

# **STATISTICAL EVALUATION OF DYNAMICS AND COHERENCE BREAKDOWN OF KEROSENE AND WATER JETS IN CROSSFLOW**

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## **ABSTRACT**

The paper presents results of experiments conducted by injecting kerosene and water in a premixing channel swept by a high-speed, high-pressure airflow, reproducing operating conditions typical of the premixing duct of LPP gas turbines. Extensive ranges of air and liquid velocity were explored using a flash shadowgraphy technique. An image statistical analysis procedures, which represent the refinement of a technique already proposed by the same authors, provided a useful tool for assessing jet trajectory and breakdown location, i.e. the point where the liquid column loses its coherence, collapsing in large fragments, which are easily captured by the local turbulent airfield and quickly aligned to the airflow. Results indicate that the breakdown location depends on liquid-to-gas velocity ratio and liquid surface tension. Empirical correlations are proposed for both transverse and downstream distance of the fracture point from the injection point. These two quantities were used to normalize the jet trajectories and it was observed that normalized trajectories group in a very narrow band. Since the dependence on operating conditions and liquid properties seems to be wholly embedded in the breakdown location components, the expressions found for these quantities act as a renormalizing factor in the mathematical formulation of a very simple empirical correlation for the trajectory of a liquid jet in crossflow.

## **INTRODUCTION**

The lean pre-vaporized and premixed (LPP) gas turbine technology is an increasingly used technology capable of achieving a good compromise between the demand for reliable heavy-duty engines and low NO<sub>x</sub> emissions. The main features of these systems are the use both of a large amount of air, exploiting its thermal inertia to contain the temperature and, as a consequence, the NO<sub>x</sub> emissions, and of largely premixed burning conditions implying higher efficiencies. A successful implementation of such technology in the case of liquid fuels relies on the development of efficient and stable fast premix systems capable of dispersing, evaporating and mixing the fuel in the airflow in small time and space intervals. In fact, the efficiency of these initial stages of the process reflects on the overall performance and stability of the system as well as on the pollutant emission. Since at the actual pressures and temperatures the ignition delay time is in the order of millisecond, characteristic time and space of atomization process represent a limiting step in achieving a satisfying level of mixing before ignition occurrence. Therefore, the design of spraying system must satisfy the constraint of the highest possible effectiveness and reliability. A common technique to achieve these results makes use of one or more plain nozzles injecting liquid fuel jets perpendicularly to the high-temperature/high-pressure air current flowing in the premixing channel. The quality of the resulting spray is expected to be enhanced by the strong interaction of the jets with the airflow whose effectiveness is even more increased due to the high air density and velocity. In addition, a good design of the jet/air-flow coupling could allow for the exploitation of the strong air dragging forces in promoting an effective dispersion of fuel droplets in the premixing duct improving vaporization and mixing process. Finally, these injection schemes make use of very simple plain nozzles, assuring the highest reliability of the spraying system.

In spite of the large number of works available in literature dealing with the air crossflow atomization only a few experimental studies were conducted on facilities reproducing conditions typical of the LPP gas turbines in the last years. Most of the experimental data are relative to distilled water jet injected in an air crossflow at pressures just above the atmospheric one [1-4]. Mazallon et al [5] studied the behavior of a liquid jet under non-turbulent crossflow conditions. Chen et al [6] studied the injection of a liquid fuel in the premixing channel at a pressure of about 2 bars. More recently Ragucci et al. [7-10] studied the effect of the high pressure on a water jet in crossflow. Becker and Hassa [11, 12] presented measurements relative to kerosene jets in relatively high pressure conditions.

The complexity of the phenomena involved and the scarce availability of experimental data makes the current degree of understanding of the interaction between a liquid jet and a gas crossflow largely unsatisfactory. This reflects also on the elaboration of thoroughly validated mathematical and/or numerical sub-models of the jet atomization and mixing processes to be implemented in the fluid-dynamic codes used in the design of new generation devices. One of the focal point in the elaboration of a satisfactory mathematical description of the jet atomization and droplet dispersion concerns the behavior of the liquid column emerging from the nozzle outlet in consequence both of its internal fluid-dynamic and of its interaction with the gaseous flow. In the near field the presence of a compact liquid core and a dense cloud of drops and irregular ligaments makes very difficult a successful investigation of the jet by means of traditional optical and laser diagnostic techniques. Oda et al. [2] reported images of the spray, collected by using a laser tomography technique, showing the light scattering of a bow-shaped cloud of drops, and suggested that bow-shaped

could be the liquid jet cross-section too, as the result of the drag forces acting on the jet analogously to the case of liquid drops flattened and then bended at the edges by an airflow. The seeming thickening of the liquid jet along its evolution, shown by the vertical tomograms presented by Oda et al. [2] and the flash shadowgraphs by Ragucci et al. [7], could be thought as an effect of the kidney shape deformation of the jet as well as an effect of fast asymmetric oscillations, already proposed to describe the behavior of strongly turbulent liquid jets in still or co-flowing gas [13]. Relying on a supposed similarity with gaseous jets in crossflow, Kihm et al. [3] described the dynamics inside the liquid phase as a couple of counter-rotating vortices giving out a kidney shaped cross section of the jet, no matter there is neither direct nor indirect evidence of that.

The experimental data presented by Wu et al. [4] suggest a similarity between the column breakup mechanisms of the jet and the aerodynamic breakup regimes observed for a spherical droplet. Depending on the value of the Weber number, defined as the aerodynamic to capillary pressure ratio, bag, multimode and shear breakup could be observed also for liquid jets. Perhaps, in the gas turbine typical conditions the Weber number should always be so large that only shear breakup is expected to settle. Depending on both the values of the Weber number, computed relatively to the gas phase, defined as  $We_G = \rho_G V_G^2 D / \sigma$ , and the liquid-to-air momentum ratio, defined as  $q = (\rho_L V_L^2) / (\rho_G V_G^2)$ , an intense surface breakup can be observed even before the growth of waves on the windward profile causes the column breakup. Becker and Hassa [12] distinguished between column and surface breakup conditions: the former appearing as a sudden collapse of the liquid column coherence due to the growth of large surface waves up to an amplitude of the same order of magnitude of the jet thickness; the latter showing a gradual mass removal from the liquid surface probably due to the development of instabilities with higher frequency and smaller amplitude. In agreement with Wu et al. [4], Becker and Hassa [12] found that the column breakup mechanism dominates when either the Weber number or  $q$  is small. In the gas turbine reference conditions the Weber number is usually quite large, due both to the high speed and high pressure of the airflow and to the low surface tension of fuels. Furthermore the  $q$  number cannot be too small, because in this case the spray would not fill the whole cross section of the premixing channel influencing mixture uniformity, so that it can be expected that in such conditions surface breakup dominates.

The penetration height is another important parameter because the jet should be able to shed droplets across the whole section of the channel as homogeneously as possible avoiding impingement onto the opposite wall. Several authors [1][4] found out that jet penetration can be correlated to the square root of the  $q$  number. Wu et al. [4] also state that the surface tension seems to have negligible effects on the penetration height, whilst it affects strongly the atomization rate and the drop size.

This paper presents the results of experiments conducted by injecting kerosene and water in a premixing channel swept by a high speed, high-pressure airflow. Extensive ranges of air and liquid velocity were explored using a flash shadowgraphy technique. The development of completely automatic procedures for the statistical analysis of acquired images, already introduced by Ragucci et al. [7], provided a useful tool for assessing both the jet trajectory and position of the breakdown point, i.e. the point where the liquid column loses its coherence, collapsing in large fragments, which are easily captured by the local turbulent airfield and quickly aligned to the airflow. Breakdown cross-stream and downstream distances are correlated, for a large Weber number value range, to the basic parameters controlling the process: initial velocity of the two phases, static pressure and liquid properties. Finally, an empirical correlation is presented for the description of a generalized jet trajectory.

## EXPERIMENTAL SETUP AND IMAGE ANALYSIS PROCEDURES

An experimental facility was designed to reproduce geometry and operating conditions of the premixing channel of a LPP gas turbine engine. The test rig consisted of a fully accessible chamber with a square cross section of 25x25 mm, capable to resist to high pressures, up to 10 MPa, and high temperatures, up to 1000 K. Three side walls of the duct hosted quartz windows for the optical access, while on the fourth side is mounted the injector with the axis normal to the channel one. The injector is a plain nozzle with a recessed hole of 500  $\mu\text{m}$ . A 45° taper introduces the liquid flow to the terminal straight section of the nozzle having an L/D ratio equal to 4. The discharge coefficient of the nozzle was preliminary determined and resulted to be 0.69. The liquid was supplied to the nozzle by means of a nitrogen-pressurized vessel and regulated by a pressure control valve. That system allowed a precise control of the liquid velocity in the field 15÷50 m/s. For all the experiments the airflow pressure was set at 1 MPa (gauge) and room temperature. The gas flow rate and the air-flow velocity were regulated by using a variable area diaphragm mounted at the end of the channel. A Pitot tube provided a single point measurement of the air velocity in the channel center.. Air velocities in the range 21÷53 m/s were exploited in the present work.

The diagnostic setup was a simple shadowgraphic scheme using a low-pressure xenon flash lamp and a Pulnix 8-bit digital camera with maximum resolution of 640x480 pixels and minimum shutter time equal to 1:32000 s. Timebase generation and synchronization between these devices is managed by a BNC delay generator. An IMAQ PCI-1422 acquisition board from National Instruments was used to store the collected images. The sampling rate was fixed to the camera maximum, i.e. 240 Hz, the flash lamp pulse duration being equal to 15  $\mu\text{s}$ . For each test condition a 1000 frame sample was collected. The implementation of image statistical analysis techniques allowed getting information on the jet trajectory and breakdown, as shown in the following section. The need for a quantitative characterization of jet behavior from the sampled images required the implementation of a set of acquisition and elaboration procedures, written in LabVIEW programming environment and based on a standard library of image manipulation algorithms (NI-IMAQ). These techniques were already introduced and discussed by Ragucci et al. [9], but in the present work the larger amount of available images provided a more statistically relevant sample, improving the performance of the analysis tools. For each measurement a background image was collected and then used to clean up the spray images, reducing the electronic noise. The spray windward profile was extracted by applying standard image morphological algorithms based on the evaluation of the maximum light dampening gradients. That profile was assumed as representative of the trajectory of the liquid jet. The average image was exploited by a further procedure, which firstly assessed the jet trajectory as the average of the 1000 trajectories obtained from single frames. The second step was to calculate the square average shift between single frame and average image trajectories, as a function of the transverse distance  $z$  from the nozzle outlet:

$$S(z) = \frac{1}{N} \sqrt{\sum_i [x_i(z) - x_M(z)]^2} \quad (1)$$

In Eq. (1)  $x_i(z)$  and  $x_M(z)$  represent the position, along the axis parallel to the airflow, of the single frame and average image trajectory respectively.

As shown in figure 1, the  $S$  profiles versus the distance from the nozzle,  $z$ , can present two different behaviors, depending on test conditions. Initially  $S$  grows with a moderate gradual slope, afterwards it can either remain always below 1.5 mm (*type I behavior*) or present at a certain  $z$  value a sudden increase up to values about 10 mm (*type II behavior*). The moderate gradual increase, observed in both cases, can be interpreted as the effect of the small jet fluctuations caused by the turbulent interaction of the two phases. The explanation of the sudden increase in the curves, observed in the second set of profiles, requires a more detailed argumentation. The increase is very steep and occurs in a very limited space interval. Typical profiles registered an increase of one order of magnitude in less than 2 mm of the transverse distance  $z$ . The position of this profile knee changes changing the operating conditions. It was observed that this peculiar phenomenology was only observed when the kinetic content of the liquid phase was sufficiently low to prevent jet impingement onto the opposite wall of the channel without significant bending. On the ground of an analogous statistical analysis of 200 images collected for each test condition, Ragucci et al. [9] proposed that the rapid boost of  $S$  occurs only when the compact structure of the liquid column collapses in large fragments and a coherent jet no longer sustains the spray. This loss of coherence was referred to as *jet breakdown*, and it corresponds to the point after which the jet does not behave as a continuous liquid column. A speculative interpretation of the  $S$  profiles relies on the hypothesis that the liquid column underneath the visible plume of drops works such as a skeleton for the spray, governing its overall shape and preventing it undergo too large fluctuations due to the aerodynamic turbulence. So as long as there is a liquid coherent structure opposing resistance to the drag force exerted by the airflow, the index  $S$  registers small fluctuation in the spray windward profile. After the jet breakdown point the jet loses its coherence, probably due to the rupture of the liquid column, and the airflow easily drags residual liquid structures, either liquid ligaments or drops. In the following the position of the point where the sudden increase in the  $S$  profiles could be observed, with respect to the nozzle outlet, will be indicated by means of a breakdown transverse and downstream distance, respectively referred to as  $z_{jb}$ . The existence of a *jet breakdown* is relevant also in the modeling of jet behavior, since the mathematical description of a jet bended by a transverse airflow is obviously quite different from the one appropriate to describe the interaction of drops or insulated liquid fragments in the same conditions.

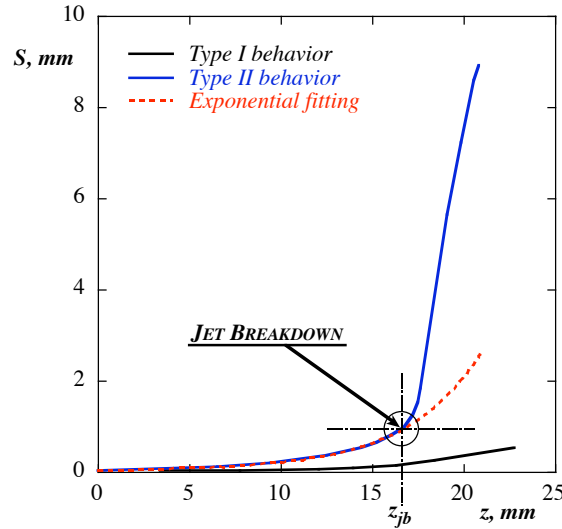


Fig. 1: Outline of the automatic procedure for the individuation of the jet breakdown point

Relying on a more statistically relevant sampling than in [9], a fully automatic procedure was developed for the determination of the jet breakdown point. This procedure is based on the experimental observation that for every test condition, up to the occurrence of the sudden increase, the behavior of  $S(z)$  is regular and well represented by an exponential curve, as shown in figure 1. Even though the parameter of the exponential law depend on the experimental condition, it was possible to build up an automatic routine for the determination of the best fitting exponential curve for each measurement. A standard statistical index was used to individuate the point on the jet trajectory after which the gap between exponential curve and  $S(z)$  becomes larger than a chosen tolerance. The coordinates  $x_{jb}$  and  $z_{jb}$  of that point are defined respectively as downstream and transverse distance of the jet breakdown point from the injection point.

In the following section empirical correlation are proposed as an analytical formulation of the jet features dependence on controlling parameters. Those equations also contain terms accounting for the effect of surface tension. Aiming to a dimensionless formulation of the problem, the scaling of this properties was referred to a baseline case characterized by the injection of liquid water, so that the reference surface tension is defined as  $\sigma_w = 0.0729$  N/m.

## RESULTS AND DISCUSSION

Results obtained by means of the statistical analysis tools presented in previous section are reported in figure 2. The plot on the left hand portraits values of  $x_{jb}$  for kerosene and water jets, normalized to the nozzle outlet size  $D$ . The following equation was found to fit well the experimental data:

$$\frac{x_{jb}}{D} = 25.6 \sqrt{\frac{\sigma_w}{\sigma} \frac{V_L}{V_G}} \quad (2)$$

The right hand plot of figure 2 reports data for  $z_{jb}$ , normalized to  $D$  plotted as a function of the ratio of liquid-to-gas velocity, for both water and kerosene jets:

$$\frac{z_{jb}}{D} = 45.5 \frac{V_L}{V_G} \quad (3)$$

The agreement with experimental data for Eqs. (2) and (3) was evaluated by means of the Pearson's correlation coefficient  $R$ , that resulted to be equal to 0.91 for both the proposed equations indicating a good fitting, as it can be clearly see by figure 2. The plot for  $x_{jb}$  shows a segregation of data for kerosene and water. This fact is simply due to the choice to experimentally explore the same range of  $V_L$  and  $V_G$  for both liquids, so that the abscissa interval results scaled by the surface tension.

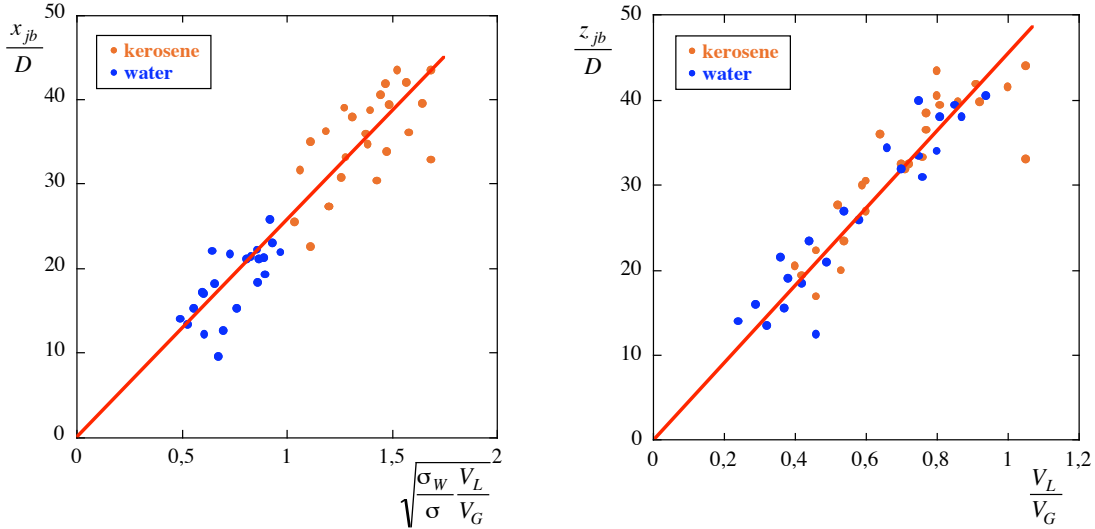


Fig. 2: Jet breakdown transverse (*left*) and downstream (*right*) distance

Wu et al. [4] suggested that  $z_{jb}$  is a function of the square root of the  $q$  number. Here a similar dependence is proposed, even though the densities of the two phases seem to have no role on the determination of the breakdown point, resulting in the extremely simplified formulation of Eq. (3). It must be stressed that the work by Wu et al. should not be considered definitive as regards the effect of densities. All the test conditions have in common the gas pressure, and liquid densities vary in a quite narrow range, as well as the present work, so that it is somewhat questionable to assess a well defined dependence on either air or liquid density based on those (or present) data.

As regards the dimensionless downstream distance  $x_{jb}/D$ , Wu et al. supposed a constant value 8.06. Measurements collected for the present work show a less than linear dependence on the velocity ratio, corrected by the square root of the surface tension. Differently from the breakdown transverse distance, the value of  $x_{jb}$  is then sensitive to the atomization process. Changing the injected liquid does not affect the cross-stream penetration of the jet, but the higher is the surface tension the weaker is the expected bending. The divergences from the Wu et al. seem to be attributable to the different atomization level. At 0.14 MPa air pressure used by Wu et al. even liquids with surface tension as low as kerosene seemingly do not have intense mass removal, so that it is possible to suppose that both the influence of surface tension on jet dynamics and the variability of  $x_{jb}$  become evident only by increasing the gas density level and so the atomization intensity. Finally, even though the basic definition of column fracture point is analogous, criteria adopted by Wu et al. to individuate that point are not the same as criteria proposed in this paper to define the automatic procedure for the search of jet breakdown point.

The measured jet breakdown distances were used to normalize the jet trajectories, aiming to study their shape independently from the position of the breakdown. The resulting trajectories for kerosene and water jets are reported separately in figure 3 in grayed colors. All the normalized trajectories appear to follow the same trend whatever are the values of the controlling parameters, suggesting that the position of the jet coherence breakdown point embed all the information about the effect of initial velocities and static pressure. The solid color line in figure 3 represents the average trajectory. For the normalized jet trajectory the standard deviation profile was evaluated along the  $x$  direction and reported as error bar plot in figure 3. For kerosene jets the average percent error is 2.8%, while for water it is 6.3%. The wider spread of water jet trajectory does not seem to represent a hurdle to the individuation of a generalized trajectory. The proof that this spread is attributable only to experimental noise is presented in figure 4, which clearly shows that the average trajectories, extracted by the two plots in figure 3 for both kerosene and water, coincide. As a consequence the normalized jet trajectory is not even dependent on liquid properties, so that a generalized analytical expression can be proposed for the dynamics of a liquid jet in crossflow from the injection up to the coherence breakdown point:

$$\frac{z}{z_{jb}} = \left( \frac{x}{x_{jb}} \right)^{0.35} \quad (4)$$

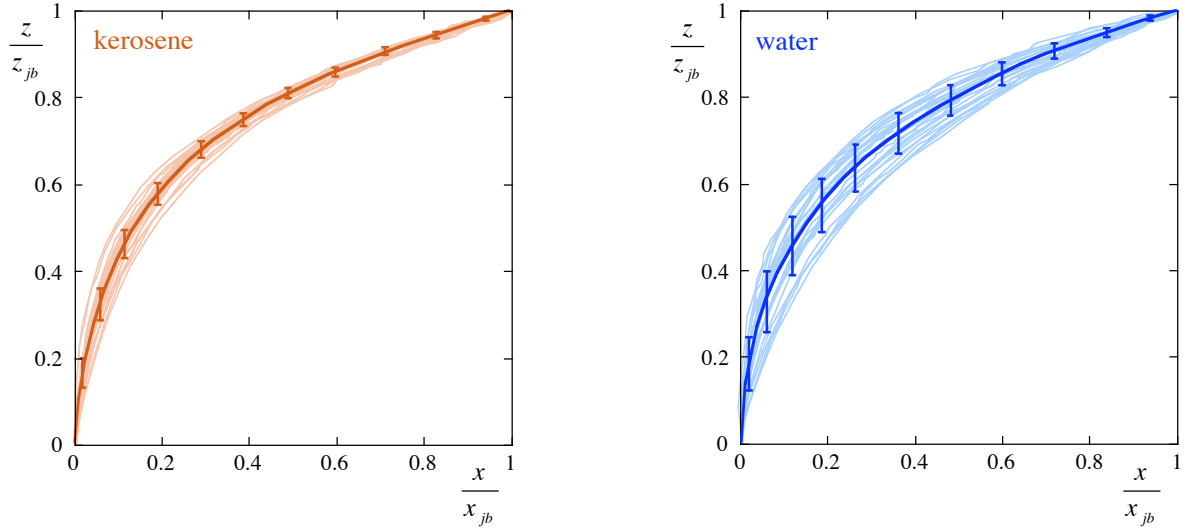


Fig. 3: Normalized jet trajectories for kerosene (*left*) and water (*right*). Gray lines stand for experimental trajectories. Black line is the average trajectory. Error bars represent the standard deviation.

The continuous line in red in figure 4, reproducing the suggested behavior, shows a good but not excellent agreement with the experimental curves. The evaluation of the Pearson's correlation coefficient provides a value of 0.954 for kerosene and 0.963 for water.

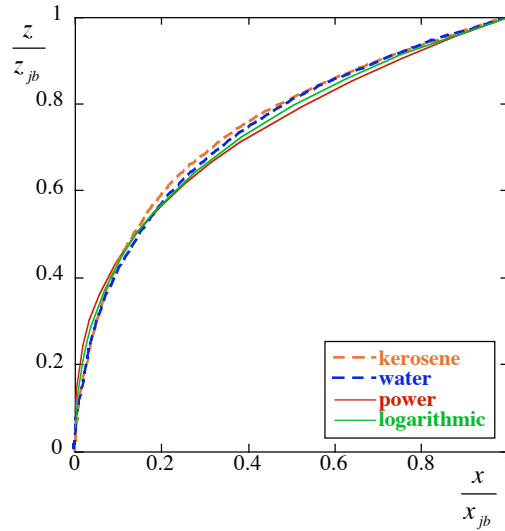


Fig. 4: Generalized trajectory for a liquid jet in crossflow

The fitting could be improved by resorting to a logarithmic dependence, as suggested by Becker and Hassa [11]. The equation giving the best results was found to be the following:

$$\frac{z}{z_{jb}} = \ln \left[ 1 + (e - 1) \left( \frac{x}{x_{jb}} \right)^{0.5} \right] \quad (5)$$

This equation is only apparently complex, due to the need for ranging between points (0,0) and (1,1). Actually the only parameter to be evaluated is the exponent of  $x/x_{jb}$  and it was found to be 0.5. In this case the correlation coefficient is 0.989 for kerosene and 0.987 for water. It must be stressed that the moderate increase in the agreement with the experimental data does not seem to justify the increase in the complexity of the mathematical formulation.

## CONCLUSIONS

The application of statistical image manipulation techniques to a large number of experimental tests on kerosene and water jets allowed for the determination of some relevant correlations between the controlling parameters and jet breakdown process. Due to unfeasibility of a direct determination of the jet fracture length a statistical determination of the point where the jet lacks its coherence and its trajectory assume a very strong oscillatory behavior was implemented. One of the most appealing features of the developed tool is that it is based on a rigorously defined procedure of elaboration of the experimental data, avoiding any subjective

and then non-repeatable interpretation of the acquired images. The values of jet breakdown location could be directly correlated to the airflow and initial jet velocities, with a simple linear relationship in the case of the cross-stream distance  $z_{jb}$  and with a square root correlation for the stream-wise distance  $x_{jb}$ . The prediction of the latter quantity also requires accounting for the influence of the atomization process on the compact jet dynamics, by means of the liquid surface tension. It has been also shown that, although the strong complexity of the crossflow atomization process, at least the trajectory followed by the liquid jet up to the coherence collapse is connected to the controlling parameters only through the two coordinates of the breakdown point. The normalized trajectories indeed appear to be independent on both velocity parameters and liquid properties. As a consequence a generalized simple functional form was proposed to correlate the two trajectory dimensionless components of a liquid jet in crossflow. By combining eqs. 2-4 the following general formulation is obtained:

$$\frac{z}{D} = 1.78 \left( \frac{\sigma}{\sigma_w} \right)^{0.5} \left( \frac{V_L}{V_G} \right)^{0.65} \left( \frac{x}{D} \right)^{0.35} \quad (5)$$

Eq. (5) can be directly adopted as an empirical tool to predict the evolution of the liquid jet. Since this kind of empirical correlations fail when one attempts to apply them for substantially different operating conditions, the proposed equation should be thought of as a summa of the experimental observation providing the necessary input, about functional dependencies, to the modeling research. The future development of physically consistent models should seek in the empirical study useful indications about the role of the several elementary mechanisms involved in the crossflow atomization problem. From the present study it is clear that even the modeling of liquid jet dynamics cannot neglect the presence of the atomization process. Task of the modeling study is to deduce from physical principles the contribution of the various phenomena, making it possible to generalize the experimental evidences into a large-scale exploitable predictive tool.

## NOMENCLATURE

$x$	downstream direction axis	mm	<b>Greek symbols</b>		
$z$	transverse direction axis	mm	$\rho$	density	kg/m3
$D$	nozzle outlet diameter	mm	$\sigma$	surface tension	N/m
$V$	velocity	m/s	<b>Subscripts</b>		
$q$	liquid-to air momentum ratio	dimensionless	$jb$	jet breakdown location	
$We$	Weber number	dimensionless	$i$	single frame value	
$S$	square average shift	mm	$M$	average value	
$N$	number of samples for each measurement		$G$	gas phase	
			$L$	liquid phase	
			$w$	water	

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