Effects of Liquid Physical Properties for Liquid Bubble Breakup Due to Airstreams

Teruhiko Yoshida[1*]
Department of Mechanical and Precision System Engineering,
Teikyo University, Utsunomiya, Japan
yoshida@uccl.teikyo-u.ac.jp

Abstract

To reduce the greenhouse gases, bio fuels are developing as an alternative fuel instead of fossil fuels. As carbon dioxide and so on are the causes of global warming, low carbon combustion is one of the crucial issues to conserve the global environment. Liquid atomization technology should contribute to the field of fuel combustion and others, such as the cooling of exhausted warm water at geothermal power plants and so on. A liquid bubble contains gas in a liquid drop. So, it has a larger surface area than a liquid drop of the same mass. It could have the advantage for fuel combustion and cooling of exhausted warm water. In order to apply it in these areas, effects of liquid physical properties for liquid bubble breakup due to airstreams were investigated. Experiments were conducted using a horizontal air-suction-type wind tunnel. Liquid and air were fed to a nozzle, and uniformly sized liquid bubbles were produced. The liquids used in the experiment were water and several kinds of ethanol in water solutions. The breakup processes and characteristic breakup patterns were precisely observed using a digital high-speed video camera. The displacement of liquid bubbles in the airstream was measured in the entire process. The deformation rate was also measured.

1. Introduction

The author has been studying on the deformation and breakup of liquid bubbles and liquid drop, such as breakup [1-3], displacement [4, 5], deformation [6, 7]. The liquid bubble has many important scientific applications such as the cooling of warm exhausted water, fuel combustion [8], and a compound liquid jet [9]. It is also useful for the study of structure of collapsing liquid drops and liquid bubbles by shock waves. By comparing a structure of collapsing liquid bubbles with liquid drops, the collapsing mechanism was revealed in a high-speed airstream behind a shock wave [10-12]. If liquid bubbles are used to combust bio-ethanol fuel, biodiesel fuel (BDF), and glycerin which is a by-product of BDF, more effective combustion could be obtained than in the case of liquid droplets.

The experiment was conducted to investigate the effects of liquid physical properties for liquid bubbles breakup due to airstreams. The breakup pattern is impacted due to surface tension, and viscosity of liquid. Several characteristic deformation and breakup patterns are observed. Some of them are never observed for liquid drops. The breakup processes and characteristic breakup patterns are precisely observed. The displacement of liquid bubbles in the airstream is measured in the entire process. The deformation rate, that is, a time variation of liquid bubble diameters is also measured. These data are obtained by analyzing the breakup processes recorded by the digital high-speed video camera.

2. Experimental Method and Conditions

2.1 Experimental apparatus

Figure 1 shows the experimental apparatus. The liquid was supplied to the nozzle by compressed air, and the liquid flow rate was measured using a measuring cylinder and a stopwatch. The airflow supplied to the nozzle was also measured by an air flow meter. Uniformly sized liquid bubbles were produced. The production rates of the liquid bubble were measured by means of a stroboscope. A horizontal air-suction-type wind tunnel is used in this experiment. It has a 60 mm × 150 mm cross section, and measures 500 mm in length. It is made of transparent acrylic plates. When uniformly sized liquid bubbles produce, the complicated breakup process can be observed precisely by adjusting the stroboscope frequency, regardless of the conditions, for example, in any continuous state and in the approximately still state. The phenomena of the deformation and breakup of liquid bubbles in the airstreams are observed using a stroboscope, and are recorded by a digital high-speed video camera or a digital still camera. The frame speed of the high-speed video camera was approximately from 1000 to 1900 frames per second. Back light source was used.

2.2 Liquid bubble and airstream

* Corresponding author: yoshida@uccl.teikyo-u.ac.jp

To produce the liquid bubbles, inner gas and the liquid are necessary. The inner gas was air, and the liquids were water, ethanol, and 70 %(vol.), 50 %(vol.), 30 %(vol.) ethanol in water solutions. By adjusting the airflow rate and the liquid flow rate, uniformly sized liquid bubbles are utilized in the experiment. The outer and inner diameters of liquid bubbles were calculated by the mass conservation law. To ensure the outer diameters, those sizes were measured on the PC display. The equations used for the calculation of the outer diameter, D_b, and the inner diameter, D_i, are listed in Eqs. (1) and (2), respectively. The liquid film thickness, δ, is calculated from Eq. (3),

$$D_b = \left\{\frac{6(Q_a + Q_l)}{\pi F_b}\right\}^{\frac{1}{3}} \tag{1}$$

$$D_i = \left(\frac{6Q_a}{\pi F_b}\right)^{\frac{1}{3}} \tag{2}$$

$$\delta = \frac{D_b - D_i}{2} \tag{3}$$

where, Q_a: airflow rate, Q_l: liquid flow rate, F_b: production rate of liquid bubble.

Figure 2 shows a sketch of a liquid bubble. A liquid bubble consists of a liquid film and inner gas. The range of airstream velocity u_a is from 0 to 40 m/s and is measured using a Pitot tube and a Betz-type manometer.

2.3 Weber number

As a liquid bubble consists of a liquid film and inner gas, the surface tension should be estimated considering the outer and inner liquid films. Therefore, the pressure difference, Δp, due to the surface tension, σ, between the atmosphere and liquid bubble should be shown

$$\Delta p = \frac{4\sigma}{D_b} + \frac{4\sigma}{D_i} = \frac{4\sigma(D_b + D_i)}{D_b D_i} \tag{4}$$

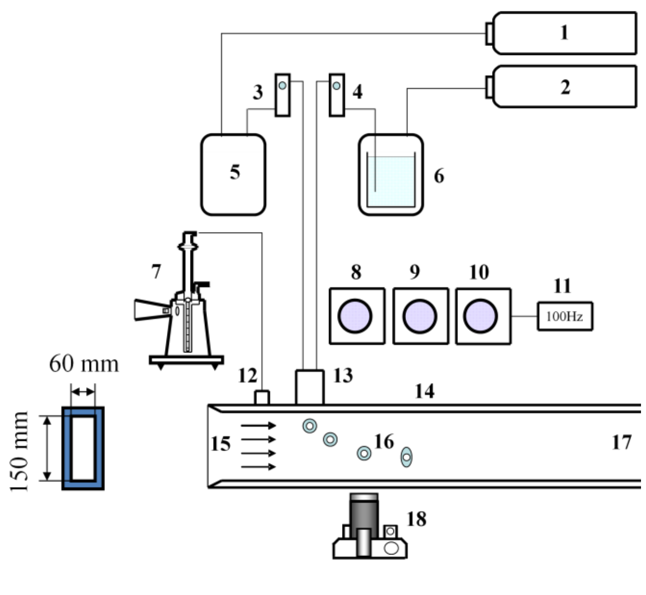
$4\sigma/D_b$ and $4\sigma/D_i$ mean the pressures due to the outer liquid film and the inner liquid film, respectively. Then the equivalent diameter, D_{eq}, is introduced as shown in Eq. (5)

$$D_{eq} = \frac{D_b D_i}{D_b + D_i} \tag{5}$$

The Weber number of a liquid bubble, We_b, is defined as Eq. (6), considering Eq. (5).

$$We_b = \frac{\rho_a u_a^2 D_{eq}}{2\sigma} = \frac{\rho_a u_a^2 D_b D_i}{2\sigma(D_b + D_i)}, \tag{6}$$

where, ρ_a: density of air, u_a: airstream velocity.



1, 2. compressed air, 3. air flow meter, 4. liquid flow meter,
5. air tank, 6. liquid tank, 7. Betz manometer, 8, 9, 10. stroboscope,
11.universal counter 12. static pressure tap, 13. Yoshida Nozzle,
14. wind tunnel, 15. airstream, 16. liquid bubble, 17. fan,
18. digital still camera or digital high-speed imaging camera

Figure 1 Experimental apparatus

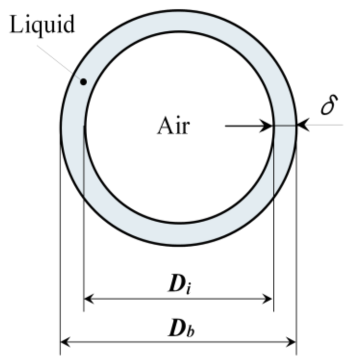


Figure 2 Sketch of a liquid bubble

Liquid bubbles deform when they are in airstreams, and the relative velocity changes with time. There are several options to choice the reference value. In this paper, the outer and inner diameters of a spherical liquid bubble are adopted as reference lengths. Reference velocity is the airstream velocity in the main flow.

2.4 Reynolds number

The bubble diameter changes with time while it is exposed to an airstream. So, the outer diameter of a spherical liquid bubble is adopted as a reference length. Reference velocity is the airstream velocity in the main flow. The Reynolds number for a liquid bubble, *Re*, is defined as Eq. (7).

$$Re = \frac{u_a D_b}{v} \quad , \tag{7}$$

where, D_b: liquid bubble diameter (outer diameter of liquid bubble), v: kinematic viscosity of air

3. Results and Discussion

3.1 Deformation and Breakup Phenomena

Figure 3 shows the deformation and breakup of liquid bubbles when they are exposed to airstreams in the wind tunnel. As the airstream velocity gradually increases from (a) to (e), the liquid bubbles show various figures. In Fig. 3(a), the air is in still condition, so the falling liquid bubbles keep approximately spherical shape. (b) The airstream velocity is u_a = 8.06 m/s. As the airstream velocity slightly has increased, the liquid bubbles slightly deform. Under this condition, all the liquid bubbles change their liquid film thickness a certain level and are changing it with time. A remarkable phenomenon can be seen on the second liquid bubble from the top. The liquid film at the stagnation point side becomes thick and the liquid is hanging down under the weight of the liquid, like a liquid bubble at the top position. At the initial stage after the liquid bubbles enter the wind tunnel, the relative velocity between airstream and liquid bubbles is high. However, as the liquid bubbles being pushed by airstream increase the velocity in the airstream direction, the relative velocity decreases. After being deformed, under this condition they tend to return to the original spherical shape as time passes. The liquid bubble is depressed at the stagnation point. In the area around the stagnation point, the figure with large curvature could be created. It could gather the liquid from other areas. Due to the liquid flowing into the area around the stagnation point, the liquid film becomes thin in the wake region. As the total pressure becomes comparatively low in the wake region, the air in the liquid bubble expands toward leeward part. Therefore, the leeward part of the liquid bubble expands with time. However, as the liquid bubbles move toward airstream direction, the relative velocity decreases as time passes. At the later stage, as the relative airstream velocity is small, the deformation gradually reduces with time. As a result, the liquid bubble regains its original spherical shape due to the action of the surface tension. The liquid bubbles have a thick liquid film around the stagnation point just after being exposed to airstreams. The region around the stagnation point receives a higher aerodynamic pressure, while the leeward part receives a lower one. Therefore the inner gas of the liquid bubble, of which pressure is higher than that at the leeward part of the liquid bubble, could move toward the downstream side of the bubble. Meanwhile, the liquid of the leeward part of the liquid bubble moves toward the upstream side of it. As a result, the liquid film thickness increases around the stagnation point, while decreases at the leeward part. However, in the fourth liquid bubble counting from the top one has just burst. The inner air breaks the liquid film and has just blown out. When the burst and non-burst liquid bubbles are observed as an intermingle state like this photo, the breakup pattern is called a transition breakup. This phenomenon shown in this photo could be the starting of the transition breakup. (c) The airstream velocity is u_a = 9.01 m/s. The tendency toward deformation is similar, but its intensity tends to increase. Many liquid bubbles begin to burst under this condition. In this photo, the third liquid bubble has already burst but the fourth one has not yet burst even though it is being exposed to the airstream for longer period than the third one. The breakup time is shorter and the breakup ratio is higher than those in (b). After bursting, the remaining liquid tends to gather due to the action caused by surface tension. As time passes, the remaining liquid forms stingy liquid chunk. The newly formed liquid chunks are also exposed to the airstream. In one case, the liquid chunks continue to move in the airstream, but another case the chunks breakup again. When they breakup again, this breakup pattern is called the secondary breakup.

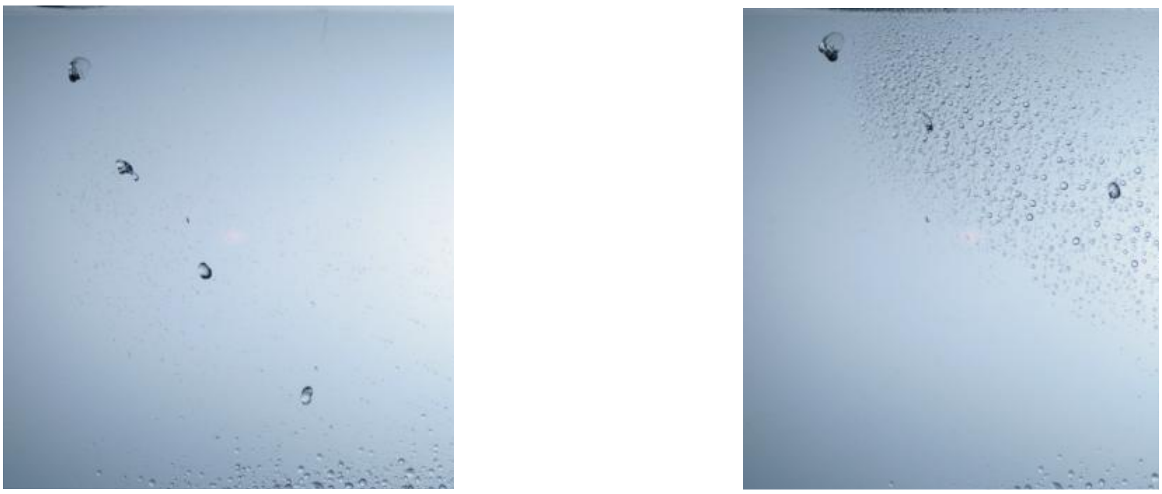
3.1.1 Transition breakup

When the airstream velocity increases gradually, several liquid bubbles begin to break up but others do not, as shown in Fig. 3(c). This is approximately the beginning of the transition breakup. Eventually all liquid bubbles begin to break up when the airstream velocity further increases. Just before all the liquid bubbles break up is the end of the transition breakup. The airstream velocity which causes the transition breakup is called the transition velocity. So, the transition velocity is defined as the airstream velocity that leads some of the liquid bubbles to breakup.

3.1.2 Critical breakup

When all the liquid bubbles begin to break up, the breakup is called the critical breakup. The lowest airstream velocity which leads all the liquid bubbles to breakup is called the critical velocity. So, the critical velocity is defined as the lowest airstream velocity that leads all the liquid bubbles to breakup. Figure 3(d) shows the situation around the critical breakup.

(a) u_a=0,We_b=0, Re=0 (b) u_a= 8.06 m/s, We_b= 1.12, Re = 36.8×10^3 (c) u_a= 9.87 m/s, We_b= 1.68, Re = 45.1×10^3



(d) u_a= 11.4 m/s, We_b= 2.24, Re = 52.1×10^3 (e) u_a= 15.1 m/s, We_b= 3.93, Re = 69.0×10^3

Water bubble, D_b= 4.57 mm, D_i= 3.99 mm, D_{eq}=2.13 mm, δ = 0.29 mm, F_b=58 pps

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Figure 3 Liquid bubble deformation and breakup due to airstreams

3.1.3 Secondary breakup

When a liquid bubble is exposed to airstreams slightly higher than the critical speed, the liquid bubble shows various breakup patterns. After bursting, the remaining liquid chunk is exposed to airstreams. In one case, the liquid chunk does not break, but in another case the liquid chunk breaks. In the transition breakup, a small particle is produced from the liquid chunk. When the remaining liquid chunk (or chunks in some cases) breaks, this breakup is called the secondary breakup. Several secondary breakup patterns are observed. In one case, a remaining liquid chunk forms a string and it would break into several small particles. In another case, when a liquid bubble is exposed to airstreams much higher than the critical velocity, the liquid bubble deforms like a balloon. After bursting the liquid film in the wake region, the remaining liquid changes into small particles and scatters. Secondary breakup patterns mainly depend on airstream velocity, physical properties of liquid, liquid bubble size, liquid film thickness and so on. The secondary velocity is defined as the airstream velocity that leads the remaining chunks to breakup.

3.2 Breakup pattern

By conducting the experiment, it is revealed that the breakup pattern depends on the physical properties of liquid. Ethanol bubbles easily break up compare to those of 50 %(vol.) ethanol in water solution. Liquid bubbles of 50 %(vol.) E/W deforms easily but hard to break up compare to the ethanol bubbles. Ethanol bubbles do not deform greatly, however, easily break up. Surface tension and viscosity is effective on deformation and breakup.

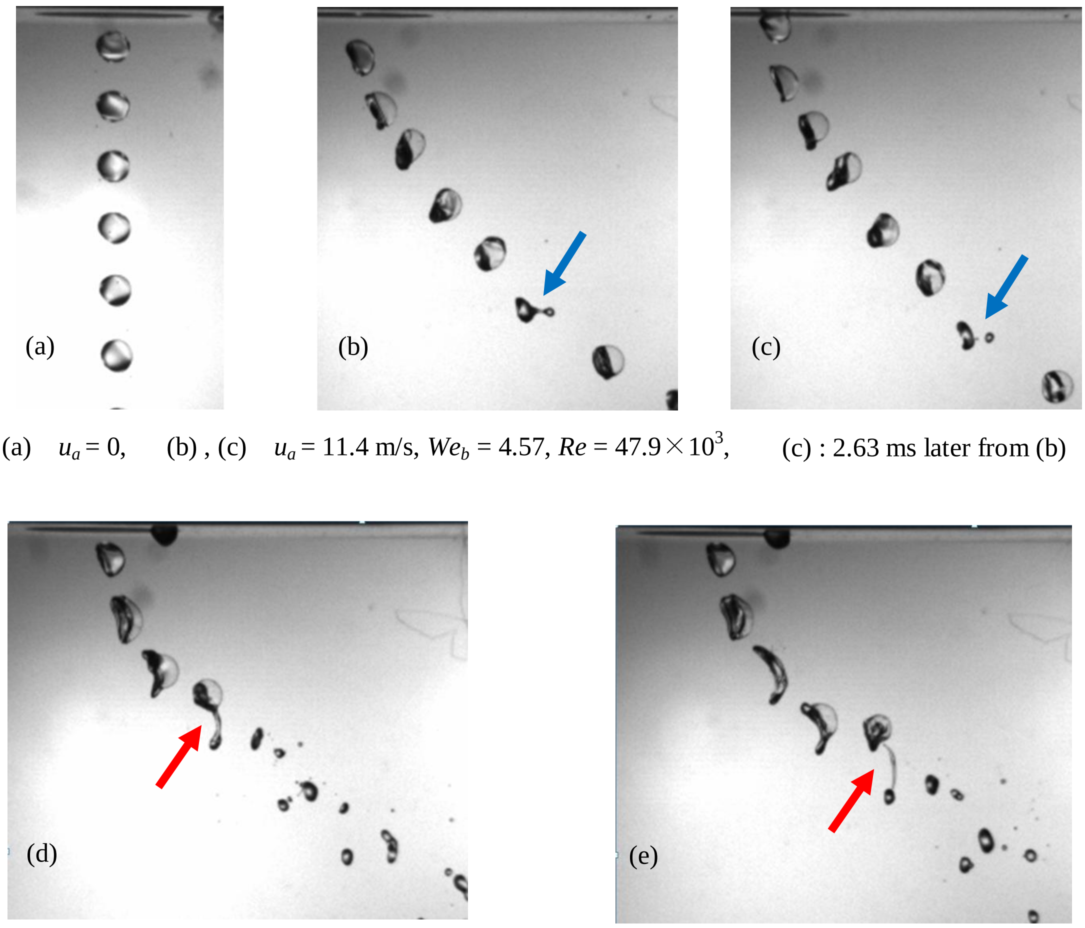
Liquid film thickness is also effective on the deformation and breakup. When the liquid film is thinner and surface tension is lower, the liquid bubble easily bursts without showing a large deformation. The liquid bubble size is also effective on the breakup. The initial transition velocity and the terminal transition velocity for smaller liquid bubbles are higher than larger ones. The small liquid bubble has a light mass compare to that of a large one. The small liquid bubble with light mass easily travels in the airstream direction compare to the heavy one. This means the rapid reduction of a relative velocity between the bubble and the airstream. This indicates that the small liquid bubble receives less aerodynamic force than that of the large one. Therefore, higher airstream velocity is required to lead the small liquid bubble to breakup compare to that of a large one.

3.2.1 Particle-separation-type breakup

Figures 4 (b), (c) show the one example of the secondary breakup, particle-separation-type breakup. This type of breakup is often seen in the transition breakup. After bursting, in many cases, the remaining main chunk and small one is connected with a string. Later, the small one is pulled closer and the two is united to form one stingy chunk. But in a few cases, the string breaks and a small particle is separated, as shown in (c).

3.2.2 Chunk-dripping-type breakup

Figures 4 (b), (c) show the chunk-dripping-type breakup. This is not a secondary breakup. When a liquid bubble is exposed to airstreams, the liquid forming the film flows into the stagnation point area, and the gathered liquid forms a chunk, and it tends to drip due to the gravity. The hanging chunk is connected with a string, but later the string breaks and it drips from the main body. This is considered to be a breakup in this paper. After dripping, the remaining liquid bubble keeps its shape in many cases. However, in some cases, the remaining liquid bubble bursts. The elongated string turns to one or a few particles. This type of breakup is observed for the



(a) $u_a = 0$, (b) , (c) $u_a = 11.4$ m/s, $We_b = 4.57$, $Re = 47.9 \times 10^3$, (c) : 2.63 ms later from (b)

(d), (e) $u_a = 12.1$ m/s, $We_b = 4.72$, $Re = 50.1 \times 10^3$, (e) : 5.26 ms later from (d)

70 %(vol.) Ethanol / water, $D_b = 4.21$ mm, $D_i = 3.76$ mm, D_{eq}=1.99 mm, $\delta = 0.23$ mm $F_b = 180$ pps

Figure 4. Particle-separation-type breakup (b), (c), and chunk-dripping-type breakup (d), (e)

liquid bubble of thick liquid film. High viscosity and relatively low surface tension also seem to be required for this type of breakup. So far, this type of breakup has not been observed against water bubbles. Water bubbles with thick liquid film enough to drip a chunk could not be easy to produce.

3.3 Liquid bubble displacement

Figure 5 and Fig. 6 show the relationships between the time and the dimensionless displacement of liquid bubbles. The displacement is normalized by the liquid bubble diameter as $X^*_{\mathrm{stag}} = x_{\mathrm{stag}}/D_b$, $X^*_{\mathrm{wake}} = x_{\mathrm{wake}}/D_b$. The liquid bubble moves in the airstream direction. The displacement of the liquid bubble with time could be useful, for example, for designing a combustion chamber. The displacement of the stagnation point side and the wake region side are traced by analyzing the data recorded by the digital high-speed video camera. The liquid bubble shows the complicated deformation, so the tracing lines of the stagnation point and the wake region sides are not smooth. The mark "×" means the breakup of the liquid bubble. In this case, the displacement means the position of scattered particles. In Fig. 5, the two results under the transition breakup, airstream velocity u_a= 8.06 m/s, are shown. One is the displacement of an unbroken liquid bubble, and another is of a broken bubble. The two displacements are similar until it bursts. When the liquid bubble bursts, the inner gas blows out. So, the bubble shrinks making its size small. As a result, the displacement of wake side reverses, while the displacement of stagnation point side continues to move forward.

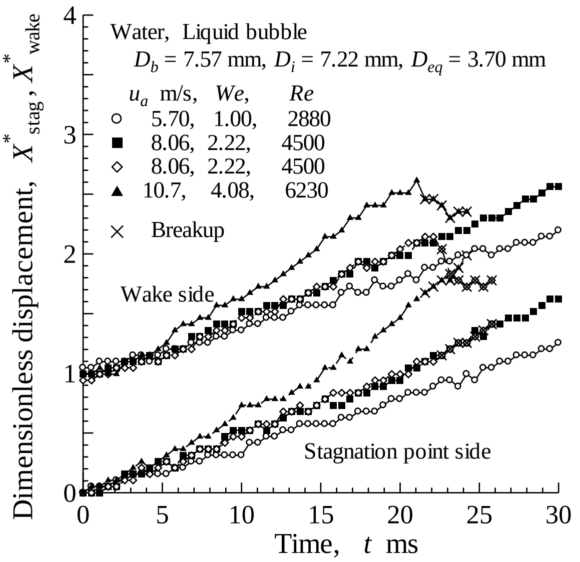


Fig. 5 Relationship between time and dimensionless displacement of LB in x direction

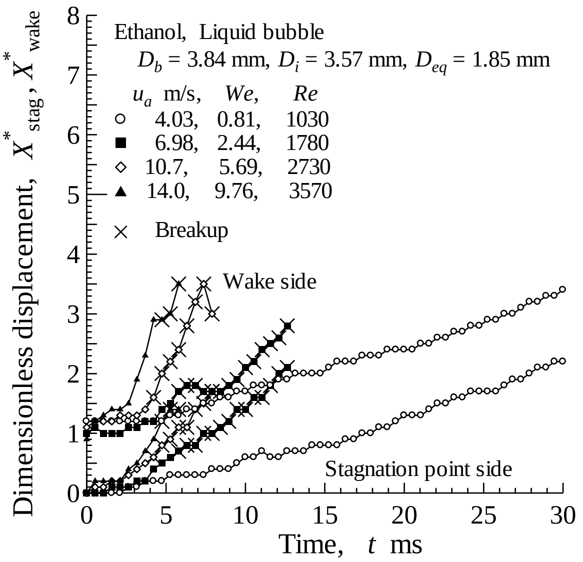


Fig. 6 Relationship between time and dimensionless displacement of LB in x direction

3.4 Deformation rate of liquid bubble

Two diameters are measured for a liquid bubble. One is the size in the normal direction against the airstream direction. This size is called a normal diameter. Another is the size in the parallel direction to the airstream. This size is called a parallel diameter.

Figures 7 and 8 show the relationships between the time, t, and the dimensionless normal diameter, D_n^*. When the airstream velocity is low, it repeats slight increase and decrease. As the time passes the deformation subsides, and finally almost no deformation is observed. When the airstream velocity is high and is enough to lead the liquid bubble to intense breakup, the normal diameter increases monotony.

3.5 Breakup time

Figure 9 shows the relationship between the airstream velocity, u_a, and the breakup time, t_b. The breakup time means a lifetime of a liquid bubble. It can be described as the period when a liquid bubble exists in airstreams without breakup. The instant when a leeward part of a liquid bubble burst is the end of a liquid bubble. When a liquid bubble is exposed to airstreams, the liquid forming a liquid film flows towards the stagnation point. The front part, or the area around the stagnation point, is depressed. And a ring-shaped figure like a donut is formed. Its curvature is so large that it could gather and keep relatively massive liquid. The gathered liquid comes from the leeward part of the liquid bubble. As a result, the leeward liquid film reduces the thickness and it tends to burst. When the airstream velocity gradually increases and reaches at a certain airstream velocity, a few liquid bubbles begin to breakup. The breakup time under the transition breakup is obtained in a wide range. As the airstream velocity increases, the breakup time range tends to concentrate to a certain value and the breakup time tends to short. Under the secondary breakup, the breakup time range further tends to concentrate to a value and its value tends to short with increasing airstream velocity. The breakup time for smaller liquid bubbles is short compare to that of larger ones. The relative velocity between the airstream and the liquid bubble is high just after the liquid bubble enters the wind tunnel, but as the time passes, the relative velocity decreases. Smaller liquid bubble has high surface tension pressure than that of larger one. So, the high airstream velocity is required to break the small liquid bubble and as a result breakup time becomes short. The breakup time tends to short as the Weber number for liquid bubbles increases. It is revealed that the deformation and breakup pattern of liquid bubbles differs greatly depending on the physical properties of liquid.

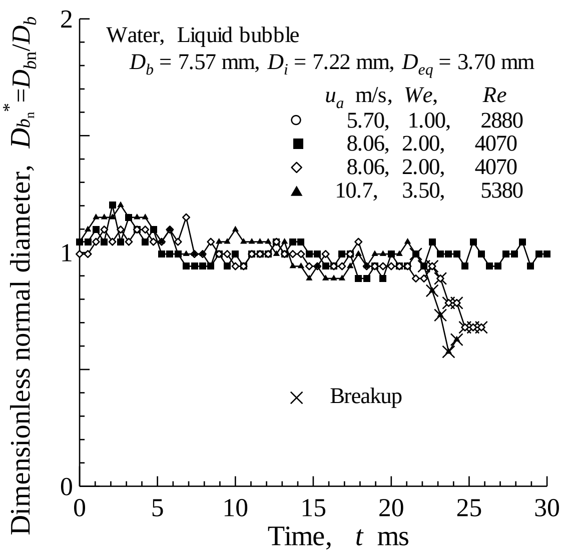


Fig. 7 Relationship between time and dimensionless normal diameter of LB

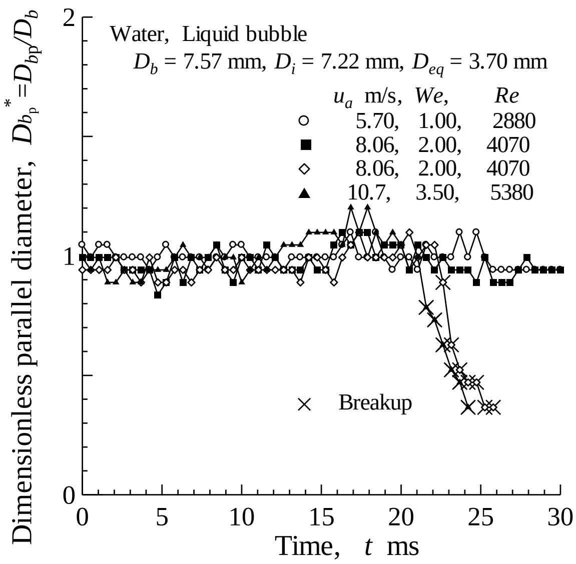


Fig. 8 Relationship between time and dimensionless parallel diameter of LB

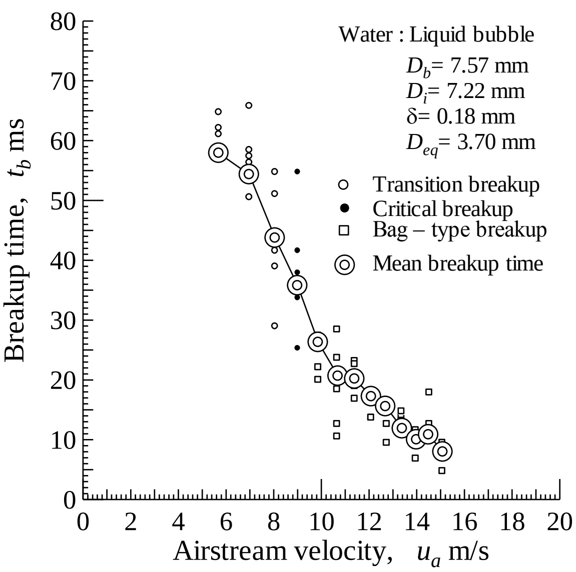


Fig. 9 Relationship between airstream velocity and breakup time

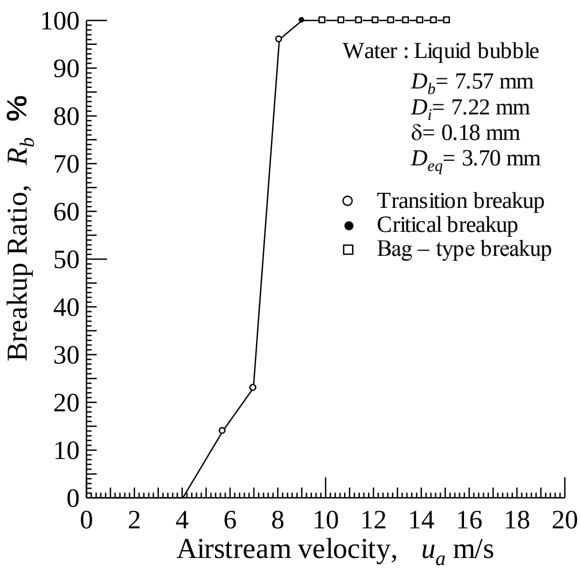


Fig. 10 Relationship between airstream velocity and breakup ratio on number of LB

3.6 Breakup ratio on number of liquid bubble

In the transition breakup, some liquid bubbles break but others do not. Figure 10 shows the relationship between airstream velocity, u_a, and breakup ratio on liquid bubble number, R_b. The breakup ratio is defined as the ratio of the number of broken liquid bubble to the number of the tested liquid bubbles. For example, when 10 liquid bubbles break up in 100 ones, the breakup ratio is defined as 10%. When the airstream velocity increases to a certain level, the phenomena is observed that a few liquid bubbles begin to breakup but others do not, as shown in Fig. 3(b). It means the beginning of the transition breakup. After the beginning of the transition breakup, the breakup ratio of the liquid bubbles increases as the airstream velocity increases. In the transition breakup, at a lower airstream velocity, a few liquid bubbles begin to break. Some liquid bubbles break, but others do not. As the airstream velocity increases, the number of broken liquid bubbles eventually increases, that is, the breakup ratio increases. Finally the breakup ratio reaches to 100 percent and it means the critical breakup. After the critical breakup, as the airstream velocity increases, the breakup ratio keeps 100 percent. The most typical breakup type in the critical condition is the burst-type breakup. As the airstream velocity increases the breakup pattern changes into the bag-type breakup or other breakup patterns. When the liquid film thickness is thin, the liquid bubble is easy to burst.

Even though the airstream velocity is zero, it is unusual, but a breakup phenomenon of falling ethanol bubbles is observed. This could mean that the low surface tension is one factor of easy breakup.

4. Conclusions

The experiment on the effect of physical proprties of liquid which forms liquid bubble was carried out. As a result, following conclusions were obtained.

1. New breakup pattern, chunk-dripping-type breakup, was observed.
2. Displacement of liquid bubble was measured on the stagnation point side and the wake side.
3. The dimensionless deformation rate of liquid bubbles was measured on the normal and the parallel directions against the airstream direction.
4. The breakup time under the transition, critical and bag-type breakups , and so on was determined.
5. The breakup ratio under the transition breakup was determined.
6. Surface tension and viscosity of liquids are effective for deformation and breakup of liquid bubbles.
7. Liquid film thickness is aslo effective for deformation and breakup of liquid bubbles .

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