ANALYSIS OF SWIRL INJECTION CHARCTERISTICS

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Abstract

The characteristics of swirl injection have been theoretically analyzed by Tanasawa and Kobayasi[1-3] and others[4]. Most such analyses are based on the ideal fluid theory, according to which the spray angle and other characteristic values of swirl injection are expressed as a function of swirl chamber coefficient *K* alone. However, as their first paper[1] pointed out, the ideal fluid theory is insufficient to describe even the swirl flow in an injector consisting of a fixed height swirl chamber, shown in Figure 1. Measurement results of spray angle flow coefficient and cavity coefficient have made clear that they are not the function of *K* alone.

The reasons that the ideal fluid theory cannot describe the measurement results is presumably as follows: Shearing friction energy loss in boundary layers along the top and bottom of swirl chamber wall surfaces, and radial flow produced in the boundary layers

According to Okazaki[4], radial flow is mostly produced in the boundary layer between the swirl chamber top and bottom walls. This suggests that, based on the angular momentum conservation law, as well as on the behavior of pressure in the swirl chamber, radial flow is an important factor regarding the presence of free vortexes. This paper describes and discusses swirl injection non-dimensional analysis taking into account the effect of radial flow in the circulating flow boundary layer. In this analysis, the functional relations among non-dimensional parameters were first presumed. Kobayasi's research results[1] were then analyzed to find the relation between swirl chamber specifications and such spray characteristics in swirl injection. Results confirmed that non-dimensional analysis was in good agreement with the measurement data.

The flow in the swirl chamber has been also analyzed with computational fluid dynamics (CFD). The CFD results certify the large radial flow in the circulating flow boundary layer.

Introduction

Flow coefficient and Reynolds number are generally used to describe the characteristics of flow from an orifice or straight tube. The flow coefficient is the product of two coefficients, each of which describes a different concept, i.e., (contraction coefficient C_c) x (velocity coefficient C_v). Two non-dimensional parameters, velocity coefficient and circulation coefficient, are here introduced to describe swirl injection. Velocity coefficient has the same physical meaning as the aforementioned C_v, and is equal to 1 when flow incurs no loss in the swirl chamber. Circulation coefficient represents the extent to which angular momentum at inlet is maintained until outlet. Velocity coefficient and circulation coefficient were added to the conventionally used non-dimensional parameters flow coefficient *C*, cavity coefficient *k*, and spray angle α. Five non-dimensional parameters were here introduced to clarify the relations of these parameters to swirl chamber configurations.

Though Reynolds number is an important parameter, as mentioned previously, this parameter was excluded from our considerations because flow coefficient and spray angle are generally stable in a turbulent flow region[3], and was turbulent flows with Reynolds numbers exceeding 10^4. The focus of our analysis was what the flow coefficient C and the spray angle α can be described with the configurations of swirl injector. Analysis of other parameters have been described in detail in the authors' paper[5].

Analysis

The swirl chamber, with a radius of r_i, and the spray configurations are shown in Fig. 1. A fluid flows into the swirl chamber at a rate of q through the inlet passage or inlet hole. The passage has a total cross-sectional area of S_i. The fluid is swirled (at circumferential velocity u) and gradually approaches the central part of the swirl chamber. It then flows out of the chamber through the outlet hole(diameter r_e) located at the bottom center. This swirling flow creates a self-swirling force at its core region, thrusting the fluid out of the injection hole at a z-axial velocity component of w and a spray angle of α. The following are the prerequisite conditions for analysis:

1. Fluid flows in tangentially to the swirl chamber, and height of swirl chamber h is constant.
2. Energy loss is produced in the region from swirl chamber radius r_i to injection nozzle radius r_e, but none is produced thereafter.

Non-Dimensional Analysis

Relations among the physical characteristics are assumed by using non-dimensional constants C_1, C_2,--- C_{12}. First, the relations of circumferential velocity u_i at inlet r_i and injection velocity w_e at injection hole r_e to flow rate q are

$$u_i = 4q / \pi d_o^2 = C_1 q / d_o^2 \quad (1) \qquad w_e = C_2 q / \{r_e^2 (1 - k^2)\} \quad (2)$$

As discussed by Okazaki[4], the radial component of circulating flow is obstructed by centrifugal force in the region distant from the top and bottom faces of the swirl chamber. However, there is no rotational flow component in the boundary layer of the circulating flow. Hence, radial flow is easily produced in the boundary layer. Basically, the angular momentum conservation law governs a free vortex. If the velocity of fluid flowing in the radial direction is low, however, accumulated shearing friction energy loss renders the angular momentum conservation law ineffective, even if there is only slight shearing friction loss produced in the free vortex. Now, circumferential flow velocity u_e at r_e is assumed to have a functional relation with circumferential flow velocity u_i at inlet hole and radial flow velocity v_e at r_e. Ue is expressed as equation (3). Exponential index n in the equation is unknown, and is used only to non-dimensional coefficient C_3. Radial flow, which is mainly observed in the boundary layer of circulating flow, is caused by pressure difference Δp_i between r_i and r_e; then, ve is expressed as equation (4).

$$u_e^2 = C_3 u_i^n v_e^{2-n} \quad (3) \qquad v_e = C_4 (\Delta P_i / \rho)^{1/2} \quad (4)$$

The pressure gradient caused by centrifugal force increases as the distance from the flow center reduces. Boundary layer thickness decreases as the distance from r_e reduces. Since radius r_e constitutes an orifice vis-a-vis flow rate q, q is described as equation (5). The thickness δ of boundary layer in circulating flow at r_e is assumed to be as equation (6).

$$q = C_5 v_e r_e \delta \quad (5) \qquad \delta = C_6 \nu / u_e \quad (6)$$

The kinematic energies of u_e and v_e are produced by Δp_i. Hence, Δp_i, including the loss, is expressed as equation (7). When the difference between total pressure and atmospheric pressure at r_i is assumed to be p_o, flow coefficient C can be described as follows (8) from its definition:

$$\Delta p_i = (C_7 u_e^2 + C_8 v_e^2)\rho \quad (7) \qquad C = q / \{\pi r_e^2 (2 p_o / \rho)^{0.5}\} \quad (8)$$

Here, C_i is expressed as equation (9). Assuming that C is a function of C_i, that is equation (10).

$$C_i = q / \{\pi r_e^2 (2 \Delta p_i / \rho)^{0.5}\} \quad (9) \qquad C = f_1(C_i) \quad (10)$$

By eliminating q, u_i, u_e, Δp_i, and δ from equations (1) through (7), v_e and w_e are expressed by

$$v_e = C_9 \nu r_e d_o^{-2}, \qquad w_e = C_{10}\nu / \{r_e(1-k^2)\}$$

and hence,

$$v_e r_e / \nu = C_9 (r_e / d_o)^2 \quad (11) \qquad w_e r_e / \nu = C_{10}(1-k^2)^{-1} \quad (12)$$

The left sides of equations (11) and (12) correspond to Reynolds numbers of v_e and w_e at r_e. If v_e is functionally related with w_e, the right sides of equations (11) and (12), or $(r_e/d_o)^2$ and $(1-k^2)^{-1}$, also have a functional relation with each other. Thus.

$$k = f_2(r_e / do) \quad (13)$$

This equation shows that cavity coefficient k can be expressed as a function of only r_e/d_o.
By arranging equations (1), (3), (4), (7), and (9)

$$C_i = \{(C_4^{-2} - C_8)C_3^{-1}C_7^{-1}\}^{1/n} C_{11}^{-1}(r_e / d_o)^{-2} = C_{12}(r_e / d_o)^{-2} \quad (14)$$

where, $C_{11} = 2^{0.5}\pi C_1 / C_4$. From equations (10) and (14),

$$C = f_3(r_e / d_o) \quad (15)$$

Consequently, flow coefficient C can also be expressed by only r_e/d_o.

Analysis Of Characteristic Value

Since there is little chance of the fluid touching the swirl chamber wall, the fluid can be assumed to have no loss in the region r_e to r_c. Hence, the angular momentum conservation law prevails with regard to flow. Accordingly,

$$u = r_c u_c / r = r_e u_e / r \quad (16)$$

If it can be assumed that kinematic energy and static pressure are constant in the region r_e to r_c in the outlet cross section, and w and v are zero at r_c,

$$\rho\left(u^2 + w^2\right)/2 = \rho u_c^2 / 2 \quad (17) \qquad q = \int_c^e 2\pi r w dr \quad (18)$$

From (16) and (17),

$$w^2 = u_c^2 - u^2 = u_c^2\left\{1 - (r_c / r)^2\right\} \quad (19)$$

Substituting equation (19) into equation (18),

$$q = \pi u_c r_e^2 C_o \quad (20) \qquad C_o \equiv \left(1-k^2\right)^{0.5} - k^2 \ln\left\langle\left\{1 + \left(1-k^2\right)^{1/2}\right\}/k\right\rangle$$

(21)

C_o is tentatively called the "flow area coefficient."
Flow coefficient C is defined as follows:

$$C \equiv q / \left\{(2p_o/\rho)^{0.5}\pi r_e^2\right\} = u_c C_o / (2p_o/\rho)^{0.5} \quad (22)$$

C_o has the same characteristics as the contraction coefficient of the hole described at the beginning of our discussion. Therefore, the flow coefficient for fluid injection from a swirl chamber has the same physical meaning as flow coefficient = (contraction coefficient) x (velocity coefficient of a fluid flowing out of a hole).

In the same manner, spray angle α can be determined from the spray momentum, on the condition that no loss is produced in the region r_e to r_c. The spray angle is an important parameter for estimating the characteristics of flow inside the swirl chamber, because it represents the flow condition inside the chamber. The spray angle has conventionally been determined only on the basis of flow velocities u and w at r_e. (velocity method). However, it is difficult to understand that the spray angle merely depends on these local velocities. It should be considered that the spray angle is determined by the combination of circumferential momentum and z-axial momentum of the entire injection spray. Based on this consideration, we focused on the momentum between r_e and r_c to determine the relation between spray angle α and cavity coefficient k. (momentum method).

$$\tan(\alpha/2)=(\text{circumferential momentum of spray})/(\text{z-axial momentum of spray})=I_u/I_w \tag{23}$$

Circumferential momentum is expressed by

$$I_u = 2\pi\rho \int_c^e ru\,w dr \tag{24}$$

After substitution of equations (16) and (19) into (24), and rearrangement,

$$I_u = 2\pi\rho r_e^2 u_c^2 k \left[\left(1-k^2\right)^{0.5} - k \tan^{-1}\left\{ \left(1-k^2\right)^{0.5} / k \right\} \right] \tag{25}$$

z-axial momentum is expressed by equation(26). When (19) is substituted into (26), equation (27) is derived.

$$I_w = 2\pi\rho \int_c^e rw^2 dr \tag{26}$$

$$I_w = \pi\rho r_e^2 u_c^2 \left(1-k^2+2k^2 \ln k\right) \tag{27}$$

From (23), (25) and (27)

$$\tan(\alpha/2) = 2k\left[\left(1-k^2\right)^{0.5} - k\tan^{-1}\left\{ \left(1-k^2\right)^{0.5}/k \right\} \right] / \left(1-k^2+2k^2\ln k\right) \tag{28}$$

In Fig. 2, the solid line represents this equation (28) and the broken line represents the results obtained by the velocity method[(1)],equation (29), in which spray angle is determined from only flow velocities u and w at r_e. We used equation (28) for our research study. As Fig. 2 shows, equation (28) can be approximated by a straight line for the range of k=0.1 to 0.8, as shown in equation (30).

$$\tan(\alpha/2) = k/\left(1-k^2\right)^{0.5} \tag{29}$$

$$\alpha = 144k + 8.6 \quad (\alpha\text{:deg})$$

(30)

Equation (28) (or equation (30), Fig. 2) allows us to estimate k from the actual measurement data for spray angle α, presented in the references introduced in the early part of this paper.

After all, theoretical equations (21), (22), and (28) {or (30)} combine flow coefficient C, cavity coefficient k, spray angle α, and flow area coefficient C_o. If cavity coefficient k can be determined from equation (13), α, and C_o can be known. Other parameters, velocity coefficient and circulation coefficient, can be also known as shown in authors' study[(5)].

Non-Dimensional Parameter and Swirl Chamber Configuration

The effect of swirl chamber configuration on flow characteristics were studied in terms of the relation of r_i, r_e, and h to d_o. According to the analysis results discussed in the preceding chapter, cavity coefficient k has an important non-dimensional parameter. However, the measured quantities were only spray angle α and flow coefficient C. First, using theoretical equation (28), or equation (30) and Fig. 2, cavity coefficient k are calculated from the actually measured spray angle α. As equation (13) shows, cavity coefficient k can be described as a function of only r_e/d_o. We derived empirical formula (31).

$$k = 5.16 / \left\{ 5.49 + \left(r_e/d_o\right)^{-1.5} \right\} \tag{31}$$

As equation (28) shows, spray angle α is also a function of r_e/d_o. The relation between the actually measured spray angle α and the corresponding r_e/d_o is shown in Fig. 3. The solid line in the figure represents equation (32), derived by substituting equation (31) into k of empirical formula (30). The result of our study on the actually measured flow coefficient C and the corresponding r_e/d_o is shown in Fig. 4. As equation (15) shows, C can be described as a function of only r_e/d_o. The empirical formula is derived as (33).

$$\alpha = 743 / \{5.49 + (r_e / d_o)^{-1.5}\} + 8.6 \quad (32) \qquad C = 0.82 / [5.9(r_e / d_o)^{1.5} + 1] \quad (33)$$

The solid line in Fig. 4 represents the equation (33). As discussed heretofore, we confirmed that cavity coefficient k, spray angle α, and flow coefficient C can be described as functions of only r_e/d_o; hence, both (circulation coefficient)/(characteristic value of swirl chamber K), and velocity coefficient are also functions of only r_e/d_o.

Verification with Numerical Analysis

In the boundary layer of circulating flow in the simple shape swirl chamber, the radial flow velocity is verified with the numerical experiment by CFD (Computational Fluid Dynamics) analysis. The computation is performed with STAR-CD that is multi-purpose thermo fluid analysis software based on finite volume methods. The VOF (Volume of Fluid) methodology, one of two-phase flow method, is applied to calculate cavity phenomenon in the swirl chamber. The standard k-ε model was selected for the turbulence model, and the log-law function was used for the flow velocity near the wall. A first-order upwind differencing scheme was selected. The computational grids are shown in Fig.5. The computational region is divided into 1/4 model (circumferential 90 degree) because of four inlets. The grid size at near lower wall and boundary region between gas and fuel is about 3.3μm. Boundary condition is 2.9MPa at inlet and 0.1MPa at injection hole. When the liquid flow rate is constant, the flow is considered a steady-state.

In arbitrary cross section swirl and radial velocity distribution are shown in Fig. 6. In swirl chamber at the region far from upper and lower wall there is very slower radial velocity and faster circumferential velocity. The smaller radius (nearer injection hole) becomes, the more circumferential velocity accelerates. But this acceleration does not quite meet the conservation law of angular momentum. On the other hand, the boundary layer of circumferential flow is formed in the region close to lower wall and radial velocity reaches maximum in the boundary layer. Further more, the smaller radius (nearer injection hole) r_i becomes, the more radial velocity v accelerates.

Circumferential u and radial velocity v distributions in near lower wall at the position of $r_i/2$ are shown in Fig. 7. As shown in Fig. 7, it is clear that radial velocity v increases in boundary layer of circumferential flow.

Conclusion

Swirl injection characteristics were non-dimensionally analyzed taking into account radial flow in a circulating flow boundary layer. The results show clearly that cavity coefficient k, spray angle α, (circulation coefficient)/(characteristic value of swirl camber K), flow coefficient C, and velocity coefficient are functions of only r_e/d_o. This result may apply not only to simple swirl chambers having a flat bottom, but also to shaft-symmetrical swirl injectors for diesel engines. And the radial flow almost stays in the boundary layer of the circumferential flow. This is the premise of the non-dimensional analysis. And it is shown correct by the numerical analysis.

Nomenclature

C :flow coefficient $\equiv q / \{\pi r_e^2 (2p_o/\rho)^{0.5}\}$ | d_o :reduced diameter of $Si \equiv 2(Si/\pi)^{1/2}$

h: height of swirl chamber | K: characteristics value of swirl chamber

k: cavity coefficient $\equiv r_c / r_e$ | p_o: total pressure at inlet relative pressure to ambient

p_i: static pressure at inlet(relative pressure to ambient) | p: static pressure relative pressure to ambient

q: flow rate | r_i: radius of swirl chamber

r_e: radius of injection hole | r_c: radius of cavity at injection hole

S_i: total cross section of inlet hole | u: circumferential velocity

u_c: circumferential velocity at r_c | u_e: circumferential velocity at r_e

u_i: circumferential velocity at inlet

v: radial velocity(plus direction is toward chamber center)

w: velocity along z-axis (downward)

α: spray angle(deg)

δ: thickness of boundary layer in circulating flow at r_e

References

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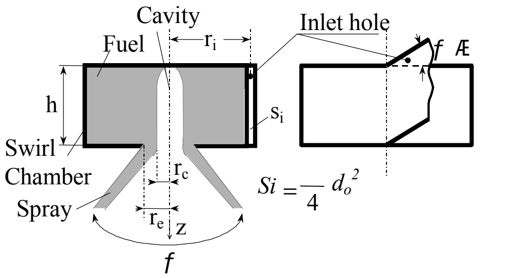

Figure 1 Swirl chamber and spray configuration

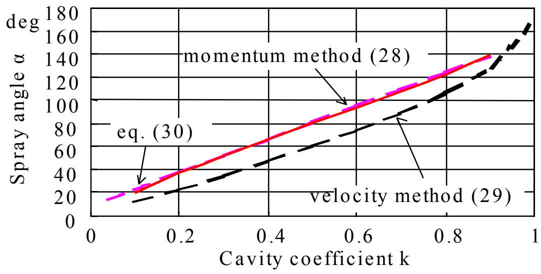

Figure 2 Spray angle α vs. cavity coefficient k

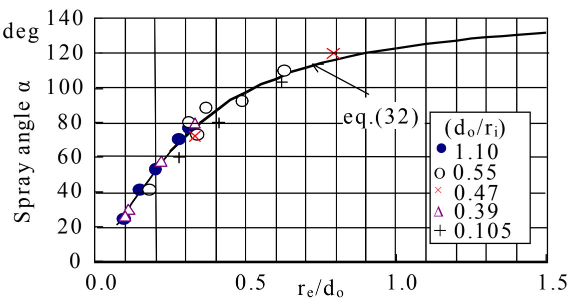

Figure 3 Spray angle α vs. r_e/d_o

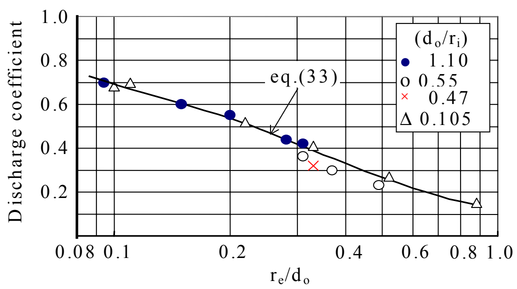

Figure 4 Discharge coefficient C vs. r_e/d_o

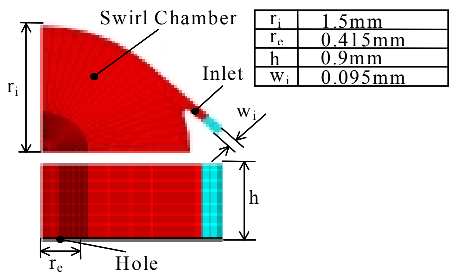

Figure 5 Computational grid and specs

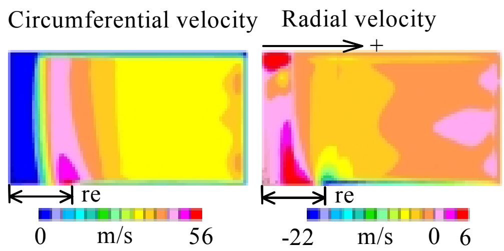

Figure 6 Flow velocity distribution

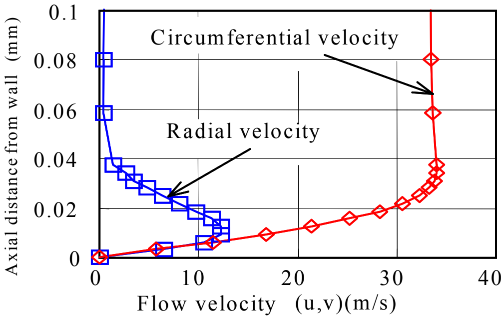

Figure 7 Flow velocity distribution near wall