \quad \varepsilon_{j l}=\frac{2}{3} \rho \delta_{j l} \varepsilon$ |  |  |  |
| $\Pi_{j l}=-c_{1} \rho \frac{\varepsilon}{k}\left(R_{j l}-\frac{1}{3} \delta_{j l} R_{k k}\right)-c_{2} \rho\left(\mathcal{P}_{j l}-\frac{1}{3} \delta_{j l} \mathcal{P}_{k k}\right)$ |  |  |  |
| $c_{S}=0.22 ; c_{\varepsilon}=0.18 ; c_{\varepsilon 1}=1.45$ |  |  |  |
| $c_{\varepsilon 2}=1.9 ; c_{1}=1.8 ; c_{2}=0.6$ |  |  |  |

Table 1: Summary of terms in the general equation for the different variables that describe the gas phase.
a resuspension of the particulate flow. A two-dimensional axisymmetric full Reynolds-stress model with either two-way or four-way coupling will be used in order to assess the performance of the model. Comparisons are presented with the experiments of García (2000) [16], which comprises particle loading ratios up to 0.9 . However, even with the highest measured mass loading ratio the effect of inter-particle collisons is very small. Therefore, a numerical study conducted for a loading ratio of 1.8 , where particle-particle interactions start to be relevant, is performed and presented.

## Summary of numerical approach

The numerical calculations of the particle-laden gas flow in a free turbulent round jet has been performed using the Euler/Lagrange methodology. The fluid flow was calculated based on the Euler approach by solving the full two-dimensional axisymmetric Reynolds stress turbulence model equations extended in order to account for the effects of the dispersed phase [11].

The time-dependent conservation equations for the fluid may be written in the general form (in tensorial notation):

$$
\begin{equation*}
(\rho \phi)_{, t}+\left(\rho U_{i} \phi\right)_{, i}=\left(\Gamma_{i k} \phi, k\right)_{, i}+S_{\phi}+S_{\phi p} \tag{1}
\end{equation*}
$$

where $\rho$ is the gas density, $U_{i}$ are the Reynolds-averaged velocity components, and $\Gamma_{i k}$ is an effective transport tensor. The usual source terms within the continuous phase are summarised in $S_{\phi}$, while $S_{\phi p}$ represents the additional source term due to phase interaction. Table 1 summarises the meaning of the quantities for the different variables $\phi$, being $P$ the mean pressure, $\mu$ the gas viscosity and $R_{j l}=\overline{u_{j}^{\prime} u_{l}^{\prime}}$ the components of the Reynolds stress tensor.

The calculation of the particle phase by the Lagrangian approach requires the solution of the equation of the motion for each computational particle. This equation includes the particle inertia, drag, gravity-buoyancy, slip-shear lift force and slip-rotational lift force because the inter-particle collisions can promote particle rotation. Other forces such as Basset history term, added mass and fluid inertia are negligible for high ratios of particle to gas densities. The change of the angular velocity along the particle trajectory results from the viscous interaction with the fluid (i.e., the torque $\vec{T}$ ). Hence, the equations of motion for the particles are given by:

$$
\begin{gather*}
\frac{d x_{p i}}{d t}=u_{p i}  \tag{2}\\
m_{p} \frac{d u_{p i}}{d t}=\frac{3}{4} \frac{\rho}{\rho_{p} D_{p}} m_{p} c_{D}\left(u_{i}-u_{p i}\right)\left|\vec{u}-\vec{u}_{p}\right|+m_{p} g_{i}\left(1-\frac{\rho}{\rho_{p}}\right)+F_{l s i}+F_{l r i}  \tag{3}\\
I_{p} \frac{d \omega_{p i}}{d t}=T_{i} \tag{4}
\end{gather*}
$$

Here, $x_{p i}$ are the coordinates of the particle position, $u_{p i}$ are its velocity components, $u_{i}=U_{i}+u_{i}^{\prime}$ is the instantaneous velocity of the gas, $D_{p}$ is the particle diameter and $\rho_{p}$ is the density of the solids. The particle mass is given by $m_{p}=(\pi / 6) \rho_{p} D_{p}^{3}$ and $I_{p}=0.1 m_{p} D_{p}^{2}$ is the moment of inertia for a sphere. The drag coefficient is obtained using the standard correlation:

$$
c_{D}= \begin{cases}24 R e_{p}^{-1}\left(1+0.15 R e_{p}^{0.687}\right) & R e_{p} \leq 1000  \tag{5}\\ 0.44 & R e_{p}>1000\end{cases}
$$

where $\operatorname{Re}_{p}=\rho D_{p}\left|\vec{u}-\vec{u}_{p}\right| / \mu$ is the particle Reynolds number.
The expressions employed for the slip-shear force $\vec{F}_{l s}$, slip-rotational lift force $\vec{F}_{l r}$ and the torque acting on a rotating particle $\vec{T}$ are identical to those in [13] and, therefore, will not be written here.


Figure 1: Radial profiles for the different variables at different axial sections for a mass loading ratio of 0.3. Axial fluid mean velocity (top left), axial particles mean velocities (top right), rms velocities (bottom left) and shear stresses (bottom right). Experiments of [16].

The equations to calculate the particle motion are solved by integration of the differential equations (Eqs. $2-4)$. For sufficiently small time steps and assuming that the forces remain constant during this time step, the new particle location, the linear and angular velocities are calculated.

The instantaneous fluid velocity components at the particle location occurring in (3) are determined from the local mean fluid velocity interpolated from the neighbouring grid points and a fluctuating component generated by the Langevin model described by Sommerfeld et al. (1993) [17]. In this model the fluctuation velocity is composed of a correlated part from the previous time step and a random component sampled from a Gaussian distribution function. The correlated part is calculated using appropriate time and length scales of the turbulence form the Reynolds stress turbulence model.

Inter-particle collisions are modelled by the stochastic approach described in detail in [15]. This model relies on the generation of a fictitious collision partner and accounts for a possible correlation of the velocities of colliding particles in turbulent flows. For the particle-particle collisions the restitution coefficient was taken as a constant equal to 0.9 and the static and dynamic friction coefficients were made equal to 0.4 .

Since inter-particle collisions will modify the particle phase properties and eventually the gas phase, the consideration of this combined effect is often referred to as four-way coupling.

## Effect of particles on gas flow

The standard expression for the momentum equation source term due to the particles has been used. It is


Figure 2: Radial profiles of the particle variables for the mass loading ratio of 0.9 with and without considering inter-particle collisions. Left: axial velocities in two axial stations of 20 and 40 diameters downstream the nozzle. Right: normal Reynolds stresses at section of 40 diameters away from the nozzle. Experimental results of [16].
obtained by time and ensemble averaging for each control volume in the following form:

$$
\begin{equation*}
\overline{S_{U_{i} p}}=-\frac{1}{V_{c v}} \sum_{k} m_{k} N_{k} \times \sum_{n}\left\{\left(\left[u_{p}\right]_{k}^{n+1}-\left[u_{p}\right]_{k}^{n}\right)-g_{i}\left(1-\frac{\rho}{\rho_{p}}\right) \Delta t_{L}\right\} \tag{6}
\end{equation*}
$$

where the sum over $n$ indicates averaging along the particle trajectory (time averaging) and the sum over $k$ is related to the number of computational particles passing the considered control volume with the volume $V_{c v}$. The mass of an individual particle is $m_{k}$ and $N_{k}$ is the number of real particles in one computational particle. $\Delta t_{L}$ is the Lagrangian time step which is used in the solution of (3).

The source terms in the conservation equations of the Reynolds stress components, $R_{j l}$, are expressed in the Reynolds average procedure as:

$$
\begin{equation*}
S_{R_{j l p} p}=\overline{u_{j} S_{U_{l} p}}+\overline{u_{l} S_{U_{j} p}}-\left(U_{j} \overline{S_{U_{l} p}}+U_{l} \overline{S_{U_{j} p}}\right) \tag{7}
\end{equation*}
$$

while the source term in the $\varepsilon$-equation is modelled in the standard way:

$$
\begin{equation*}
S_{\varepsilon p}=C_{\varepsilon 3} \frac{1}{2} \frac{\varepsilon}{k} S_{R_{j j} p} \tag{8}
\end{equation*}
$$

where $C_{\varepsilon 3}=1.0$ has been used and the sum is implicit in the repeated subindex $j$.

## Configuration and simulation conditions

The described strategy is applied to calculate the jet flow described in García (2000) [16]. The experimental configuration was characterized by an air jet laden with glass particles ( $\rho_{p}=2450 \mathrm{~kg} \cdot \mathrm{~m}^{-3}$ ) flowing downwards without swirl, issuing from a 12 mm diameter nozzle into a 480 mm -square cage assembly. Data were obtained in several axial positions, both in the near nozzle and in the far nozzle regions. The data at the exit plane serve as inlet conditions for the numerical calculation.

All the calculations have been performed with an axisymmetric mesh of $150 \times 60$ control volumes in the axial and stretched in the radial direction, respectively. Such a resolution was found to be sufficient for producing grid-independent results. A converged solution of the coupled two-phase flow system is obtained by successive solution of the Eulerian and Lagrangian part, respectively. Initially, the flow field is calculated without particle phase source terms until a converged solution is achieved. Thereafter, a large number of parcels are tracked through the flow field (typically 120000) and the source terms are sampled. In this first Lagrangian calculation inter-particle collisions are not calculated, since the required particle phase properties are not yet available. Hence, for each control volume the particle concentration, the local particle size distribution and the size-velocity correlations for the mean velocities and the rms-values are sampled. The particle phase properties are sampled in the standard way for each transverse cell when the computational particle crosses this location. These properties are updated each Lagrangian iteration in order to allow correct calculation of inter-particle collisions. From the second Eulerian calculation, the source terms of the dispersed phase are introduced using an under-relaxation procedure. For the


Figure 3: Radial profiles of the particle variables for the mass loading ratio of 1.8 with and without considering inter-particle collisions. Left: axial velocities in two axial stations of 20 and 40 diameters downstream the nozzle. Right: normal Reynolds stresses at section of 40 diameters from the nozzle.
present calculations typically about 20 coupling iterations have been performed with an under-relaxation factor of 0.20 .

## Results

Figure 1 shows the performance of the presented approach for the very dilute two-phase jet flow of [16] (mass loading ratio $L R=0.3$ ), where inter-particle collisions can be safely neglected. The evolution of mean axial velocities for both phases, air (top left) and particles (top right) is quite well captured up to 40 diameters downstream the nozzle. The normal stresses for the gas phase also show a reasonable agreement with the experiments. As an example, the normal stresses for both phases are plotted at the axial station $X=30 D$ (bottom left figure); it is necessary to point out that the underprediction of the particles $u^{\prime}$ value is typical of the lagrangian calculation of jet flows [18] where the anisotropy of particle turbulence is much higher than that of the gas phase. The calculated particle radial stresses, however, agree much more better with the experimental data. Finally the shear stresses for both phases are drawn in the bottom right graphic for two different axial sections.

The considered experiments provide measurements also for a loading ratio $L R=0.9$. In a pipe or channel flow laden with glass particles, due to the particle-wall interactions which resuspend the solids, the inter-particle collisons cannot be disregarded [13]. However, in the free jet, due to the absence of walls confining the flow, the consideration of the interaction among solids harly makes any difference with the situation where collisions are not taken into account (Fig. 2).

In order to demonstrate the influence of inter-particle collisions on the particle phase variables in a turbulent round jet flow, we consider the higher loading ratio of $L R=1.8$. In this case the mass of solids is large enough to expect that the interaction among particles play an appreciable role, but the solids volume fraction is still low enough to have a dilute flow. Unfortunately, for this case no experimental Measurements were available. The results are shown in Figure 3. The left plot presents the particle axial mean velocities in the axial sections of 20 and 40 diameters far away from the nozzle. It is seen that the effect of collisions is to reduce the values of the centerline axial mean velocity. On the right part the particle normal Reynolds stresses are plotted at the axial station of 40 diameters. Here, there is a decrease of the axial normal stresses regarding the simulation where no collisions were taken into account, while the normal radial stresses experiment a slight increase, resulting in a decrease of the anisotropy of the particle phase turbulence. Both tendencies are similar to the phenomena occurring in a channel flow [13].

## Conclusions

The simulation of a free turbulent round jet laden with solids has been performed taking into account the four-way coupling, i.e., considering inter-particle collisions. The obtained results have been compared with the experimental measurements of García (2000) [16]. Unfortunately, for the highest mass loading ratio ( $L R=0.9$ ) considered in these experiments the role of particle-particle interactions was found to be almost negligible; therefore, in order to illustrate such an influence in the particle phase variables, a mass loading ratio of 1.8 has been simulated. In this last case, the effects of collisions among the solids are in line with what happen in confined flows
such as pipes or channels: the collisions tend to isotropize the particle turbulence and flatten the profile of particle mean axial velocity.

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