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NUMERICAL STUDY ON BINARY DROPLET COLLISION CONSIDERING THE
COLLISION-INDUCED BREAKUP AND THE EFFECTS OF LIQUID PROPERTY

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ABSTRACT The main purpose of the present study is to perform numerical study on the binary droplet collision considering the collision-induced breakup and the effects of liquid property. In this study, the collision model proposed in our previous study is used for simulation of collision-induced breakup, and we modify this model to consider the effect of liquid property on the behavior of droplet-droplet collision. This collision model is based on the conservation laws for mass, momentum, and energy between before and after collision and provides several formulae for post-collision characteristics of colliding droplets and satellite droplets. Improving the accuracy of the model, in this study, appreciate criterion for bouncing collision is added and dissipation energy during collision process is newly modeled. To validate the new model, numerical calculations are performed and their results are compared with experimental data published earlier for binary collisions of water, propanol, and tetradecane droplets. It is found from the results that the new model shows good agreement with experimental data for the number of satellite droplets. It can be also shown that the predicted mean diameter by the new model decrease with increasing the Weber number because of the collision-induced breakup, whereas the O'Rourke model fails to predict the size reduction via the binary droplet collision.

Keywords: Collision-Induced Breakup, Bouncing Collision, Satellite Droplet, Mutual Impinging Spray

1. INTRODUCTION

Collision dynamics of liquid droplets is important in evolution of sprays used in various industrial applications. In dense spray system and inter-spray impingement system [1, 2], droplet collision can significantly affect the spray characteristics, such as drop size and velocity distribution. The outcomes from binary droplet collision are classified four different types: bounce, coalescence, reflexive separation, and stretching separation, as seen in Fig. 1. Many researchers [3-5] observed that the separations produce the satellite droplets from the interacting parts between two colliding droplets, and in turn results in the size reduction in droplets (see Fig. 1(c) & 1(d)). This procedure is called a collision-induced breakup process whose effects are more pronounced as Weber number increases. In addition, it is well known that the droplet collision phenomena highly rely upon the droplet property. By the contrast in the collision of water droplets, the bouncing phenomena occur over the wide range of Weber number, and also the surface tension energy loss becomes more significant in the collision of hydrocarbon fuel droplets.

In most of numerical simulation on spray dynamics, the model of O'Rourke [1] has been commonly used for the droplet collision process. However, there are some problems that the O'Rourke model includes only coalescence and grazing separation. It means that the O'Rourke model cannot mimic other important processes related to the binary collision, e.g., reflexive and stretching separations, and corresponding formation of satellite droplets. Moreover, because the O'Rourke model was derived on the basis of binary collision of water droplets, it

may not be suitable for calculation of collision processes of fuel droplets. Recently, we [6, 7] proposed the new droplet collision model considering droplet collision-induced breakup process with the formation of satellite droplets. This model consists of several equations to determine the post-collision characteristics of colliding droplets and satellite droplets. These equations were derived from the conservations of droplet mass, momentum, and energy between before and after collision, and made it possible to predict the number of satellite droplets, and the droplet size and velocity in the analytical way. In this model, however, the liquid property effects including bouncing phenomena and surface tension energy loss of fuel droplets are hardly considered.

The main purposes of the present study are thus to perform numerical study on the binary droplet collision considering the collision-induced breakup and the effects of liquid property. The collision model proposed in our previous study [6, 7] is used for simulation of collision-induced breakup, and this model is modified to consider the effect of liquid property on the behavior of droplet-droplet collision. The comparisons of numerical predictions with experimental data [3, 4] are conducted for post-characteristics of binary droplet collision process.

2. DROPLET COLLISION MODEL

2.1 Binary Droplet Collision Phenomena

Generally, the droplet collision process is described by three non-dimensional parameters as follows [3-5];

$$We = \rho D_1 |\vec{u}_{rel}|^2 / \sigma \quad (1)$$

$$\Delta = D_1 / D_2, \quad (2)$$

$$b = 2B / (D_1 + D_2), \quad (3)$$

where We is the Weber number based on droplet diameter, Δ the droplet size ratio, and b is the impact parameter. ρ and σ are the density and the surface tension of liquid phase, and the subscripts 1 and 2 represent smaller and larger droplets, respectively. B is calculated by taking the distance from the center of one droplet to the relative velocity vector, $\vec{u}_{rel}$, placed on the center of the other droplets, as illustrated in Fig. 1. The outcomes from collision are classified four different types: bounce, coalescence, reflexive separation, and stretching separation, as seen in Fig. 2. As two droplets impinge each other, the gas between them is trapped and the pressure increases inside this gap. If relative velocity of droplets is not enough to overcome the pressure force, two droplets do not impinge and go away from each other. This is referred to as bouncing. For higher relative velocity, on the other hand, one droplet contacts another directly and in turn coalescence process takes place between them. At high Weber numbers, the droplets have excess kinetic energy, which leads to the separation of droplets from droplet coalesced tentatively. The temporarily coalesced droplets tend to undergo a reflexive separation and a stretching separation at low and high impact parameters, respectively. Many researchers [3-5] observed that the separations produce the satellite droplets from the interacting parts between two colliding droplets, and in turn results in the size reduction in droplets. This procedure is called a collision-induced breakup process whose effects are more pronounced as Weber number increases.

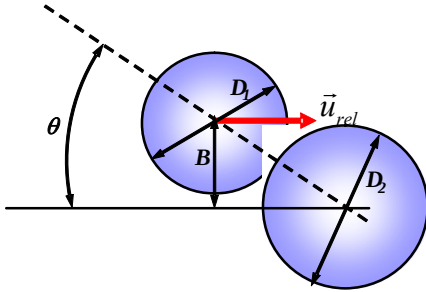


Fig. 1 Kinetic and geometric parameters of the droplet collision.

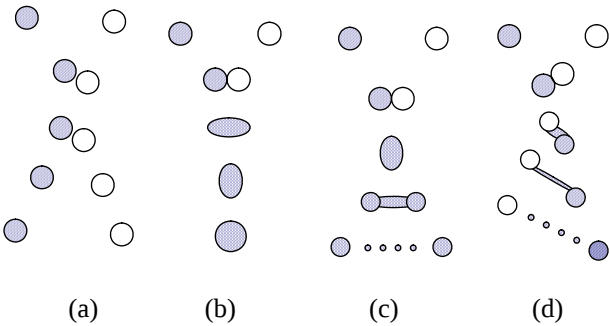


Fig. 2 Diagram of collision regimes: (a) bouncing; (b) coalescence; (c) reflexive separation; (d) stretching separation.

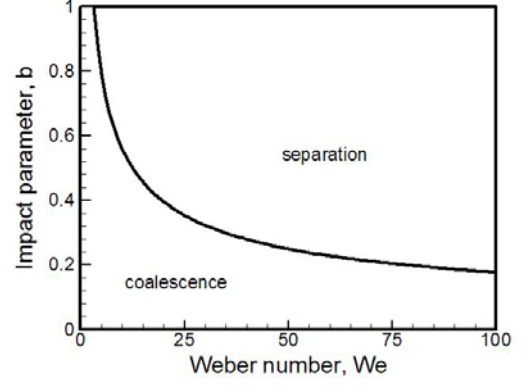


Fig. 3 Boundaries between collision regimes adopted in the O'Rourke model [1] model for $\Delta=1$.

2.2 O'Rourke Model

Among outcomes of droplet collision, the O'Rourke [1] model considers two regimes of permanent coalescence and separation, but ignores the formation of the satellite droplets. Figure 3 shows the boundaries between the regimes adopted in the O'Rourke model for the collision of equal-sized droplets. Followings explain the relationships in O'Rourke model for the criteria and the post-collision characteristics of three regimes. The transition criterion from coalescence to separation is given in terms of coalescence collision efficiency E_{coal} as follows [8].

$$E_{coal} = \min \left[1, \frac{2.4f(\gamma)}{We_s} \right], \quad (4)$$

where, $f(\gamma) = \gamma^3 - 2.4\gamma^2 + 2.7\gamma$, $\gamma = D_2 / D_1$ and $We_s = \rho |\vec{u}_{rel}|^2 (D_1 + D_2) / 2\sigma$. In addition, the subscripts 1 and 2 represent smaller and larger droplets, respectively. The regime boundary between coalescence and separation is therefore determined by $b = E_{coal}$. In other words, coalescence occurs if impact parameter b is less than E_{coal} and otherwise the separation occurs.

The post-collision properties of the collision regimes such as droplet diameters and velocities are determined from the conservation equations of mass, linear momentum, and angular momentum [1]. In coalescence regime, the droplet mass and the velocity are expressed as

$$D_{1a} = \frac{(\rho_1 D_1^3 + n \rho_2 D_2^3)^{1/3}}{\rho_{1a}}, \quad (5)$$

$$\vec{u}_{1a} = \frac{(\rho_1 D_1^3 \vec{u}_1 + n \rho_2 D_2^3 \vec{u}_2)}{\rho_{1a} D_{1a}^3}, \quad (6)$$

where the subscript a denotes a value after collision and n is the number of coalescence. In the separation regimes, no mass is assumed to exchange between two colliding droplet parcels. The velocities are determined as

$$\bar{u}_{1a} = \frac{\rho_1 D_1^3 \bar{u}_1 + \rho_2 D_2^3 \bar{u}_2 + \rho_2 D_2^3 (\bar{u}_1 - \bar{u}_2) \left(\frac{b - \sqrt{E_{coal}}}{1 - \sqrt{E_{coal}}} \right)}{\rho_1 D_1^3 + \rho_2 D_2^3}, \quad (7)$$

$$\bar{u}_{2a} = \frac{\rho_1 D_1^3 \bar{u}_1 + \rho_2 D_2^3 \bar{u}_2 + \rho_2 D_2^3 (\bar{u}_2 - \bar{u}_1) \left(\frac{b - \sqrt{E_{coal}}}{1 - \sqrt{E_{coal}}} \right)}{\rho_1 D_1^3 + \rho_2 D_2^3}, \quad (8)$$

For more details, good summary is well documented in references [1, 9].

2.3 The Present Model

The present model is a modified version of the collision model, which is recently proposed by the authors [6, 7]. Contrary to the O'Rourke model, the present model takes account for the stretching and reflexive separation regimes including the formation of satellite droplets. In addition, a criterion for bouncing regime is introduced and the dissipation energy during collision process is newly modeled to calculate more accurately the collision process of fuel droplets. Figure 4 shows the boundaries between the regimes adopted in the present model for the binary collision of equal-sized droplets.

In the present model, the criterion proposed by Estrade et al. [10] is used to distinguish the bouncing collision from other regimes. When the following condition is satisfied, the bouncing occurs;

$$We < \frac{\Delta(1+\Delta^2)(4\phi' - 12)}{\phi_2(\cos(\arcsin b))^2}, \quad (9)$$

where

$$\phi_2 = \begin{cases} 1 - (2 - \lambda)^2(1 + \lambda)/4 & \text{for } \lambda > 1.0 \\ \lambda^2(3 - \lambda)/4 & \text{for } \lambda < 1.0 \end{cases}$$

and $\lambda = (1 - b)(1 + \Delta)$. The shape factor ϕ' is given a value of 3.351 by Estrade et al. [10].

As seen in Fig. 5, the reflexive separation takes places for the head-on or near-centre collisions of two droplets,

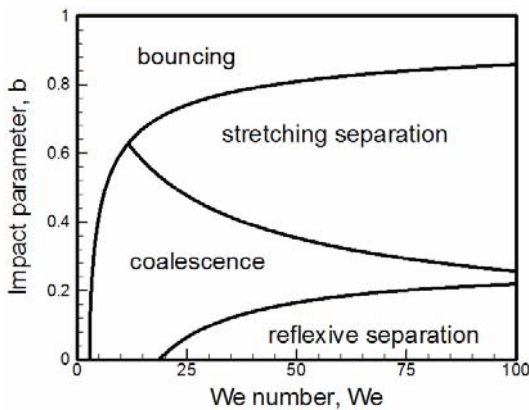


Fig. 4 Boundaries between collision regimes adopted in the new model for $\Delta=1$.

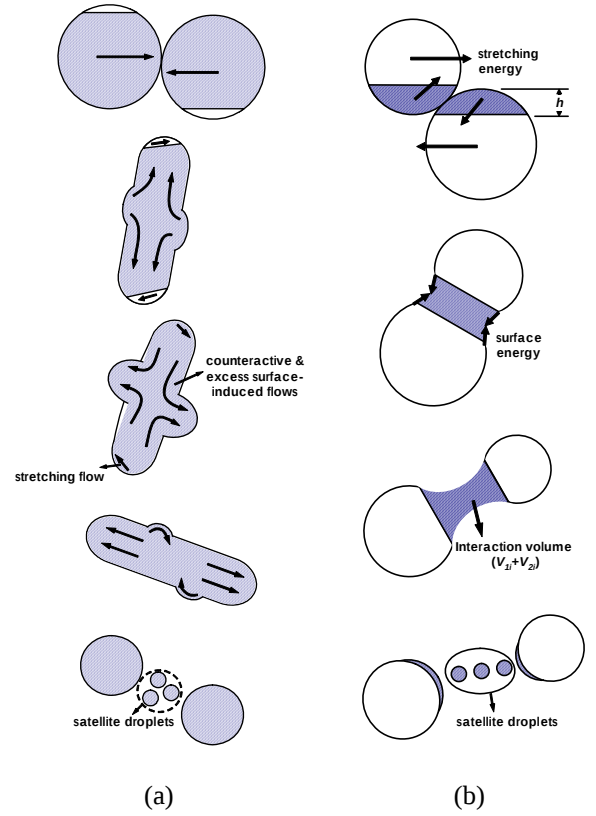


Fig. 5 Schematic of the (a) reflexive and (b) stretching separations after collision between two droplets.

and the stretching separation occurs at the high impact parameter. Ashgriz and Poo [3] explained two separation processes using the balance between the effective kinetic energy and the surface energy of the temporarily coalesced droplet. They proposed the boundaries from coalescence to reflexive and stretching separations as follows;

$$We > 3[7(1 + \Delta^3)^{1/3} - 4(1 + \Delta^2)] \frac{\Delta(1 + \Delta^3)^2}{\Delta^6 \eta_1 + \eta_2} \quad (10)$$

(for reflexive separation),

$$We > \frac{4(1 + \Delta^3)^2 [3(1 + \Delta)(1 - b)(\Delta^3 \phi_1 + \phi_2)]^{1/2}}{\Delta^2 [(1 + \Delta^3) - (1 - b^2)(\phi_1 + \Delta^3 \phi_2)]} \quad (11)$$

(for stretching separation),

where the parameters $\eta_1 = 2(1 - \xi)^2(1 - \xi^2)^{1/2} - 1$ and $\eta_2 = 2(\Delta - \xi)^2(\Delta^2 - \xi^2)^{1/2} - \Delta^3$, and $\xi = (1/2)b(1 + \Delta)$. In addition, ϕ_1 and ϕ_2 denote the portions of interaction region (the hatched parts in Fig. 4(b)) of two colliding droplets for stretching separation, i.e.,

$$V_{ji} = \phi_j V_j, \quad (12)$$

$$\phi_1 = \begin{cases} 1 - \frac{1}{4\Delta^3} (2\Delta - \lambda)^2 (\Delta + \lambda) & \text{for } h > \frac{D_1}{2} \\ \frac{\lambda^2}{4\Delta^3} (3\Delta - \lambda) & \text{for } h < \frac{D_1}{2} \end{cases}, \quad (13)$$

$$\phi_2 = \begin{cases} 1 - \frac{1}{4}(2 - \lambda)^2(1 + \lambda) & \text{for } h > \frac{D_2}{2} \\ \frac{\lambda^2}{4}(3 - \lambda) & \text{for } h < \frac{D_2}{2} \end{cases}, \quad (14)$$

where $\lambda \equiv (1 - b)(1 + \Delta)$ and the interaction height, $h = 0.5(D_1 + D_2)(1 - b)$. V is the volume of droplet, Δ the size ratio of two droplets, b is the impact parameter, subscript i the interaction portion of droplets, and subscript $j = 1, 2$. Equations (10) and (11) showed good agreements with their experimental data [3] for the regime boundaries between coalescence and separations. However, Ashgriz and Poo [3] did not offer any empirical or theoretical relationship on the droplet velocities and sizes, which are essential for describing the separation processes numerically.

On the basis of Ashgriz and Poo [3]'s theory, the present study newly derives the mathematical formulae for the post-collision characteristics in the reflexive and stretching separations. Both reflexive and stretching separation processes accompanies with the formation of satellite droplet as shown in Fig. 5. In order to determine the volume separated from the colliding droplets, the present study introduces the new parameter C_V , called the separation volume coefficient that is defined as the ratio of separating volume to interaction volume (the hatched parts in Fig. 4) between two colliding droplets. It is assumed that this coefficient C_V is proportional to the ratio of energy required for separation to total energy of two colliding droplets in the following manner.

$$C_V = \frac{KE_{sep} - SE_{coal}}{KE_{sep} + SE_{coal}}, \quad (15)$$

where KE_{sep} represents the effective kinetic energy inducing the separation of the temporarily combined droplets, and SE_{coal} is the effective surface tension energy retaining the coalescence between two droplets. The present study determines these effective energies for two different separation processes using the equations of Ashgriz and Poo [3] as follows:

$$KE_{sep} = \begin{cases} \sigma\pi D_2^2 \left[(1 + \Delta^2) - (1 + \Delta^3)^{2/3} + \frac{We(\Delta^6\eta_1 + \eta_2)}{12\Delta(1 + \Delta^3)^2} \right] & \text{(for reflexive separation)} \\ \frac{1}{2} \rho u_{rel}^2 V_2^2 \left\{ \frac{\Delta^3[(1 + \Delta^3) - (1 - b^2)(\phi_1 + \Delta^3\phi_2)]}{(1 + \Delta^3)^2} \right\} & \text{(for stretching separation)} \end{cases} \quad (16)$$

$$SE_{coal} = \begin{cases} 0.75\sigma\pi(D_1^3 + D_2^3)^{2/3} & \text{(for reflexive separation)} \\ \sigma[2\pi V_2 D_2 \lambda (\Delta^3\phi_1 + \phi_2)]^{1/2} & \text{(for stretching separation)} \end{cases} \quad (17)$$

Using above equations, the separating volume and the size of colliding droplets after collision are calculated as follows:

$$V_s = C_V(V_{1i} + V_{2i}), \quad (18)$$

$$D_{ja} = (1 - C_V\phi_j)^{1/3} D_j, \quad (19)$$

where subscript a means a value after collision, and $j = 1, 2$. ϕ is determined by Eqs. (13) and (14) for stretching separation, and is used the value of unity for reflexive separation because the interaction between two droplets occur over the whole volume of temporarily combined droplets (see Fig. 4 (a)).

For simplicity, all satellite droplets are assumed to have same properties. In turn, conservation equations for mass, momentum and energy are rewritten as

$$\sum_{j=1}^2 D_j^3 = \sum_{j=1}^2 D_{ja}^3 + N_{sa} \cdot D_{sa}^3, \quad (20)$$

$$\sum_{j=1}^2 D_j^3 \vec{u}_j = \sum_{j=1}^2 D_{ja}^3 \vec{u}_{ja} + N_{sa} \cdot D_{sa}^3 \vec{u}_{sa}, \quad (21)$$

$$\sum_{j=1}^2 \left(\frac{\pi}{12} \rho D_j^3 \vec{u}_j^2 + \sigma \pi D_j^2 \right) = \sum_{j=1}^2 \left(\frac{\pi}{12} \rho D_{ja}^3 \vec{u}_{ja}^2 + \sigma \pi D_{ja}^2 \right) + KE_{sa} + SE_{sa} + EL, \quad (22)$$

where N is the number of droplets and subscript sa means the satellite droplet. Additionally, $\vec{u}_1$ and $\vec{u}_2$ are the velocities of the smaller and larger droplets in the mass-centre coordinates, respectively, as given by $\vec{u}_1 = \vec{u}_{rel}/(1 + \Delta^3)$ and $\vec{u}_2 = -\Delta^3 \vec{u}_{rel}/(1 + \Delta^3)$, where $\vec{u}_{rel}$ is the relative velocity of two droplets. The total kinetic energy of satellite droplets can be represented by

$$KE_{sa} = N_{sa} \cdot \left(\frac{\pi}{12} \rho D_{sa}^3 \right) \vec{u}_{sa}^2 = \frac{1}{2} \rho V_s \vec{u}_{sa}^2. \quad (23)$$

Also, EL denotes the energy loss consisting of dissipation energy and surface tension energy loss as follow:

$$EL = \iint \mu \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 dV dt + \Delta S \sigma. \quad (24)$$

where ΔS is the additional surface area associated with the deformation during collision process [4].

In the our previous studies [6, 7], the following equation is derived on the basis of the relationship suggested by Jiang et al. [11] who expressed the viscous dissipation in terms of the kinetic energy of interaction volume:

$$EL = \alpha \left(\frac{1}{2} \rho \sum_{j=1}^2 \phi_j V_j u_j^2 \right), \quad (25)$$

where α denotes the energy loss coefficient. For water droplets, α has been empirically determined to be around 0.5 by Jiang et al. [11].

In this formula, the viscosity is excluded and the surface tension energy loss is also neglected. Qian and Law [4], however, reported that the critical Weber number for the reflexive separation is dependent on the Ohnesorge number,

representing the ratio of the viscous to surface energies. This means that the viscous dissipation and surface tension energy loss are significantly affected by the liquid properties in the reflexive separation regime. In this study, thus, a new formula is derived on the Qian and Law [4]'s relationship for the energy loss during the reflexive separation process as follow:

$$EL = \frac{\pi D_1^2 \sigma}{24} \left\{ \frac{\alpha}{2} We + 16\sqrt{2}(1-\alpha)\beta Z + (1-\alpha)\gamma \right\} \quad (26)$$

where β and γ are the empirical constants and adopted as 30 and 15 from Qian and Law [4], respectively, and $Z = \mu/(\rho R \sigma)^{1/2}$ is the Ohnesorge number.

Considering the kinetic energy loss for the reflexive separation, the velocities of colliding droplets can be written as follows:

$$\bar{u}_{ja} = (1-\alpha)^{1/2} \bar{u}_j. \quad (27)$$

For the stretching separation, because the loss of kinetic energy is involved within the interaction portion which becomes the satellite droplets, the colliding droplets can be assumed to retain their velocities, i.e.,

$$\bar{u}_{ja} = \bar{u}_j. \quad (28)$$

By solving the Eqs. (20) ~ (22) simultaneously, the characteristics of satellite droplets can be finally obtained as follows:

$$D_{sa} = \left[\frac{C_v (\Delta^3 \phi_1 + \phi_2)}{N_{sa}} \right]^{1/3} D_2. \quad (29)$$

$$\bar{u}_{sa} = \frac{\Delta^3 \phi_1 u_1 + \phi_2 u_2}{(\Delta^3 \phi_1 + \phi_2)}, \quad (30)$$

$$N_{sa} = \left[\frac{SE_{sa}}{\sigma \pi C_v^{2/3} (\Delta^3 \phi_1 + \phi_2)^{2/3} D_2^2} \right]^3, \quad (31)$$

Now, the post-characteristics of the separation collisions can be calculated using Eqs. (19), and (27) ~ (31).

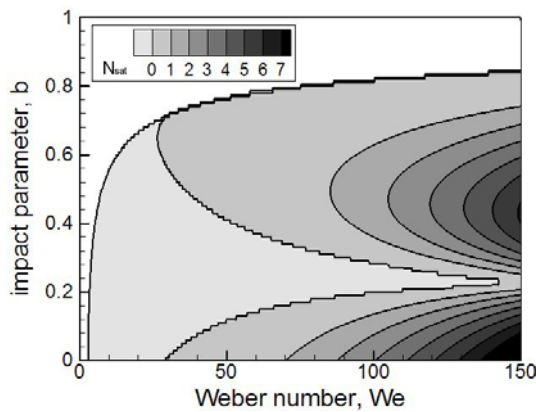


Fig. 6 Calculated number distribution of satellite droplets for binary collision of two water droplets for $\Delta=1$.

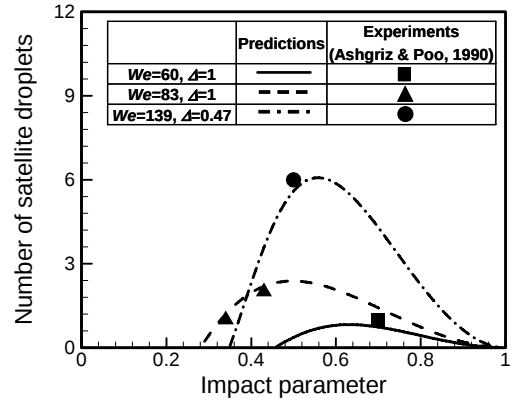


Fig. 7 Comparison of the calculated satellite droplet number with experimental data [3] for stretching separation collision of two water droplets.

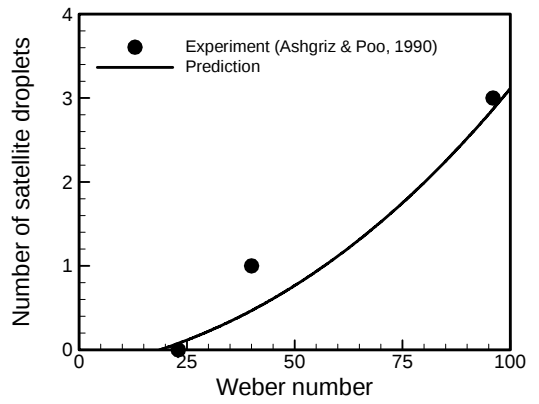


Fig. 8 Comparison of the calculated satellite droplets number with the experimental data [3] for the head-on collision of two equal-sized water droplets.

3. RESULTS AND DISCUSSION

This section examines the predictability of new model by comparing the predicted results with earlier published experimental data [3, 4] for the binary collision of two droplets. Figure 6 shows the predicted We - b maps for the number of satellite droplets in case that the binary collision of two equal-sized water droplets takes place. In this calculation, the energy loss coefficient α is 0.5 suggested by Jiang et al. [11]. In the stretching separation regime, the number of satellite droplets is of the highest value in the range from 0.4 to 0.6 in impact parameter. Until the impact parameter reaches to 0.4, the number of satellite droplets increases because of the increase in the stretching energy. When the impact parameter ranges from 0.6 to 1.0, on the other hand, the probability of satellite droplet formation is reduced due to the decrease in interaction region between two colliding droplets. In the reflexive separation regime, in addition, the satellite droplets are formed most actively in the case of the head-on collision. As mentioned previously, it can be supported by the fact that the stretching effects induced by off-centre collision reduce the reflexive separation. In Figs. 7 and 8, comparisons are made between the predictions and the experimental data of Ashgriz and Poo [3] for the number of satellite droplets. As seen in Figs.

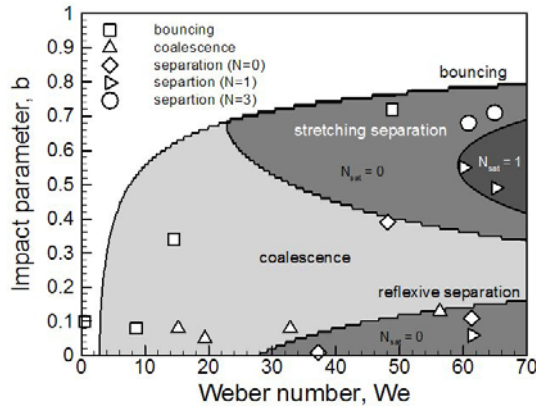


Fig. 9 Comparison of the calculated number distribution of satellite droplets with experimental data [4] for the collision of equal-sized tetradecane droplets.

7 and 8, the present model predicts well compared to the experimental data for the stretching separation and for the head-on collision at different conditions. For the head-on collision, it is found that the number of satellite droplets increase with the Weber number.

Figure 9 compares the predicted We - b map for the collision of equal-sized tetradecane droplets with experimental data of Qian and Law [4]. Although some discrepancy between the predicted boundary of bouncing regime and experiments are shown at low Weber number, the predicted We - b map represents well the overall distribution of collision regimes observed from experiment [4].

In Fig. 10, the normalized mean diameter is presented for different Weber numbers and it is averaged over the whole range of impact parameter at a given Weber number. Here, D_{10} and SMD denote the arithmetic and Sauter mean diameters, respectively. They are often used to analyze the characteristics of droplet size distributions in applications of liquid spray. Over the range of low Weber number, both models predict the increase of mean diameter after collisions because of droplet coalescence. As the Weber number increases, however, both D_{10} and SMD computed by the new model decrease gradually, indicating that the droplet breakup via collision increases with Weber number. Since the O'Rourke model ignores the reflexive separation process at low impact parameters as well as the change in droplet size even in stretching separation process, the predictions of the O'Rourke model are rarely varied over the whole range of Weber number. From these results, it is concluded that the new model is more reasonable than the O'Rourke model for simulation in the droplet collision process leading to the droplet breakup.

4. CONCLUSIONS

The present study has performed numerical study on the binary droplet collision considering the collision-induced breakup and the effects of liquid property. In the present model, the post-collision characteristics of droplets were determined by the formulae based on the conservation equations between before and after collision. For the validation of the new model, the calculated results are first compared with experimental data [3, 4] on the binary

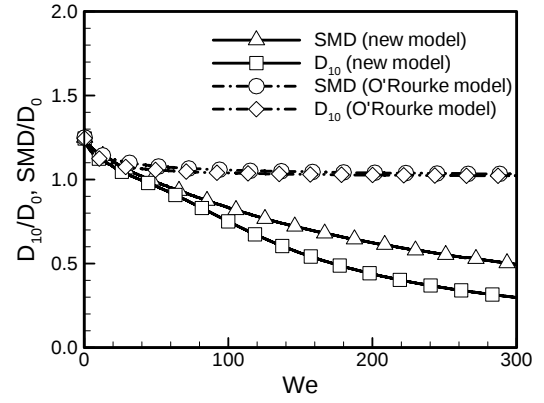


Fig. 10 Comparison between the predictions by both models for the D_{10} and SMD normalized by the initial droplet diameter D_0 for the binary collision of equal-sized water droplets.

droplet collision. In the stretching separation regime, the number of satellite droplets is of the highest value in the range from 0.4 to 0.6 in impact parameter. In the reflexive separation regime, in addition, the satellite droplets are formed most actively in the case of the head-on collision. The predicted mean diameter by the new model decreased with increasing the Weber number because of the collision-induced breakup. The O'Rourke model, on the other hand, failed to predict the size reduction via the binary droplet collision. From these results, it can be concluded that the new model for droplet collision is more reasonable than the O'Rourke model for simulation in the collision-induced breakup process accompanied with formation of satellite droplets.

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NOMENCLATURE

b	impact parameter
C_V	separation volume coefficient
D	droplet diameter [m]
DE	dissipation Energy, J
h	interaction height [m]
KE_{sep}	effective kinetic energy inducing separation [J]
N	number of droplets
n	number of collision
R_{crit}	critical radius [m]
SE_{coal}	surface tension retaining coalescence [J]
V	volume of droplet [m ³]
V_s	separating volume from coalesced droplet [m ³]
We	Weber number

Greek letters

α	energy loss coefficient
β	empirical constant
Δ	droplet size ratio
μ	viscosity [N·s/m ²]

ρ	density [kg/m ³]
ϕ	volume ratios of interaction region
σ	surface tension [N/m]

Subscripts

1	smaller one of two colliding droplets
2	larger one of two colliding droplets
a	after collision
i	interaction portion of droplets
s	separation portion of droplets
sa	satellite droplet

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