On Computational Investigation of the Supercooled Stefan Problem

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Abstract

Supercooled droplets often occur in clouds which are located at heights which aircrafts usually have to pass during start and landing. When these droplets impact on the surface of the aircraft, the crystallization of the water is triggered and this results in the formation of ice layer on the body of the aircraft. This phenomenon well known as airframe icing is recognized as a significant aviation hazard. It leads to increased aerodynamic drag and weight, associated with a reduction in lift and thrust. Exemplarily only in the U.S. airframe icing accidents led to 819 deaths within the last 19 years.

Supercooled water-ice transition results to be at least a three step process: nucleation, cooperative birth of critical nuclei in the whole sample and growth of the macroscopic solid phase. Concerning the last step two phases are to be considered, a supercooled liquid and a solid phase, and in between of these an interface or moving boundary separating the adjacent phases. The solidification process is described by the energy equation in each phase. The solidification rate is determined by the balance of the heat fluxes at the solidification front. It is governed by the Stefan condition.

In the present work a computational model for the macroscopic freezing mechanism under supercooled conditions relying on the physical and mathematical description of the two-phase Stefan problem is formulated. The relevant numerical algorithm, based on the finite volume method, is implemented into the open source software OpenFOAM©. For the numerical capturing of the moving interface between the supercooled and the solidified liquid an appropriate level set formulation is utilized. The heat transfer equations are solved in both the liquid phase and solid phase independently from each other. At the interface a Dirichlet boundary condition for the temperature field is imposed and a ghost-face method is applied to ensure accurate calculation of the normal derivative needed for the jump condition, i.e. for the interface-velocity in the normal direction. For the sake of updating the level set function a narrow-band around the interface is introduced. Within this band, whose width is temporally adjusted to the maximum curvature of the interface, the normal-to-interface velocity is appropriately expanded. The physical model and numerical algorithm are validated along with the analytical solution.

Understanding instabilities is the first step in controlling them, so to quantify all sorts of instabilities at the solidification front the Mullins-Sekerka theory of morphological stability is investigated. If there is only a low supercooling of the fluid, the interface is stable, i. e. the amplitude of the pertubation decays exponentially, whereas the amplitude rises if the supercooling reaches a critical value. In case the supercooling is greater than the critical value pertubation grows exponentially, i. e. the flat interface is not stable. If the frequency of the pertubation is raised in the case of an unstable interface it will at some point make the interface stable again, which is a result of the Gibbs-Thomson relation. This effect could be simulated with the described code as well.

In summary, we show excellent agreement between our computations and the theoretical results of Stefan's freezing model. Furthermore, we quantitatively check the morphological instability of the pertubated solidification front and compared it against the Mullins-Sekerka theory, obtaining agreement within a few percent. In future work, we intend to demonstrate that our computational model is also able to describe dendritic growth.

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