VORTEX RING-LIKE STRUCTURES IN GASOLINE FUEL SPRAYS: MODELLING AND OBSERVATIONS

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ABSTRACT

Automotive gasoline fuel sprays exhibit complex structures, within which vortex rings can be clearly traced. These structures have been studied experimentally and theoretically. The experimental data used for the analysis were obtained in the following conditions. Liquid iso-octane was injected at a frequency of 1 Hz, pressures of 3.5 bar (Port (PFI) injector) and 100 bar (Direct (G-DI) injector) into air at atmospheric pressure and a temperature of 20 °C. Phase Doppler Anemometry was performed over a measurement grid that covered the whole spray. The decaying phase showed the most clearly defined vortex ring-like structures. To model the observed properties of these structures, a generalised vortex ring model was applied. In this model, the time dependence of the vortex ring thickness ℓ is given by the relation $\ell = at^b$, where *a* is an arbitrary positive number, and $1/4 \le b \le 1/2$. The predictions of the model were compared with the results of experimental studies, described earlier. The data analyses were focused on the determination of the time evolution of the locations of the regions of maximal vorticity and the axial velocities in these locations. Most of the values of the axial velocity of the vortex ring development and b=1/4 (fully developed turbulence) and 1/2 (laminar case). The results for the PFI injector seem to be not compatible with model predictions.

INTRODUCTION

Several key parameters are commonly used to describe the dynamics of sprays in internal combustion (IC) engines. These are the spray tip penetration, break-up length, spread or cone angle and the atomisation quality (the spatial and temporal distribution of droplet sizes and velocities). These parameters can be used to predict the rate at which the mixing proceeds and have been well documented in the literature for port fuel and direct injections [1, 2]. However, the effects of spray induced vortex ring-like structures have been generally overlooked, although these play an important role in the rate at which the liquid evaporates.

It is therefore important to perform further studies of engine fuel sprays taking into account the formation and dynamics of vortex ring-like structures. Some preliminary results of these studies were reported in [3], where it was shown that vortex ring-like structures behaviour in gasoline engine-like conditions could not be adequately described in terms of the laminar vortex ring models [4]. Instead, it was suggested that the observed features of these structures were more suited to the turbulent ring model developed by Lugovtsov [5, 6]. In this paper, the most important recent results of these studies (experimental and theoretical), obtained by the authors, are summarised. The results of the comparison of the experimentally observed and predicted axial and radial translational velocities in the vortex ring regions of maximal vorticity are discussed. The focus of the paper is on the fluid dynamics characteristics of sprays. A review of recently developed models for spray heating and evaporation is given in [7].

In the next section, the experimental methods used for vortex ring analysis are described. Then, earlier developed and the most recent vortex ring models are summarised. After that, comparison between the experimental results and the model predictions is presented and discussed. Finally, the main results of the paper are summarised.

EXPERIMENTAL SET-UP

Two modern production gasoline injectors; a low pressure, port fuel injector (PFI: injector A) and a high pressure, direct fuel injector (G-DI: injector B) were chosen for the analysis. The choice of fuel injection systems was adopted to highlight the difference in the spray dynamics and to assess the robustness of the vortex ring models. Fuel was delivered using a production fuel pump and pressure-regulated fuel rail in both cases.

Previous studies [3, 8, 9] have shown that the identification of specific structures in gasoline fuel sprays is often obscured by the turbulent gas boundary layer around sprays or through the interaction of sprays with the incoming airflows. A typical example of complex vortex ring-like structures in a high-pressure, direct-injection gasoline fuel spray (G-DI), in a motored, optically-accessed, single cylinder research engine at 1000 rpm, is shown in Fig. 1. In this study, the formation of vortex ring-like structures was observed in a large, atmospheric pressure and temperature, quiescent chamber. For both injectors, homogenous engine operating conditions occurred when the gas pressure at the start of fuel injection is close to atmospheric at wide-open throttle. These conditions were used to determine a typical fuel injection duration for stoichiometric engine operation. The specifications of the fuel injection systems are given in Table 1.

The experimental set-up is illustrated in Fig. 2. Each injector was mounted vertically in the quiescent chamber of

square cross-section. The large windows permitted a full view of the spray without impingement or boundary interference with the walls. The injector orientation was selected to ensure that the measurement plane bisected the injection axis. A programmable traverse was used to move the instruments through the spray. The location of the protruding tip of the injector was taken as the origin for all measurements. The axes are defined vertically downwards along the spray axis and radially, perpendicular to the spray axis as shown in Fig. 2.



Fig. 1. A sequence of high-speed photographs showing the formation of vortex ring-like structures in a G-DI spray in a motored optical engine at 1000 rpm. The start of injection timing was 65 CA ATDC non-firing. The photographs were

recorded every 3 CA intervals starting with 67 CA ATDC. Fuel injection starts from the top left-hand corner.

A series of high-speed ciné films and short exposure digital still photographs were used to observe the formation of vortex ring-like structures. A combination of a pulsed laser light sheet and backlit shadowgraphy were applied. A Phantom V7.1 high-speed camera was used for the ciné photography with framing rates in the range of 14.4 kHz to 100 kHz. In addition, a high-resolution charge-coupled device (CCD) camera (1280x1024 pixels) was used to re-construct the injection event from a series of sequential time-stepped still images captured over consecutive injections. At each step, 20 images were acquired to produce a single average image. The optimum camera exposure time used was 1 µs and the laser sheet thickness was limited to approximately 1mm. Both cameras were synchronised to the start of injection (SOI) trigger pulse, provided to the engine management system.

В

Injector A

Fuel injector type	
Port (PFI)	Direct (G-DI)
Nominal fuel pressure	
3.5 bar	100 bar
Fuel temperature	
22 °C	22 °C
Fuel type	
Iso-octane (2,2,4 TMP)	Iso-octane (2,2,4 TMP)
Injection frequency	
1 Hz	1 Hz
Injection duration	
5 ms	2 ms
Air pressure	
1 bar	1 bar
Air temperature	
20 °C	20 °C
Orifice size	
200 µm	250 μm

Table 1. Fuel injection equipment specifications.



Fig. 2. Schematic of the quiescent spray chamber, locations of the Phase Doppler Anemometer (PDA) and traverse.

A Dantec Dynamics Classical two-component Phase Doppler Anemometer (PDA) was used to measure the droplet size and velocity distributions over a 1 mm by 5 mm measurement grid close to the nozzle and a 2 mm by 10 mm grid further downstream. The extents of the grid were determined from a series of preliminary measurements performed across the vertical injector axis of symmetry to the periphery of the spray. The PDA processor internal clock was synchronised to the SOI trigger pulse. The time taken from the start of injection was recorded as t. Non-coincident data was collected for either 10,000 measurements or 60 s duration for the axial velocity component (V_x) and 5,000 measurements or 60 s duration for the radial velocity component (V_r) . The velocity and droplet size measurement range was optimised to ensure that all of the acquired, validated data was within the range of ± 3 standard deviations of the respective mean values. The total data validation rates were greater than 90% in both sprays except for locations close to the nozzle exit and either side of the vertical injector axis of symmetry (for $x \le 10$ mm). In these regions, the data validation rates were of the order of 70%.

MODEL

Since the pioneering papers by Helmholtz [10] and Lamb [11] the theory of vortex rings has been extensively developed and the results have been reported in several review papers and monographs (e.g. [4, 12]). Among more recent publications we can mention [13-20]. The analysis of various modelling approaches is beyond the scope of this paper. In what follows, previously reported formulae describing the axial translational velocity of the vortex rings, under various approximations, will be summarised, and some features of the new models are outlined.

Although some approaches to modelling of turbulent vortex rings have been suggested by Lugovtsov [5, 6], the quantitative models have been developed mainly for laminar rings. Another assumption, commonly used in modelling is the smallness of the vortex ring Reynolds number, defined as $\text{Re}=\zeta_0 \ell^2/v$, where ζ_0 is the characteristic vorticity, ℓ is the characteristic vortex ring thickness (see Fig. 3). In the case of laminar vortex rings, $\ell = \sqrt{2\nu t}$, where *t* is time and *v* is the kinematic viscosity of the fluid.



Fig. 3. A schematic view of a vortex ring.

Using these assumptions and ignoring changes with time of the radius of the vortex ring (distance from its axis to the point where the fluid velocity is equal to zero) described by parameter R_0 (see Fig. 3), the following general equation for the normalised vortex ring axial translational velocity has been obtained [16]:

$$U_{x} = \frac{V_{x}}{v_{n}} = \sqrt{\pi\theta} \left\{ 3 \exp\left(-\frac{\theta^{2}}{2}\right) I_{1}\left(\frac{\theta^{2}}{2}\right) + \frac{\theta^{2}}{12} \right.$$

$$\times_{2} F_{2}\left[\frac{3}{2}, \frac{3}{2}; \frac{5}{2}, 3; -\theta^{2}\right]$$

$$\left. - \frac{3\theta^{2}}{5} {}_{2} F_{2}\left[\frac{3}{2}, \frac{5}{2}; 2, \frac{7}{2}; -\theta^{2}\right] \right\}, \qquad (1)$$

where the generalised hypergeometric function

$$_{2}F_{2}[a_{1},a_{2};b_{1},b_{2};x]$$
 is defined as:
 $_{2}F_{2}[a_{1},a_{2};b_{1},b_{2};x] = \sum_{k=0}^{\infty} \frac{(a_{1})_{k}(a_{2})_{k}x^{k}}{(b_{1})_{k}(b_{2})_{k}k!},$ (2)

with the coefficients defined as:

$$(\alpha)_0 = 1; \ (\alpha)_1 = \alpha; ,$$

$$(\alpha)_k = \alpha(\alpha + 1)....(\alpha + k - 1) \ (k \ge 2),$$

$$\theta = \frac{R_0}{\ell}, \quad v_n = \frac{M}{4\pi^2 R_0^3} = \frac{\Gamma_0}{4\pi R_0},$$

 $\kappa = 4\pi R_0 - 4\pi R_0$ $\Gamma_0 = M/(\pi R_0^2)$ is the initial circulation of the vortex ring, *M* is the vortex ring momentum divided by density of the ambient fluid. Equation (1) was obtained under the assumption that $\text{Re} = \xi_0 \ell^2 / \nu$ is small. This assumption can be justified by weak dependence of U_x on Re, which follows from the direct comparison of predictions of Equation (1) with the results of numerical simulations [21] and experimental data [22].

In the limit of long times (small θ), Equation (1) is simplified to:

$$U_{x} = \frac{7\sqrt{\pi}\theta^{3}}{30} \propto t^{-3/2}.$$
 (3)

Equation (3) is identical to the one obtained by Rott and Cantwell [13]. A more general analysis, taking into account the effect of turbulence, leads to the same dependence on *t* but with the power 3/2 being replaced by 3/4 [5,6].

In the limit of short times (large θ), Equation (1) is simplified to:

$$U_{x} = \ln \theta + \frac{3 - \gamma}{2} - \psi(3/2),$$
(4)

where $\gamma \approx 0.57721566$ is the Euler constant, $\psi(x)$ is the digamma function defined as:

$$\psi(x) = \frac{d\log\Gamma(x)}{dx},\tag{5}$$

 $\Gamma(x)$ is the Gamma function. Note than $\psi(1) = \gamma$. (This result was obtained in collaboration with Professor Y. Fukumoto). Equation (4) is identical to the one obtained earlier by Saffman [23].

One of the important limitations of the models described above is that they are based on the assumption that the value of R_0 does not change with time. This limitation was overcome in the model developed by Saffman [23] who based his analysis on simple dimensional considerations rather than on rigorous solution of the underlying equations.

Although Equation (1) was originally derived, assuming that $\ell = \sqrt{2\nu t}$, it remains valid in the more general case when

$$\ell = at^b, \tag{6}$$

where a and b are constants. Our generalised vortex ring model is essentially based on Equation (6).

It can be shown that a physically correct solution can be expected when

$$1/4 \le b \le 1/2.$$
 (7)

When $a = \sqrt{2\nu}$ and b=1/2, ℓ predicted by Equation (6) reduces to the one predicted by the conventional laminar vortex ring model. When b=1/4, then some predictions of the model are similar to those which follow from the turbulent vortex rings model [5, 6]. From this point of view, the generalised model is expected to incorporate both laminar and turbulent vortex ring models, developed earlier.

In the limit of large times, the model predicts the following value of U_x :

$$U_x = \frac{7\sqrt{\pi}}{30} R_0^3 a^{-3} t^{-3b}.$$
 (8)

Equation (8) is a straightforward generalisation of Equation (3). Note that the velocity U_x , predicted by Equation (8), is different from the velocities of the fluid in the region of maximal vorticity, $U_{\omega x} = V_{\omega x}/v_n$. In the case of small θ (large *t*), they are linked by the following equation:

$$U_{ax} = U_x + 2\pi\theta^2 \int_0^\infty \mu \operatorname{erfc}\left(\frac{\mu}{\sqrt{2}}\right) J_1(\theta\mu) J_0(\mu) \mathrm{d}\mu,$$
(9)

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-t^2) dt,$$

and J_1 and J_0 are Bessel functions of the first and zero order respectively. U_x is defined by Equation (8). Note that for sufficiently large μ_0 , the contribution of $\mu > \mu_0$ in the integral in Equation (9) can be ignored. On the other hand, for $\theta << \mu_0^{-1}$, we can write

$$J_1(\theta\mu) = \frac{1}{2}\theta\mu,\tag{10}$$

Combining Equations (3), (9) - (10) we obtain

$$U_{ax} = \left[\frac{7\sqrt{\pi}}{30} + \pi \int_{0}^{\infty} \mu^{2} \operatorname{erfc}(\frac{\mu}{\sqrt{2}}) J_{0}(\mu) \mathrm{d}\mu\right] \theta^{3}. (11)$$

Comparing Equations (3), (8) and (11), we can see that both U_x and $V_{\omega x}$ are proportional to $\theta^3 \sim t^{3b}$, that is

$$U_{ax} \propto t^{-3b}.$$
 (12)

As in the case of the conventional vortex ring model, the radial component of its velocity, both in the region where the velocity is close to zero and in the region when the vorticity is maximal, predicted by the generalised model, is equal to zero.

RESULTS VERSUS MODEL PREDICTIONS

The fuel sprays produced by both low pressure and high pressure injectors were observed to comprise three phases. Initially, a developing phase was observed where a poorly atomised, unsteady, high velocity jet was formed, combined with small dispersed droplets. This was followed by the main, quasi-steady period, where the mass flow of liquid fuel was approximately constant and the position of the leading edge of spray penetration was observed to move almost linearly with time. In the final phase, the spray momentum decayed and vortex ring-like structures were seen to be formed, translated and destroyed as the jet collapsed inwards towards the injector axis. The main difference between the sprays produced by both injectors was observed in the relative phasing and duration of these three periods, for a given injection pulse width, and the mean droplet velocities and diameters associated with each phase. The interpretation of these phases is complicated by the position of the measurement probe in the spray.

The vortex ring-like structures were observed in both sprays over a wide range of timescales. In the PFI case, these structures were seen to occur shortly after injection, close to the nozzle, but were not easily identified for experimental investigation. In the G-DI case, reverse flow structures of differing scales were observed over the entire injection duration. In both cases, however, the vortex ring-like structures observed in the decaying phase were used as these could be identified with sufficient precision for quantitative analysis.

In the first instance, the high-speed photographic results were used to identify the regions of investigation for the PDA study. Then the centres of the vortex ring-like structures were located by identifying the regions corresponding to maximal vorticity magnitude computed from the interpolated PDA measurement grid. These occurred from approximately 1.75 ms to 5 ms after SOI in the high-pressure, G-DI fuel sprays, and 12 ms to 15 ms after SOI in the low-pressure, PFI spray.

An example of the vortex ring-like structures is shown in Fig. 1 for the G-DI spray photographed within the optical engine. Several vortex ring-like structures can be clearly traced despite the complex nature of the spray. These images can be used to estimate the radii of these structures, and their distance from the spray axis. The translational and radial

displacements and the velocities of these structures can be estimated from a sequence of images. In [3], the points where the fluid velocities are close to zero were manually traced between frames. In our present study, computer tracing of the regions of maximal vorticity, calculated from a velocity field measured by the PDA, is considered instead.

In the PFI case, vortex ring-like structures were clearly observed to form close to the nozzle exit. In the latter stages of fuel injection and in regions far from the nozzle, the low concentration of droplets did not provide sufficient illumination for photographic evidence of vortex ring-like structures. However, the presence of rotating structures could be indirectly identified using the PDA droplet velocity data. The normalised turbulence intensity components, defined as the ratio of the amplitude of the velocity component oscillation to the absolute value of the average velocity component, were used to visualise the results.

In each time interval, the mean vorticity magnitude of the observed spatial velocity distribution was calculated where clearly defined vortex centres were identified. An example of the spatial field of computed vorticity magnitude is shown in Fig. 4 and Fig. 5 for the PFI spray at 13.75 ms, and G-DI spray 4.25 ms, respectively.



Fig. 4. The distribution of the vorticity magnitude for the PFI spray for t=13.75 ms. The cross shows the location of the region of maximal vorticity.

The regions corresponding to maximal vorticity magnitude are denoted by crosses. The time evolution of the axial $(V_{\omega x})$, and radial $(V_{\omega r})$, velocity components at these regions were obtained by direct measurement. Similar plots were obtained for 14.00 ms and 14.25 ms after SOI for the PFI spray, and for 4.50 ms and 4.75 ms after SOI for the G-DI spray.

The maximum vorticity magnitude recorded for the PFI injector for $13.75 \le t \le 14.25$ ms was 0.72 s^{-1} . In the G-DI injector case for $4.25 \le t \le 4.75$ ms it was 2.78 s^{-1} . It is interesting to note that the regions corresponding to maximal vorticity consistently showed the smallest mean droplet diameters within each incremental time step for the G-DI

injector spray. The uncertainty in the determination of the regions of maximal vorticity was estimated to be between ± 0.2 mm and ± 0.5 mm for the PFI and G-DI injectors respectively. The greatest uncertainty in the velocity as a result of this positional error in the interpolated field was ± 0.5 ms⁻¹ for the PFI case and ± 1 ms⁻¹ for the G-DI spray.

The following properties of the radial $V_{\omega r}$ and translational $V_{\omega x}$ velocities in the regions of maximal vorticity were observed for both sprays.



Fig. 5. The distribution of the vorticity magnitude for the G-DI spray for t=3.75 ms. The cross shows the location of the region of maximal vorticity.

Firstly, V_{or} shows the most scatter in data. In most cases, V_{or} is close to zero, being either positive or negative. This is particularly clearly visible in the case of the G-DI injector for *t* greater than approximately 3.5 ms after SOI.

Secondly, in both cases, $V_{\omega x}$ is positive for all values of t and tends to decrease with time over the range identified. There is more scatter in the data for $V_{\omega x}$ for the port injector than for the direct injection case but a trend in each data set can be traced. The experimental results were approximated as $V_{\omega x}(t) = At^{B}$, where t is the time after SOI. Constants A and B were found using a linear least squares curve fitting technique by minimising the sum of the squares of the vertical distance of the points from the curve. In the PFI case, it was obtained that $A = 1239 \text{ ms}^{-1}$ and B = -2.21. In the G-DI case, $A = 74.1 \text{ ms}^{-1}$ and B = -1.57.

Thirdly, oscillations in both the axial and radial velocities, with a frequency of approximately 1 kHz can be seen in both sprays, especially far from the nozzle during the decaying phase of injection. The analysis of these oscillations, which are likely to be related to spray instabilities, is beyond the scope of our analysis.

Although vortex ring-like structures, observed in gasoline engines (see Fig. 1) look rather different from the structures of the classical vortex rings (see [4]), we can expect that the integral characteristics of these structures, such as translational axial and radial velocities, are similar. This can be supported by the fact that although the finite values of the Reynolds number lead to noticeable changes in vorticity distribution, especially far from the ring core, the effects of this number on these velocities is weak. This can justify our attempt to explain the properties of these velocities based on classical vortex ring models.

A comparison of the experimental data with the available models of the vortex rings identified several unknown parameters including the values of R_0 and initial time. To eliminate the effect of these values, the axial velocities were normalised by the times at which the vortex rings were first observed, $t = t_{init}$. Also, the non-dimensional time, $\overline{t} = t/t_{init}$ was introduced. The non-dimensional axial velocity component then becomes, $\overline{V}_{\omega x}(\bar{t}) = V_{\omega x}(\bar{t} t_{init})/V_{\omega x}(t_{init})$. The plots of $\overline{V}_{\alpha x}$ versus \overline{t} are shown in Fig. 6 for both G-DI and PFI cases. In both cases, the normalised axial velocity component decreased with increasing normalised time. Also, during the period where vortex rings were identified in each spray, the rate of decay in the range $1 \le t \le 1.25$ is approximately 1.5 times greater in the PFI spray than that observed in the high-pressure spray case. It should be noted, however, that the very high scatter of experimental results for the PFI sprays indicates that these are less reliable than those for the G-DI sprays.



Fig. 6. Normalised experimental values of the axial velocity component $\overline{V}_{\alpha x} = V_{\omega x}(t)/V_{\omega x}(t_{init})$ against normalised time $\overline{t} = t/t_{init}$ for the G-DI and PFI injectors (filled and unfilled diamonds). Approximations of the experimental results are given by $\overline{V}_{\alpha x} = \overline{t}^{-1.14}$ (G-DI injector) and $\overline{V}_{\alpha x} = \overline{t}^{-2.97}$ (PFI injector). The values of $\overline{V}_{\alpha x} = \overline{t}^{-\alpha}$ predicted by the model in the limit of long times (thick dash-dotted curves) for $\alpha=3/4$ and 3/2 (indicated near the curves).

As in the case of non-normalised velocities, data presented in Fig. 6 are approximated by a power function. A restriction $\overline{V}_{\alpha\alpha}(\bar{t}=1) = 1$ is applied. Hence this approximation is presented in the form $\overline{V}_{\alpha\alpha}(\bar{t}) = \bar{t}^{-C}$. The value of constant *C* was calculated by the use of the least squares best fit method. For the PFI case it was found that C = 2.97 and C = 1.14 was found for the G-DI spray. The plots of $\overline{V}_{\omega x}(\overline{t}) = \overline{t}^{-2.97}$ and $\overline{V}_{\omega x}(\overline{t}) = \overline{t}^{-1.14}$ are included in Fig. 6. The best curve fit approximation was achieved for the G-DI spray data.

As follows from Equation (12), $\overline{V}_{\alpha x} \sim \overline{t}^{-\alpha}$, in the long time limit, where $\alpha = 3/2$ in the laminar case and $\alpha = 3/4$ in the turbulent case. The plots $\overline{t}^{-\alpha}$ for $\alpha = 3/4$ and $\alpha = 3/2$ are also shown in Fig. 6. It can be seen that for the high-pressure sprays, approximately 60% of the experimental points occur within the boundaries of the theoretical curves, $\bar{t}^{-3/2}$ and $\bar{t}^{-3/4}$. In this case, the experimentally predicted value of the power is $\alpha = 1.14$, which falls between the predictions for laminar and turbulent limits. In contrast, only two of the data points for the PFI spray are located near to the curve corresponding to $\alpha = 3/2$. The remainder are far from the prediction of the models; the measured values of the normalised axial velocity component are consistently less than that predicted by both the laminar and turbulent models. The experimentally determined value of $\alpha = 2.97$ suggests a much greater rate of decaying of vortex ring axial translational velocity with time. It remains unclear, however, whether this should be attributed to the different underlying physics of the process, compared with the one described earlier, or to excessive scatter of experimental data.

CONCLUSIONS

The results of recent experimental and theoretical studies of vortex ring-like structures in gasoline engine-like conditions are summarised. The focus has been on port-injected (lowpressure) and direct injection (high-pressure) fuel sprays. In both cases, fuel has been injected into an ambient pressure and temperature, quiescent, optical chamber. High-speed photography has been used to show that both fuel sprays comprised complex, vortex ring-like structures that exhibit a range of spatial scales that occurred over a broad range of timescales. An optimised, classical, forward scatter PDA has been used to measure the spray droplet diameters and velocities over a fine measurement grid.

An analysis of the temporal evolution of droplet axial velocities and diameters has shown that both sprays comprise of the classical three phases; initial unsteady, main quasisteady and exponential trailing phase. The results have shown that the timing and main features of these phases are highly dependent upon the location of the probe volume in the spray; the relative phasing of these periods has been different between the injectors studied. The vortex ring-like structures have been observed mainly during the decaying phase of spray development.

The mean vorticity magnitude has been calculated within consecutive time intervals where vortex ring-like structures could be identified. In the PFI spray, the range has been 13.75 to 14.25 ms after SOI. In the G-DI case, the range has been 3.75 to 4.75 ms after SOI. The maximum vorticity in the G-DI case has shown to be approximately 4 times greater than that computed in the PFI spray. The direct measurement of the axial and radial velocity components in the region corresponding to maximal vorticity was used. In both sprays, the radial component has shown the most scatter in data and has been close to zero for both the G-DI and PFI injectors. The observations in the G-DI case have been shown to be consistent with the predictions of the available vortex ring models. The axial components of velocity were positive for all values of t and the PFI injector has shown the most scatter in data. In the PFI case, the axial data has been approximated as $V_{\omega x}(t) = 1239t^{2.21}$. In the G-DI case, the axial data has shown a better curve fit that has been approximated as $V_{\omega x}(t) = 74.1t^{1.57}$. In both cases, *t* refers to time elapsed after the start of injection. Periodic oscillations of about 1 kHz have been noted in both sprays' axial and radial vortex velocity components.

The results have been presented by normalising the time with respect to the initial time at which the vortex ring-like structures were first observed (t_{init}) , with \bar{t} defined as t/t_{init} . The axial translational velocity component of the structures has been normalised by its value at the initial time, such that $\overline{V}_{\alpha\alpha}(\bar{t}) = V_{\alpha\alpha}(\bar{t} t_{init})/V_{\alpha\alpha}(t_{init})$. A curve fitting routine has been used to compare the experimental data to the model prediction. In the PFI case, the experimental data has been approximated as, $\overline{V}_{\alpha\alpha}(\bar{t}) = \bar{t}^{-2.97}$. For the high-pressure fuel spray, the best curve fit has been achieved for $\overline{V}_{\alpha\alpha}(\bar{t}) = \bar{t}^{-1.14}$

A conventional laminar vortex ring model is generalised by assuming that the time dependence of the vortex ring thickness ℓ is given by the relation $\ell = at^b$, where *a* is an arbitrary positive number, and $1/4 \le b \le 1/2$. In the case when $a=\sqrt{(2\nu)}$, where ν is the laminar kinematic viscosity, and b=1/2, the predictions of the generalised model are identical with the predictions of the conventional laminar model. In the case of b=1/4 some of its predictions are similar to the turbulent vortex ring models. In the long time limit, the translational velocity of the vortex ring and the translational velocity of fluid in the region of maximal vorticity are proportional to t^{3b} .

Although the experimental results have shown some scatter of data, the time evolution of \overline{V}_{ax} for the G-DI case has shown good agreement with the model that predicts the time evolution of \overline{V}_{ax} between $t^{-3/2}$ (laminar case) and $t^{-3/4}$ (turbulent case). In contrast, the agreement of time dependence of \overline{V}_{ax} predicted by the model and observed experimentally for the PFI injector has been poor. It is not clear whether this should be attributed to physical processes different from those described by the available models, or to excessive scatter of experimental data.

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NOMENCLATURE

Symbol	Quantity			SI Unit
а	constant definition c	in of ℓ	the	m s ^{-b}
A	constant approximat $= At^{B}$.	in ion of	the $V_{\omega x}(t)$	m s ^{-B-1}
b	Constant definition of	in of ℓ	the	dimensionless
В	constant approximat $= At^{B}$.	in ion of	the $V_{\omega x}(t)$	dimensionless

С	constant in the	dimensionless
	approximation of	
	$V_{\omega x}(\bar{t}) = \bar{t}^{-C}$	
$_{2}F_{2}$	generalised	dimensionless
	hypergeometric	
_	function.	
l	characteristic vortex	m
14	ring thickness	4 -1
M	vortex ring momentum	m's '
K_0	vortex ring radius	m dimonsionloss
ĸe	vortex ring Reynolds	dimensionless
	number ($\operatorname{Re} = \xi_0 \ell^2 / \nu$)	
t	time taken from the	S
	start of injection	
t _{init}	time at which vortex	S
	ring-like structures	
-	were first observed	dimensionless
I	$\frac{1}{4} = t/t$	dimensionless
I I	$l = l/l_{init}$	dimensionless
O_x	avial translational	unnensionness
	velocity	
<i>U</i>	normalised axial	dimensionless
	velocity of the fluid in	annensterness
	the region of maximal	
	vorticity	
v_n	$M/(4\pi^2 R_0^3)$	ms^{-1}
V	radial component of the	me ⁻¹
V _r	fluid velocity	1115
V	axial component of the	ms ⁻¹
, x	droplet velocity	1115
$V_{\rm our}$	the axial component of	ms ⁻¹
654	the translational	
	velocity in the regions	
	corresponding to	
	maximal mean vorticity	
	magnitude	
$V_{\omega x}(\bar{t})$	non-dimensional axial	dimensionless
	component of the	
	translational velocity in	
	corresponding to	
	maximal mean vorticity	
	magnitude	
Γ_{θ}	initial circulation of the	$m^2 s^{-1}$
0	vortex ring	
γ	Euler constant	dimensionless
θ	R_0 / ℓ	dimensionless
μ, μ ₀	dimensionless	dimensionless
	parameters used in the	
	definition of $U_{\omega x}$	
v	kinematic viscosity of	$m^2 s^{-1}$
	the fluid (mixture of air	
6	and droplets)	-1
ζ_0	characteristic vorticity	S
	used in the definition of	
	Revnolds number	
)//	di-gamma function	dimensionless
γ Subscripts	or Summa milenon	4111011510111055
init	initial	n/a
r	radial component.	n/a
x	axial component.	n/a
ω	maximal vorticity	n/a

0 initial n/a

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