A pressure-based numerical method for the simulation of compressible two-phase flow.

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Abstract

Our motivation is the extension of an originally incompressible multi-phase flow solver to the compressible regime. We aim at the simulation of evaporating droplets under extreme ambient conditions of high pressure and temperature where the compressibility effects inside the liquid phase can no longer be neglected. As a consequence, the flow equations require additional thermodynamic conditions given by an equation of state (EOS). This leads to stronger coupling effects at the material interface, which can cause spurious oscillations in the vicinity of the interface that have to be dealt with numerically by a special interface treatment. In order to stay as close as possible to the original incompressible numerical method, a pressure-based algorithm for the compressible multiphase flow simulation is chosen. Therefore, as it is the case for incompressible methods, pressure is used as primary variable. In a first approach, viscosity is neglected and we restrict ourselves to the compressible Euler equations. To circumvent their singular incompressible limit, the multiple pressure variables (MPV) scheme for compressible and incompressible flows is used in a conservative formulation. It is based on an asymptotic expansion of the pressure in terms of a global flow Mach number parameter M

$$p(x,t) = p^{(0)}(t) + M^2 p^{(2)}(x,t). \tag{1}$$

The pressure is split into a background pressure $p^{(0)}$ that formally satisfies the EOS in the incompressible limit case $M = 0$ and a hydrodynamic pressure $p^{(2)}$.

The above pressure decomposition is inserted into the Euler equations and the energy equation is reformulated in terms of pressure and kinetic energy using an appropriate EOS. The flow equations are discretized in a semi-implicit manner, similar to incompressible schemes. The spatial discretization is carried out on a Cartesian staggered grid.

With respect to the extension of the scheme to multi-phase flows, the material interface has to be tracked. This is done with a level set approach. Furthermore, the ideal gas EOS and the Tait EOS for liquids are used to describe the thermodynamic behavior of the different materials. It is well-known from the literature that density-based numerical schemes can suffer from unphysical spurious pressure and velocity oscillations at the interface location. Often complex interface treatment techniques are required to control this spurious behavior. It can be shown that our pressure-based scheme prevents this kind of oscillations due to the fact that pressure is used as primary variable in combination with an adequate spatial discretization. The material interface can therefore be treated as a contact discontinuity, which avoids more complex interface treatments like the ghost fluid technique.

The numerical scheme proves to give excellent results for one-dimensional multi-material shock tube test cases. The corresponding results agree very well with the exact, analytical solution of the considered Riemann problems. The method has also been assessed for three-dimensional flows such as the simulation of a planar shock wave moving through air and finally impinging on a water droplet. The results show typical wave structures inside and outside the droplet. The wave propagation inside the droplet is clearly visible including wave reflections at the gas-water interface.

The scope of the ongoing work is directed to further enhance the tracking and the resolution of the interface. At present, the discretization of the level set transport equation with a high-order discontinuous Galerkin method is investigated. A more sophisticated and accurate interface tracking scheme is a prerequisite for the inclusion of additional physical phenomena like surface tension to our simulation framework. Moreover, we plan to consider the effects of viscosity, extending the approach to the Navier-Stokes equations.

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