

# ***SPRAY SIZING BY IMAGING: FOCUSING AND DROPLET IMAGE SUPERPOSITION***

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## **ABSTRACT**

The drop size distribution is an important feature of a spray and also a tool to characterize the efficiency of an atomisation device. Non-intrusive optical techniques are nowadays largely used to measure drop size distributions. Among these techniques, image based techniques present the advantage of allowing the analysis of the drop shape and so the measurement of non spherical drops. The ability to describe the size and the morphology of non-spherical droplets is very suitable in order to analyse not totally atomised sprays, during the successive phases of the atomisation process. Nevertheless two main limitations exist : unfocused droplets characterisation and partial or total overlapping projection of the droplet images. The out of focus effect on the droplet sizing is corrected by an imaging model lying on the point spread function and is used to define the 3D location of the droplets in the space. Partially overlapping droplet images are discriminated through the analysis of the grey level gradient at the outline of the image. The effect of totally overlapping droplets on the drop size distribution estimation is corrected by a statistical model.

## **STATE OF THE ART**

Out-of-focus and overlapping effects are main disadvantages of spray sizing by image analysis that need to be corrected in order to obtain reliable drop size distribution estimations. These two problems have been explored in the literature. Lee and Kim [1] presented an interesting reviewing on this subject at the last ICLASS'03 meeting. Different empirical focusing criteria can be found in the literature to select the focused droplets. But the selection of only perfectly focused droplets leads to an important filtering that yield to a few measured droplets in one image. Thus, a great number of images must be treated in order to calculate a statistically correct droplet size distribution. This could be very time consuming. To overcome this problem Blaisot & Ledoux [2] and more recently Malot & Blaisot [3] proposed a model based on the representation of the imaging optical system through its point spread function (PSF). This model permits to correct the diameter measured on out-of-focus droplets with a precise dynamical threshold level and the knowledge of the image contrast. This model is briefly presented in this paper and is extended here to the 3D localisation of the droplets. The advantage of this model on the filtering of focused droplets methods is that the criterion for 3D localisation is not dependent on the droplet size. Indeed, empirical methods are often unable to analyse correctly the focusing of the largest droplets because these large droplets are focused over a larger distance from the focus plane than smaller droplets. The population of the largest droplets is then overestimated in the droplet size distribution as the actual measurement volume increases with the droplet size.

The problem of the detection and separation on spray images of the superimposed spherical droplet images has also been studied in the past [1]. Droplet image superposition occurs when the depth of field is large enough or when droplet concentration is high enough. Commonly proposed solutions for the identification of overlapped images are based on image contour analysis. For example Kim & Lee [4] proposed the detection of the overlapped droplets by analysing the discontinuity of the droplet contour function. Kim & Lee [5] also proposed a method based on the Hough transform to identify portion of circular contour in order to localize each droplet centre and radius. However, these techniques are only available for separation of the spherical droplets and are applicable on two two-level images only, after image threshold application. A new method for the detection of superimposed droplets is proposed in this paper. This one is based on the analysis of grey level images and is able to distinguish the superimposed spherical droplets from complexes morphological liquid objects.

No works have been found in the literature on the effect of totally overlapping images of droplets. Indeed, when the droplet concentration is very high, this phenomenon can lead to a misinterpretation of the images for the determination of the drop size distribution. The smallest droplets are most likely concerned with this phenomenon. Indeed, the probability for a droplet to be overlapped by another one decreases when the droplet size increases due to the reduction of the number of droplets able to do it. These three developments of the image-based granulometry have been developed more extensively in a recent thesis at the CORIA laboratory [6] and applied to the Diesel spray analysis. During the fuel injection, the results show more accurate droplet size distribution measurements with this technique than with PDPA.

## CORRECTION OF THE OUT OF FOCUS EFFECT

The image formation model used by Malot & Blaisot [3] assumes that the illumination in the image plane  $i(x,y)$  can be described by the convolution product of the irradiance distribution in the object plane  $o(x',y')$  and the point spread function (PSF) of the imaging system  $s(x,y)$  Eq.(1) :

$$i(x,y)=o(x',y')\otimes s(x,y) \quad (1)$$

In this equation, the droplets are considered as transmitting plane discs. In the cylindrical coordinates associated to the droplet centre, the irradiance distribution is defined as Eq. (2) where  $a$  is the object radius,  $\tau$  is the transmission coefficient,  $\gamma$  the lateral magnification of the imaging set-up and  $circ(r)$  a function of the radial distance  $r$ , equals to 1 if  $r<1$  and equals to 0 otherwise.

$$o(r)=1-(1-\tau)circ(r/\gamma a) \quad (2)$$

For a diffraction-limited optical system of circular aperture under non-coherent polychromatic light, the PSF can be modelled by a Gaussian shape :

$$s(r)=s_0 \exp\left(-2\left(\frac{r}{\chi}\right)^2\right) \quad (3)$$

with  $\chi$  the PSF half-width and  $s_0$  a normalization constant.

The blurring effect of the droplet image  $i(x,y)$  is directly linked to the ratio  $a/\chi$ . For a given droplet size  $a$ , the more the PSF width is large, the more the droplet is out of focus. So, the  $\chi$  parameter increasing with the droplet distance of the focus plane reveals the position of the droplets along the optical axis.

According to Eqs. (1-3) it is possible to virtually describe the grey level profile of an unfocused droplet in the image. Then it is possible to theoretically define the local contrast revealing the dynamic of the grey levels on the image of the droplet Eq. (4) :

$$C=\frac{i_{\max}-i_{\min}}{i_{\max}+i_{\min}}=\frac{(1-\tau)(1-\exp(-2(a/\chi)^2))}{2-(1-\tau)(1-\exp(-2(a/\chi)^2))} \quad (4)$$

The grey level profile of an unfocused droplet also permits to calculate the apparent droplet size  $r_l$  obtained for a given relative threshold  $l$  of the image. The relative threshold level  $i_{thr}(l)$  is defined between the minimum and maximum grey level value of the image profile according to equation (5) :

$$i_{thr}(l)=i_{\min}+l\cdot(i_{\max}-i_{\min}) \quad (5)$$

It was shown [3] that for a given relative threshold level  $l=0.61$ , the ratio between the true droplet radius and the measured one is a strictly increasing function of the local image contrast (dash line in Fig. 1-a). So, it is possible to experimentally measure the contrast linked to a droplet image and its apparent size obtained for the chosen threshold level. Then, these two parameters are used to calculate the real size of the droplets. A calibration procedure of the optical apparatus must be done using a reticule on which calibrated discs are engraved. This reticule is shifted along the optical axis  $z$  in order to consider the out-of-focus effect. The calibration results presented in Fig. 1-a show that the correction made on the measured diameters using the PSF model is not sufficient in this particular optical configuration.

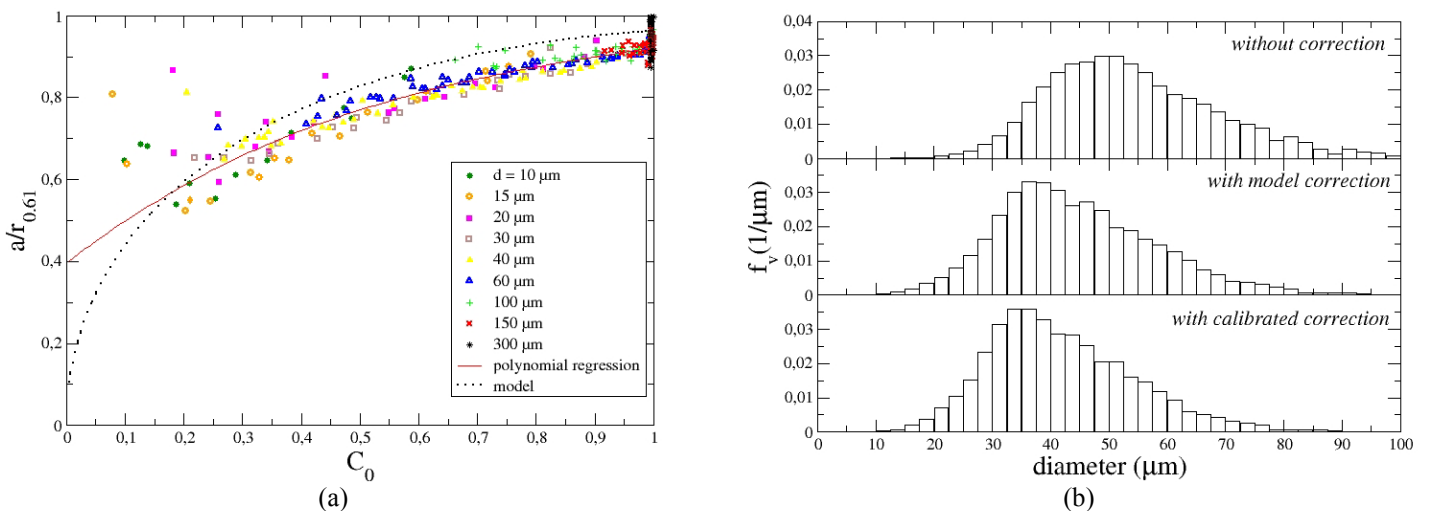


Figure 1 : Experimental validation of the PSF model used for the droplet size correction (a) and its effect on volume based distribution (b)

Thus a polynomial regression is computed on the experimental data to obtain a calibrated correction law in place of the one predicted by the model. Figure 1-b presents the effect of the correction applied to the measured diameter with the model and with the calibrated curve on the volume based droplets size distribution. This shows the strong effect of the proposed correction law.

The PSF imaging model is also used to determine the droplet distance from the focus plane, measuring the PSF half-width parameter  $\chi$ . A method for experimentally calculating this parameter consists in the measurement of two apparent radius associated to different thresholds levels ( $l_1=0.77$ ,  $l_2=0.25$ ). The PSF model indicates that the ratio  $(i_{thr}(l_1)-i_{thr}(l_2))/\chi$  is directly related to the droplet image contrast  $C$ . Then, by measuring  $i_{thr}(l_1)-i_{thr}(l_2)$  and  $C$  and using the function obtained with the PSF model, it is possible to determine an estimation for  $\chi$ , representative of the defocusing effect. To correlate the value of  $\chi$  to the position of the object in the measurement volume, the optical system must be calibrated as it has been done for the out-of-focus correction. Results are presented in Fig. 2-a. The focus plane is defined by the position where  $\chi$  is minimum ( $z=0$ ). The more distant of this focus plane the droplets are, the less visible they become. Therefore, in order to construct a correct droplets size distribution without overestimation of the largest droplets, a filtering has to be done in order to select only drops that are included in a precise volume of measurement. This filtering is obtained preserving only droplets for which  $\chi$  is less than a fixed maximum value  $\chi_{max}$ . The choice of  $\chi_{max}$  is directly linked to the deep of the volume of measurement  $\Delta z$  (Fig. 2-a) and is obviously dependant of the optical system. The Figure 2-b explicitly shows that this filtering has an important effect on the drop size estimation. It considerably reduces the larger droplets trail in the drop size distribution. This has a non negligible effect on the estimation of the SMD.

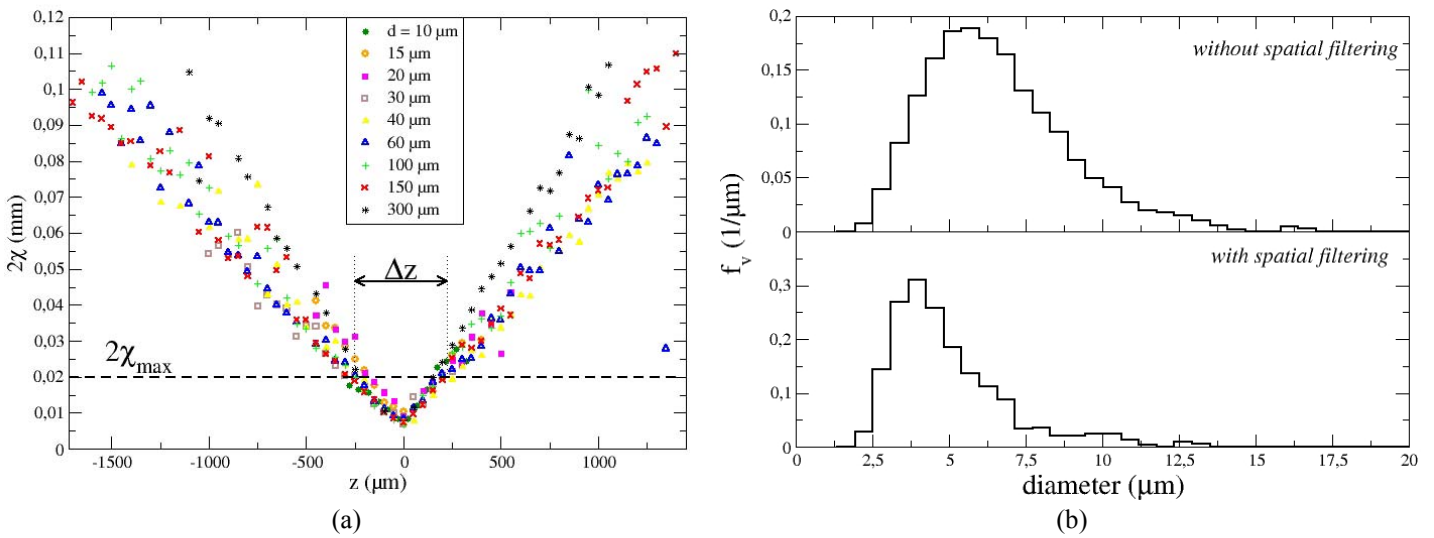


Figure 2 : Experimental calibration of the PSF half-width measurement (a) and its effect on volume based distributions (b)

### PARTIALLY OVERLAPPED DROPLETS DETECTION

A new method for partially overlapped spherical droplets is presented. This one uses the grey level information for the determination of the droplets centres, diameters and defocusing. The main idea is to use the grey level gradient vectors obtained with Sobel linear filtering on the droplets images. Indeed, each pixel of the droplets image positioned at the droplets outline is characterized by a grey level gradient normal to the local interface and oriented toward the exterior part of the droplet as shown in Fig. 3-a. Thus, if the droplets are spherical, all the grey level gradient vectors intersects themselves at the droplet centre. The calculation of a density map of intersection permits to reveal the localisation of the droplets centres. Of course, only upstream vectors intersections has to be taken into account as seen in Fig. 3-b and 3-c.

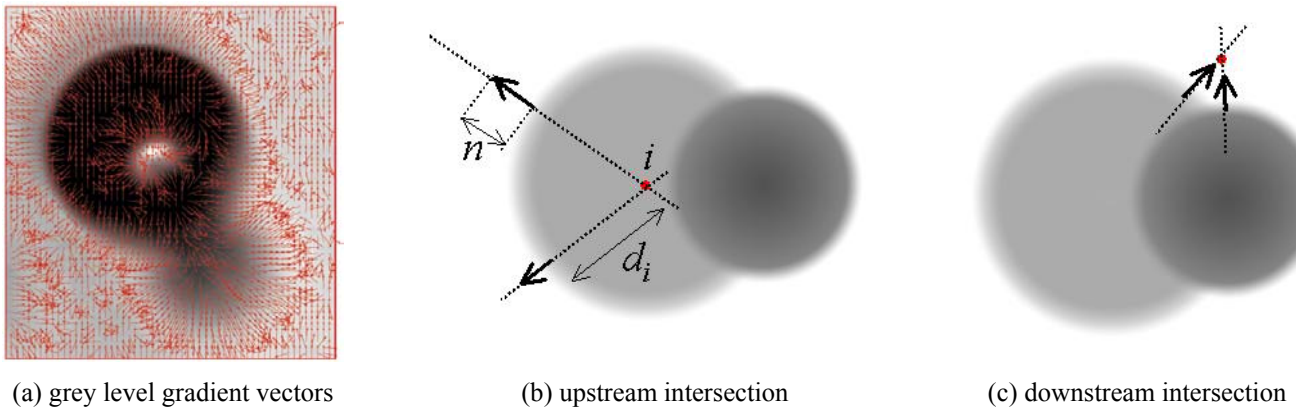


Figure 3 : The detection of the centres of spherical droplets from the intersection of the grey level gradient vectors

Each intersection  $i$  is localized on the picture and associated to a mean diameter  $d_i$  (that correspond to the distance between the intersection point and the vector application point) and to a mean norm  $n$  of the two vectors (directly linked to the droplet out of focus). The density of vector intersections is directly linked to the number of intersections  $N_i$  caused by the  $N_v$  vectors calculated at the droplet outline. The density map calculated thanks to the  $N_i$  intersections is lower for a small focused droplet than for a large unfocused one that is characterized by a greater perimeter and so a greater number of vectors  $N_v$ . The relation between  $N_i$  and  $N_v$  is given in Eq. (6) :

$$N_i = \frac{N_v(N_v-1)}{2} \approx \frac{N_v^2}{2} \quad (6)$$

The number  $N_v$  is proportional to the surface at the droplet outline according to Eq. (7), where  $i_{back}$  and  $i_{min}$  are two reference levels and  $n$  is the mean grey level gradient:

$$N_v \propto \pi d_i \frac{(i_{back} - i_{min})}{n} \quad (7)$$

Thus the density map is corrected using the parameter  $\rho$  that takes into account the size and the blurry effect of the overlapped droplets. The parameter  $\rho$  is defined Eq.8 by the ratio of the measured number  $N_v$  given by Eq. (6) and its estimation given by Eq. (7) :

$$\rho = \frac{2\sqrt{2N_i \cdot n}}{d_i(i_{back} - i_{min})} \quad (8)$$

The Figure 4 presents the density maps obtained for overlapping of three droplets of different diameters and different blurring effects. In this example, we clearly observed a good detection of the droplet centres with the  $\rho$ -based density map.

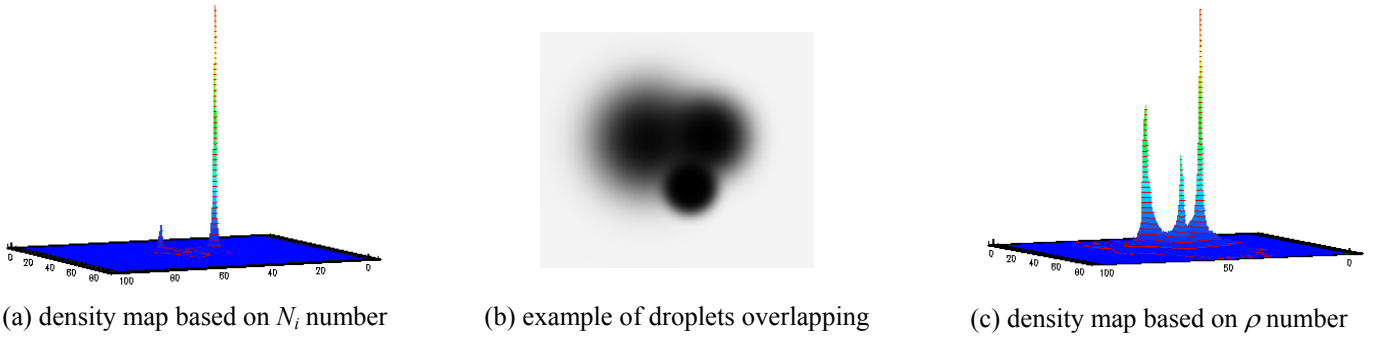


Figure 4 : The detection of overlapped droplet centres

Each partially overlapped droplet detected is associated to a mean diameter  $d_i$  and to a mean grey level gradient value  $n$  representative of the blurry effect of the drop. Thus, it is possible to define a criterion that uses these parameters in order to differentiate overlapped droplets from a liquid element of complex morphology. In the case of droplets that present different values for the parameter  $n$  as in Fig. 5-a, the droplet separation is applied (Fig. 5-b). In the other case (Fig. 5-c), it is possible that the detected liquid element presents a complex geometry and then no separation is applied (Fig. 5-d).

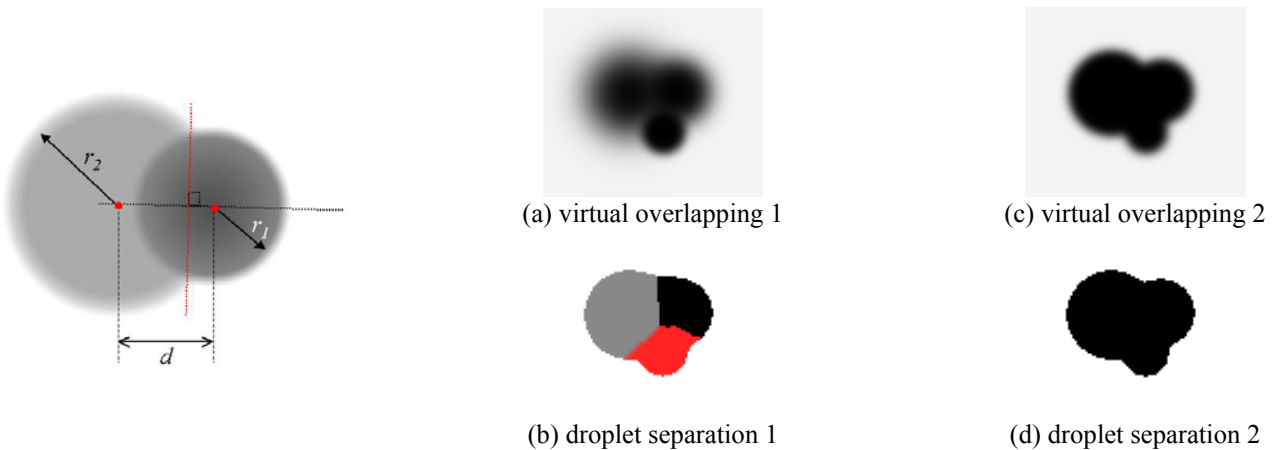


Figure 5 : The separation of overlapped droplets

## EFFECT OF TOTALLY OVERLAPPED DROPLETS ON THE DROPLET SIZE DISTRIBUTION

Due to the optical projection on the image recording detector, some droplets can be totally hidden by larger ones in the picture. This total overlapping can not be detected a priori with image processing tools. We propose a correction law that permits to determine a more realistic droplet size distribution in such a case.

Let us consider  $N$  droplets randomly and homogeneously distributed in the volume of measurement defined by height  $h$ , width  $w$  and the depth of field  $\Delta z$ . Let us now consider two droplets of diameter  $D$  and  $X$  ( $D < X$ ). The set of droplet centres for which the droplet  $D$  is totally overlapped by the droplet  $X$  is the surface  $S$  (Eq. 9) :

$$S = \pi \left( \frac{X}{2} - \frac{D}{2} \right)^2 \quad (9)$$

Considering  $f_n$  the real numerical droplet size distribution, the overlapping density  $d\xi$  on the image of the droplet of diameter  $D$  by the droplets of diameters between  $X$  and  $X+dX$  is given by (Eq. 10) :

$$d\xi(X, D) = \frac{N \cdot \pi}{4 \cdot w \cdot h} f_n(X) (X - D)^2 dX \quad (10)$$

This overlapping density can be generalized for all the droplets that are able to overlap the droplets of diameter  $D$ . The overlapping density of a droplet which diameter is  $D$  by other droplets is then given by Eq. (11) :

$$\xi(D) = \frac{N \cdot \pi}{4 \cdot w \cdot h} \int_D^\infty f_n(X) (X - D)^2 dX \quad (11)$$

The value  $\xi(D)=2$ , for example, means that, on average, the droplet of diameter  $D$ , is overlapped by two larger droplets in the volume of measurement. The maximum value for  $\xi(D)$  is the total number of droplets  $N_D$  in the measurement volume, with a diameter larger than  $D$ . The probability  $P(D)$  for a droplet of diameter  $D$  to be overlapped by the droplets that are able to do that is given by Eq. (12) :

$$P(D) = \xi(D) / N_D \quad \text{with} \quad N_D = N \int_D^\infty f_n(X) dX \quad (12)$$

Considering that  $P(D)$  is low and  $N_D$  is high enough, the probability  $P(D)$  can be modelled by the Poisson's law in order to define the probability  $p'(D)$  for a droplet of diameter  $D$  to be overlapped at less once time (Eq. 13) :

$$p'(D) = 1 - \exp(-\xi(D)) \quad (13)$$

Thanks to the equation 13, it is possible to estimate the number of droplets of diameter between  $D$  and  $D+dD$  to be hidden on the picture (Eq. 14) :

$$N f_n(D) p'(D) dD \quad (14)$$

Thus, the number of droplets of this size which can be detected on the pictures of the spray is :

$$N f_n(D) (1 - p'(D)) dD = N f_n(D) \exp(-\xi(D)) dD \quad (15)$$

Using equation 15 it is possible to define the relationship between the real numerical droplet size distribution  $f_n$  and the apparent one  $f_m$  :

$$f_m(D) = \frac{f_n(D) \exp(-\xi(D)) dD}{\int_0^\infty f_n(D) \exp(-\xi(D)) dD} \quad (16)$$

It has been shown that, in most practical cases, the totally overlapped droplets have a weak effect on the measured droplet size distribution. This is due to the fact that when total overlapping effect is big, the droplet image density is high and images are often not analysable. This implies that the volume of measurement must contain a relatively small quantity of fluid, and in this case, the total overlapping occurrence is rare. Nevertheless, in certain cases, the total overlapping of droplets can have an effect on the qualitative interpretation of the measured drop size distributions. This is illustrated in figure 6.

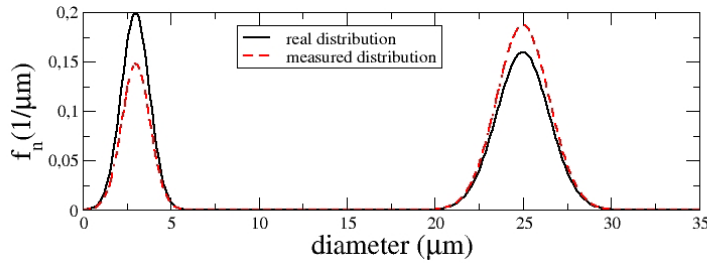


Figure 6 : Theoretical effect of the total droplet overlapping effect on droplet size distributions

The figure 6 presents a bimodal numerical droplet size distribution and the one that would be measured with image analysis due to the overlapping effect. The picture size is  $200 \times 200 \mu m^2$  and the depth of field is  $400 \mu m$ . In this calculation the fluid represents only 2.5% of the total volume of measurement. The optical configuration implies that the droplet images cover 70% of the picture area. In this critical but realistic example, there is an incidence of the overlapping effect on the droplet size distribution. This is particularly observed when the most populated mode is characterized by little droplets but in this case, the misinterpretation induces only a weak error on the calculation of the representative diameters of the distribution. Indeed, in our example, the relative error on the SMD is only about 0.3%.

## CONCLUSION

The present work is concerned with the improvement of an image processing technique used for spray sizing. Two important defects of image-based techniques induced by the out-of-focus effect are corrected using an model for the image formation. One concern the overestimation of the diameter of unfocused droplets. This defects is corrected by the measurement of the local contrast and the use of a correction law deduced from model for the calibrated optical system. The second defect is due to the fact that bigger droplets are observed further from the focus plane than smaller ones. This effect leads to the overestimation of the big diameters in the droplet size distributions. A filtering of the detected droplets is achieved based on the measurement of the point spread function width of the optical system which is linked to the droplet position along the optical axis.

The problem of the droplet overlapping on the image of the spray has also been taken into account. A new method for partially overlapped spherical droplets, based on the grey level gradient vectors has been proposed. This one is used to distinguish overlapped spherical droplets from a non spherical one. Finally, the effect of the totally overlapped droplets on the image-based granulometry has been studied. The proposed calculation shows that this effect seems to be negligible for the measurement of characteristic diameters representative of droplet size distributions, in the case of analysable images of a spray. Nevertheless this effect induces a rearrangement of the droplet size population that can lead to qualitative misinterpretation of the spray granulometry.

## NOMENCLATURE

$a$	object real radius [m]	$r_l$	apparent radius obtained with the $l$ level of threshold [m]
$D$	droplet diameter [m]	$p'(D)$	probability for the droplet to be overlapped at less once time []
$C$	local image contrast []	$P(D)$	overlapping probability []
$d_i$	measured mean diameter [m]	$X$	droplet diameter [m]
$f_m$	measured numerical droplet size distribution [ $m^{-1}$ ]	$x, x', y, y', z$	Cartesian localization [m]
$f_n$	real numerical droplet size distribution [ $m^{-1}$ ]	$\chi$	PSF half-width [m]
$i$	image grey level []	$\gamma$	magnification of the imaging system []
$l$	relative level of threshold []	$\rho$	droplet centre detection cartography [ $m^{-2}$ ]
$n$	norm of the grey level gradient vector [ $m^{-1}$ ]	$\tau$	transmission coefficient []
$N$	number of droplets in the volume of measurement []	$\xi$	overlapping density []
$N_i$	number of vector directions intersections []		
$N_v$	vectors number []		
$r$	radial localization [m]		

## REFERENCES

1. S. Y. Lee and Y. D. Kim, Sizing of spray particles using image processing, *Proc. Symp. ICLASS*, Sorrento, Italy, 2003.
2. J-B Blaisot and M. Ledoux, Simultaneous measurement of diameter and position of spherical particles in a spray by an original imaging method, *Applied Optics*, vol. 37, N°22, pp. 5137-5144, 1998.
3. H. Malot and J-B Blaisot, Droplet size distribution and sphericity measurement of low density sprays through image analysis, *Part. Part. Syst. Charact.*, N°17, pp.146-158, 2000.
4. Y. D. Kim, S. Y. Lee and J. H. Chu, Separation of overlapped particles using boundary curvature information, *Proc. Symp. ILLAS-Asia*, pp. 259-264, 2001.
5. Y. D. Kim and S. Y. Lee, Application of the Hough transform to image processing of the heavily overlapped particles with spherical shaped, *Atomization and Sprays*, Vol. 12, pp. 451-461, 2002.
6. J. Yon, Jet Diesel Haute Pression en Champ proche et lointain : Etude par imagerie, Ph.D dissertation of the university of Rouen, France, 2003.