INFLUENCE OF AN INJECTION OF ELECTRIC CHARGES ON THE STABILITY OF A VISCOUS JET OF DIESEL OIL IN ATOMISATION REGIME

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ABSTRACT

The present study is a linear analysis of the stability of electrified viscous jets of diesel oil. In a first part we study the growth of initial perturbations on the liquid surface taking into account the effects of inertia, surface tension, viscous, aerodynamic and electric forces. We start from the linear analysis of the stability of viscous jets done by Reitz and Bracco who found the dispersion relation of the non electric problem. We add the electric terms of Maxwell's tensor corresponding to the injection of electric charges in the liquid and give the dimensional and non-dimensional dispersion relations of the electric problem.

In a second part we solve the dispersion equation numerically for different configurations of the hydrodynamic and electric parameters. Then we present the results and analyse the role of the electric parameters on the destabilization of the jet.

In the last part of the article we give a few experimental results which confirm the destabilizing role of the electrification process.

1. INTRODUCTION

Improving the quality of the atomization of fuels (reducing the size of the droplets for instance) is one of the most important concerns for motor industries. Because of the actual regulation on the environmental impact of the combustion the car manufacturers are forced to find new processes that better atomize the fuels. For typical boiler burners it has been proved that reducing the mean size of the droplets by 10% would result in an unburned carbon reduction of 35% [1]. One possible way to reduce the size of the droplets of the atomization is to electrify the fuel by injecting electric charges in the liquid. It is the process that we study in this article.

The effect of the electric charges on liquid jets has been already studied by Rayleigh [2] and many other researchers like Basset [3] for instance. Their works have shown clearly how the electric charges modify the stability of the jets.

The present work is a linear stability analysis of a charged cylindrical liquid jet. It includes the effect of the liquid viscosity and the effect of the electrostatic forces. Following the approach of Reitz, Bracco [4], [5] and Saville [6], the stability of the liquid surface is examined using a first-order linear theory, which leads to the dispersion relation.

2. LINEAR STABILITY ANALYSIS

Let us consider an axisymetrical, viscous, incompressible liquid jet of diameter $2a$ moving with velocity U through a quiescent, inviscid, incompressible gas medium. It is assumed that the liquid is charged and that the potential on the surface of the jet is constant. The densities of the liquid and the gas are ρ_1 and ρ_2 respectively and the viscosity of the liquid is μ_1. The cylindrical polar coordinate system (r,θ,z) which has been chosen because of the natural geometry of the jet moves with the same velocity U. We impose on the initial steady motion the following infinitesimal axisymmetric perturbation :

$$\eta = \Re\left(\eta_0 \exp(ikz + \omega t)\right) \tag{1}$$

where η_0 is the initial wave amplitude, $k = 2\pi/\lambda$ is the wave number and $\omega = \omega_R + i\,\omega_I$ is the complex growth rate.

The perturbation produces fluctuating velocities and pressures u_1, v_1 and p_1 for the liquid and u_2, v_2 and p_2 for the gas, u and v being the components of the velocity on z and r axis respectively.

To obtain the dispersion relation we solve the linearized continuity and momentum equations in the liquid with the following linearized boundary conditions at the interface between the liquid and the gas:

$$v_1 = \frac{\partial \eta}{\partial t} \tag{2}$$

$$-p_1+2\mu_1\frac{\partial v_1}{\partial r}-\frac{\sigma}{a^2}\left(\eta+a^2\frac{\partial^2\eta}{\partial z^2}\right)+p_2-\frac{\sigma_0{}^2}{\varepsilon_0}\frac{\eta}{a}\left(ka\frac{K_1(ka)}{K_0(ka)}-1\right)=0 \tag{3}$$

$$\mu_1\left(\frac{\partial u_1}{\partial r}+\frac{\partial v_1}{\partial z}\right)=\frac{\sigma_0{}^2}{2\varepsilon_0}ik\eta \tag{4}$$

where σ is the surface tension, σ_0 is the initial surface charge density. Equations (2) – (4) are the kinematic, the normal stress and the tangential stress first order equations with the Maxwell's tensor terms.

The linearized hydrodynamic equations for the liquid phase are:

$$\begin{aligned}
&\frac{\partial u_1}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv_1)=0\\
&\frac{\partial v_1}{\partial t}=-\frac{1}{\rho_1}\frac{\partial p_1}{\partial r}+\nu_1\left(\frac{\partial^2 v_1}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_1}{\partial r}\right)\right)\\
&\frac{\partial u_1}{\partial t}=-\frac{1}{\rho_1}\frac{\partial p_1}{\partial z}+\nu_1\left(\frac{\partial^2 u_1}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial r}\right)\right)
\end{aligned} \tag{5}$$

Equations (5) are solved by separating the liquid velocity in two parts according to Helmholtz decomposition, one part representing the irrotational solution and the other one the effect of the viscosity. The fluctuating velocities can be written as follows:

$$\begin{aligned}
v_1&=\frac{\partial\Phi_1}{\partial r}+\frac{1}{r}\frac{\partial\Psi_1}{\partial z}\\
u_1&=\frac{\partial\Phi_1}{\partial z}-\frac{1}{r}\frac{\partial\Psi_1}{\partial r}
\end{aligned}$$

We introduce the following stream function Ψ_1 and velocity potential Φ_1 :

$$\Phi_1=\phi_1(r)e^{ikz+\omega t},\quad \Psi_1=\psi_1(r)e^{ikz+\omega t}$$

If we inject them in (5), we obtain two differentials equations for ψ_1 and φ_1. The solutions, free from singularities on the axis, are:

$$\begin{aligned}
\Phi_1&=-\frac{\eta_0}{kI_1(ka)(k^2-l^2)}\left(\omega(k^2+l^2)-\frac{k^2\sigma_0{}^2}{2\varepsilon_0\mu_1}\right)I_0(kr)e^{ikz+\omega t}\\
\Psi_1&=-\frac{ik\eta_0}{I_1(la)(k^2-l^2)}\left(2\omega-\frac{\sigma_0{}^2}{2\varepsilon_0\mu_1}\right)rI_1(lr)e^{ikz+\omega t}
\end{aligned}$$

where l represents a wave number defined by $l^2=k^2+\dfrac{\omega}{\nu_1}$.

The stream function and the velocity potential give the liquid velocity and the pressure p_1 in the liquid can then be found from the relation :

$$p_1=-\rho_1\frac{\partial\Phi_1}{\partial t}$$

The gas pressure p_2 is found from the linearized inviscid equations of the motion of the gas :

$$\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}(rv_2)+\frac{\partial u_2}{\partial z}=0\\
&\frac{\partial v_2}{\partial t}+U_2\frac{\partial v_2}{\partial z}=-\frac{1}{\rho_2}\frac{\partial p_2}{\partial r}
\end{aligned} \tag{6}$$

$$\frac{\partial u_2}{\partial t} + v_2 \frac{\partial U_2}{\partial r} + U_2 \frac{\partial u_2}{\partial z} = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial z}$$

The gas phase boundary conditions require that $v_2 = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial z} U_2$ at $r = a + \eta$ and that u_2, v_2, $p_2 \to 0$ as $r \to \infty$.

We define the stream function of the gas phase as :

$$\Psi_2 = \left[U_2(r) - i\frac{\omega}{k} \right] \eta f(r)$$

Introducing this stream function in the gas equation gives a differential equation for f. This equation can be solved if the boundary conditions for the gas are adapted to the problem of f. For our case of slip at the gas-liquid interface, we assumed that $U_2(r)$ is constant. The solution for f is:

$$f(r) = r\frac{K_1(kr)}{K_1(ka)}$$

The gas pressure at the jet surface is given by:

$$p_2 = -\rho_2 \eta \left(U - i\frac{\omega}{k} \right)^2 k \frac{K_0(ka)}{K_1(ka)}$$

The solutions of the gas and liquid pressure are injected into Eq. (3) and the following dispersion relation between ω and k is obtained for the case of a charged jet:

$$\omega^2 + 2\nu_1 k^2 \omega \left[\frac{I_1'(ka)}{I_0(ka)} - \frac{2kl}{l^2+k^2}\frac{I_1(ka)}{I_0(ka)}\frac{I_1'(la)}{I_1(la)} - \frac{\sigma_0^2}{4\varepsilon_0 \mu_1 \nu_1}\frac{1}{l^2+k^2} \right] =$$
$$\frac{\sigma k}{\rho_1 a^2}\left(1-k^2a^2\right)\left(\frac{l^2-k^2}{l^2+k^2}\right)\frac{I_1(ka)}{I_0(ka)} + \frac{\rho_2}{\rho_1}\left(U - i\frac{\omega}{k}\right)^2 k^2 \left(\frac{l^2-k^2}{l^2+k^2}\right)\frac{I_1(ka)K_0(ka)}{I_0(ka)K_1(ka)} \tag{7}$$
$$+\frac{\sigma_0^2 k^2}{\varepsilon_0 \rho_1}\left[\frac{k^2}{l^2+k^2}\frac{I_1'(ka)}{I_0(ka)} - \frac{kl}{l^2+k^2}\frac{I_1(ka)}{I_0(ka)}\frac{I_1'(la)}{I_1(la)} + \left(\frac{l^2-k^2}{l^2+k^2}\right)\frac{I_1(ka)}{I_0(ka)}\left(\frac{K_1(ka)}{K_0(ka)} - \frac{1}{ka}\right)\right]$$

Equation (7) is exactly the one obtained by Reitz and Bracco with two additional terms. These terms correspond to the interaction between the viscosity, the surface tension and the electric forces.

If we write the dispersion relation in non dimensional form we obtain :

$$\beta^2\left(1 + F_0\right) + 2Zk^2a^2\beta\left(F_1 - E_1F_1'\right) = ka\left(1 - k^2a^2\right)F_2 + We_2\, k^2a^2F_3 + E_2F_4 \tag{8}$$

where $\beta = \omega_R \sqrt{\rho_1 a^3/\sigma}$, $Z = \mu_1 / \sqrt{\rho_1 \sigma a}$, $We_2 = \rho_2 U^2 a/\sigma$, $E_2 = \sigma_0^2 a/\varepsilon_0 \sigma$ and $E_1 = \sigma_0^2 a^2 \rho_1^2 / \varepsilon_0 \mu_1^2$. Z is the Ohnsorge's number and We_2 is the Weber's number. The F_i are dimensionless ratios of Bessel functions and wave numbers.

3. THE DISPERSION EQUATION

In this section we study the effects of the physical parameters (surface charge density, surface tension and the kinematic viscosity) on the stability of the jet. We work on diesel oil. The influence of a parameter is destabilizing if an augmentation of its value provokes an increase of the amplitude of the disturbance (through the increase of the growth rate). The following figures show the behaviour of the dimensionless growth rate β as a function of the dimensionless wave number ka for different values of the physical parameters.

Figure 1 shows the influence of the injection of electric charges into the liquid on the stability. We solve the dispersion equation for three different cases : $\sigma_0 = 0$ C/m^2, $\sigma_0 = 1.0\ 10^{-4}$ C/m² and $\sigma_0 = 3.0\ 10^{-4}$ C/m².

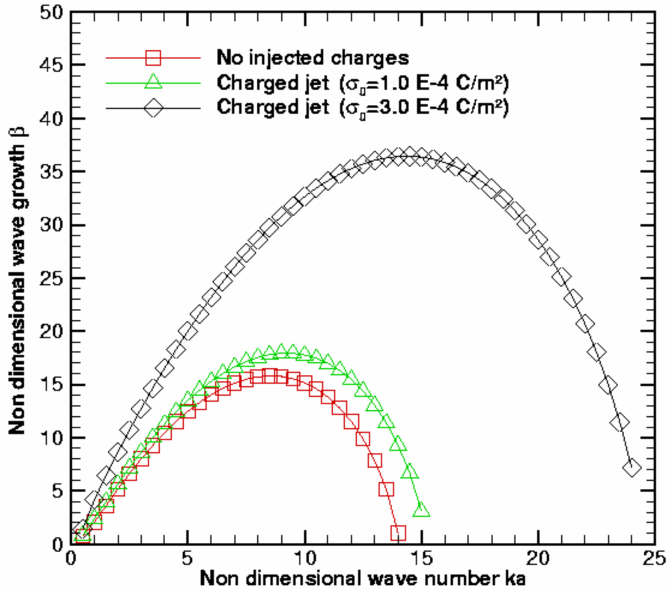

Figure 1: β as a function of ka for different surface charge densities.

We can observe that for a fixed wave number the growth rate of the charged jet is always higher than the one of non charged jet. This means that for a fixed wave number the disturbance grows faster with the electric charges than without and that the droplets corresponding to this wave number appears earlier in the jet. An injection of electric charges in the jet (all the other parameters remaining unchanged) induces smaller primary droplets. Moreover figure 1 shows that when ka>15 we do not have droplets if the jet is not electrically charged whereas we have some if it is charged.

We can also see that the maximum of the curves is always higher when the jet is electrified. The wave number of the maximum is the one which gives the first droplets. It is higher with a charged jet than with a non charged jet, which means that it will give smaller droplets. From these results it is clear that when we electrify a diesel oil jet we better atomize it.

The next case gives the influence of the surface tension on the stability of an electrically charged jet. The surface electrical charge density has been fixed at 3.0 10^{-4} C/m². We solve the dispersion equation for three different values of the surface tension (32 10^{-3} N/m for oil, 50 10^{-3} N/m for diesel oil and 73 10^{-3} N/m for water). We can see on figure 2 that the most destabilised jet is the one with the lowest surface tension. It is well known that we have the same kind of results with a non charged jet. The surface tension stabilizes the jet shall it be charged or non charged even if the surface tension and the electric charges act in two opposite ways. Additional calculus have shown that we obtain the same curve with a charged jet (surface tension σ and charge density σ_0 =3.0 10^{-4} C/m²) and a non charged one if the surface tension of the non charged jet is σ/2.

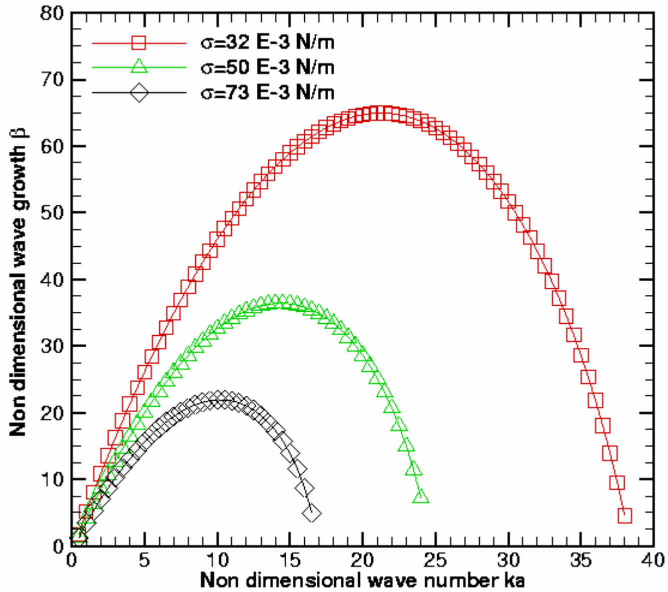

Figure 2: β as a function of ka for different σ.

The last case gives the influence of the kinematic viscosity on the stability of an electrically charged jet. The value of the surface charge density is still 3.0 10^{-4} C/m². We solve the dispersion equation for three different viscosities (1 10^{-3} kg/m s for water, 2.2 10^{-3} kg/m s for diesel oil and 70 10^{-3} kg/m s for oil).

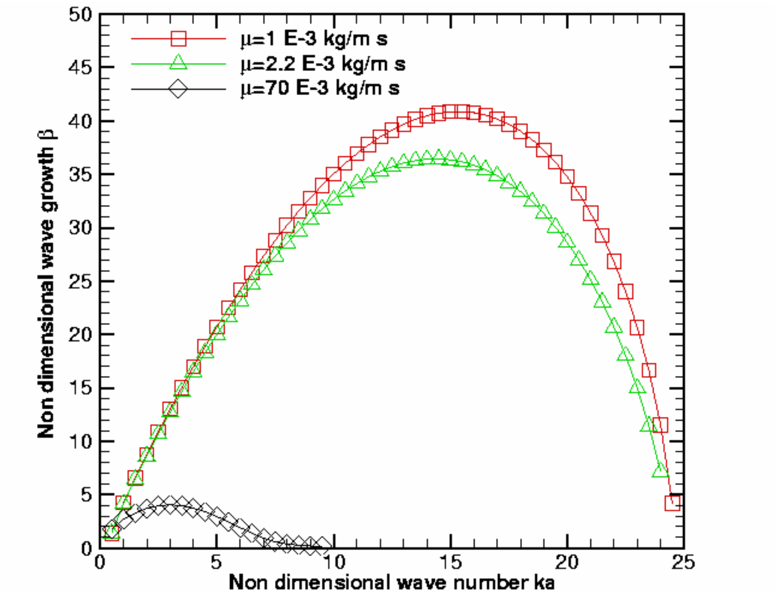

Figure 3: β as a function of ka for different kinematic viscosities.

In figure 3 we can see that the kinematic viscosity, like the surface tension, stabilizes the jet. An increase of viscosity leads to a decrease of the maximum of *ka*.

Finally we can say that the global effect of the physical parameters is the same in both cases (non charged or charged jets) but that the injection of electric charges destabilizes the jet in all the cases and gives best atomisation.

4. EXPERIMENTS

In the experimental device the injector comprises a needle connected to the high voltage source and a grounded counter electrode. The liquid which is electrified inside the injector goes through the orifice of the nozzle whose diameter is 500µm. It arrives in a collector vessel and discharges there.

To observe the influence of the injected charges on the jet a Malvern's Spraytec system has been used. The following results show the influence of the electrification on the droplet diameter distribution for 120 bars. The measurements are done at 20 mm from the nozzle in the centre of the spray.

In figure 4 we have the histograms of the diameters with the charges (on the right) and without the charges (on the left). On each graph the scale on the right gives the volume of the classes of the diameters in percent and the scale on the left the cumulate volume distribution (curve) of the diameters of the droplets in percent.

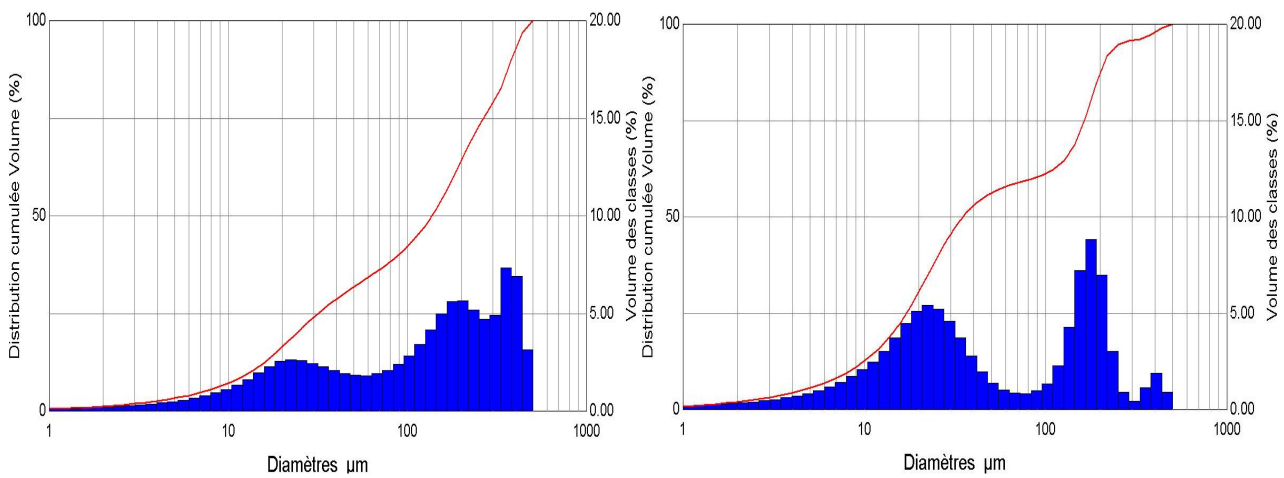

Figure 4: Size distribution for 120 bars, 0 kV (left) and -25 kV (right).

We can observe in figure 4 that the volume occupied by the biggest droplets (more than 200 µm) decreases because of the application of the electric potential and that the volume occupied by the classes between 20 µm and 30 µm increases at the same time. The injection of the electric charges in the diesel oil diminishes the diameter of the droplets globally.

In figure 5 each curve represents the evolution in time of different diameter Dv. For instance Dv [70] = 152.19 µm means that 70% of the entire measurements volume is occupied by less than 152.19 µm diameter droplets. We clearly see in figure 5 that the application of the electric potential provokes a discontinuity discontinuity of all the parameters Dv. For example the injection of the electric charges makes Dv[70] changing from 375 µm to 150 µm. 70% of the measurement volume occupied by less than 375 µm diameter droplets without the charges is occupied by less than 150 µm diameter droplets when the charges are injected.

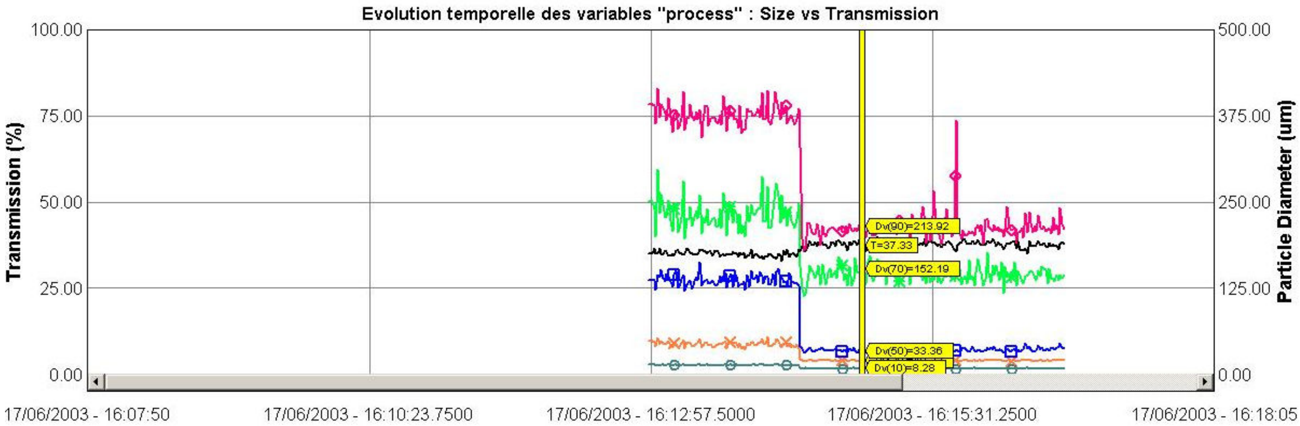

Figure 5: Temporal evolution of different mean diameters of droplets.

The injection of electric charged in the jet diminishes the diameter of the droplets globally.

5. CONCLUSION

Charging jets electrically is one efficient process that leads to a better atomization. The theoretical analysis has shown that smaller droplets appear with the injection of charges into the jet. This result has been confirmed with experiments done on jets electrified by injection of charges. From Malvern granulometry measurements we have shown that for a negative applied potential on the needle, the volume of the biggest droplets decreases and the volume of the smallest droplets increases. The jet because of the electrification is globally better atomized.

NOMENCLATURE

a	liquid jet radius (m)	r	radial coordinate (m)
Dv	diameter of droplets (m)	t	time (s)
$E_{1,2}$	dimensionless number	u	axial velocity component (m s^{-1})
$F_{1,2,3,4}$	dimensionless ratios of Bessel functions	v	radial velocity component (m s^{-1})
k	wavenumber (m^{-1})	We_2	Weber number (dimensionless)
l	wavenumber (m^{-1})	z	axial coordinate (m)
p	pressure (kg m^{-1} s^{-2})	Z	Ohnsorge number (dimensionless)
β	dimensionless wavegrowth	σ	surface tension (kg s^{-2})
η	surface wave amplitude (m)	σ_0	initial surface charge density (C m^{-2})
λ	wavelength (m)	Φ	velocity potential
μ_1	liquid viscosity (kg m^{-1} s^{-1})	Ψ	stream function
$\rho_{1,2}$	density (kg m^{-3})	ω_R	real growth rate (s^{-1})

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