# Linear oscillations of viscoelastic drops used for measuring the polymer retardation time

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**Abstract**

Linear oscillations of liquid drops have been of interest since the time of Lord Rayleigh [1], who derived the equation for the oscillation frequency of an inviscid drop in a vacuum. Theoretical analyses following this work accounted for the drop viscosity and the density of the ambient gaseous medium [2]. A further generalisation was achieved by account for the viscosity of the ambient medium [3]. Nonlinear drop oscillations were investigated, among others, by Tsamopoulos and Brown [4], and by Becker et al. [5]. Despite the wide literature about drop oscillations available nowadays, literature on oscillations of viscoelastic drops is still sparse [6]. In our contribution to the conference we first treat the linear oscillations of a viscoelastic drop in a vacuum theoretically. In our linearised treatment, the Oldroyd 8-constant model, which we use as the rheological constitute equation, reduces to the Jeffreys model. The characteristic equation derived is analogous to the equation obtained by Chandrasekhar for a globe of a Newtonian liquid [7]. The material law in our analysis is characterized by two time scales - the stress relaxation time and the deformation retardation time of the polymer molecules in the solution. The material law represents the viscoelastic liquid behaviour by a frequency-dependent dynamic viscosity, so that a quasi-Newtonian behaviour is obtained. While the relaxation time (of spinnable liquids) can be conveniently measured using a filament stretching rheometer, the retardation time is not readily obtained experimentally. In simulations of viscoelastic flows it is state-of-the-art to set the retardation time to 1/10 or 1/8 of the relaxation time. The present contribution to the conference proposes a method for determining the polymer retardation time from damped linear oscillations of drops of the polymeric liquid. For this purpose, the deformations of an individual drop in damped oscillation are measured by high-speed imaging. Two parameters are obtained by image processing: the frequency and the damping rate of the oscillations. Making use of these two values, two parameters involved in the characteristic equation of the drop may be derived: the dynamic viscosity in the Jeffreys model, and the polymer deformation retardation time. First tests of this new method with aqueous solutions of two different polyacrylamides at different concentrations revealed reasonable data for the two quantities. They show that, for the polymer solutions investigated here, the ratio of the relaxation time, measured with a filament stretching rheometer, to the deformation retardation time is not a constant and lies outside the range between 1/10 and 1/8. This finding casts doubt on the state-of-the-art method used in simulations. The paper will present the derivation of the characteristic equation of the oscillating viscoelastic drop, the differences of its behaviour from the Newtonian case, and the deformation retardation times derived from measurements on damped oscillations.

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**References**

[1] Lord Rayleigh, J.W.S., *Proceedings of the Royal Society London A* 29: 71-97 (1879)
[2] Lamb, H., Hydrodynamics, 6th edn., Cambridge Univ. Press 475, 1997
[3] Miller, C. A., Scriven, L. E., *Journal of Fluid Mechanics* 32: 417-435 (1968)
[4] Tsamopoulos, J. A., Brown, R. A., *Journal of Fluid Mechanics* 127: 519-537 (1983)
[5] Becker, E., Hiller, W. J., Kowalewski, T. A., *Journal of Fluid Mechanics* 231: 189-210 (1991)
[6] Khismatullin, D. B., Nadim, A., *Physical Review E* 63: 061508 (2001)
[7] Chandrasekhar, S., *Proceedings of the London Mathematical Society* 9: 141-149 (1959)

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