To the Theory of Drop Shattering in a Speedy Gas Flows

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Abstract

The general differential equations of shattering kinetics for mass efflux from fragmented in speedy gas flow drop and for quantity of stripped daughter droplets are derived on a base of mechanism of gradient instability in conjugated boundary layers on drop surface. At some assumptions the system is integrated and laws of parent drop mass diminishing, shattering drop motion, as well as distribution function for stripped droplets by sizes, are obtained theoretically. Intermediate and final distributions of stripped droplets by sizes are calculated and discussed for various values of definitive parameters of the problem. Comparison of approximated results with those obtained by more precise numerical scheme get evidence of good enough agreement. Some general peculiarities of dispersion kinetics are described.

Distribution function of quantity of torn-off daughter droplets by sizes is obtained at arbitrary ratio $h=A/3H$ of mass efflux rate $A=0.46(1+(\alpha\mu)^{1/3})^{-1}(\mu^2/\alpha)^{1/6}$ to rate H of relaxational decreasing of relative velocity of parent shattering drop and gas flow. The result is based on earlier investigation of local surface instability with due regard to changing of velocity profile across the boundary layers, and along drop surface. For weak-viscosity liquids it revealed a new type of hydrodynamic instability – "gradient instability". Mechanism of this type differs from that of Kelvine – Helmholtz type and it is caused by large enough velocity gradient inside liquid boundary layer. The theory explains the "stripping" mode of breakup as quasi-continuous high-freequency dispersing from unstable part $\varphi_{\mathrm{cr}}<\varphi<\pi/2$ of drop surface. In speedy flows, when $\mathrm{GI}\gg\mathrm{GI}_{\mathrm{cr}}\approx 0.3$, values of polar angle of critical point are small: $\varphi_{\mathrm{cr}}\ll\pi$, so, most part of drop surface generates a mist of droplets (here $\alpha=\rho_{\mathrm{g}}/\rho_l$ and $\mu=\mu_{\mathrm{g}}/\mu_l$ are the media density and viscosity ratios, $\mathrm{GI}\equiv\mathrm{We}_{\mathrm{d}}\mathrm{Re}_{\mathrm{d}}^{-0.5}$ is criterion of gradient instability).

The equations of drop mass efflux (ablation) and of torn-off droplets quantity demand simultaneous solution of equation of drop motion in order to determine relative velocity, and equation for critical conditions of instability – to determine $\varphi_{\mathrm{cr}}(\tau)$. For speedy flows and nearly spherical shape of drop we have obtained: $M(\tau)=\left(1-A\left(\tau-\alpha^{0.5}X_{\mathrm{d}}(\tau)\right)/3\right)^3$, that indicates direct influence of drop motion law $X_{\mathrm{d}}(\tau)$ on its ablation law. Then, at approximation of experimental data for drop velocity versus time in the form $W=1-\exp(-H\tau)$, $H=2\sqrt{\alpha}$, or $\sqrt{\alpha}X_{\mathrm{d}}(\tau)=\tau-(1-\exp(-H\tau))/H$, we have integrated the equation of drop mass efflux and have obtained drop mass history: $M=\left(1-h(1-\exp(-H\tau))\right)^3$.

Distribution function was then obtained by integrating the equation of torn-off droplets quantity and the distributions $\Delta n(\tilde{r})$ for various $h,\mathrm{Re}_{\mathrm{d}}$ were calculated. Analysis showed, that values of h slightly higher than $h=1$ are inherent to flows behind shock and detonation waves, the values $h>4$ correspond to ablation of liquid meteoroids and the case $h<1$ – to incomplete shattering of viscous drops. When $h>1$, the whole drop is dispersed to the moment $\tau_{\mathrm{b}}=H^{-1}\ln\left(h/(h-1)\right)$. When $h<1$ dispersion terminates before drop is completely shattered, because of quick reducing of main reason of dispersing – relative velocity.

In view of lack of empirical data about laws of drop motion for various gas-droplets systems the main relations of shattering drop kinetics were obtained on reliable ground of theoretical laws. They were found analytically as solutions of system of non-linear differential equations of drop motion, drop mass efflux and quantity of torn-off droplets in a speedy uniform gas stream at neglecting by drop deformation. The two approaches of determination of drop motion law, which are based on empirical and theoretical methods, have led eventually to similar distributions, but theoretical approach has advantage being independent from lack of empirical data, so obtained formula is applicable to any gas – droplets system.