To the Theory of Drop Breakup at a Relatively Small Weber Numbers

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Abstract

The theoretical explanation of "bag" and "bag-with-a-stamen" generation is suggested. Hydrodynamic instability of drop surface due to inertia forces of drop acceleration is enlisted as a hypothetical mechanism of breakup at low Weber numbers, and a simplified model of deformed drop, regarded as a thin liquid accelerating layer, is elaborated for instability treatment. The linear non-viscid instability analysis shows, that in gas flows with a relatively small Weber numbers development of aperiodic unstable disturbances can be the real mechanism of generation of "bag" and "bag-with-a-stamen", which is the preliminary of drop disintegration. Results of present investigation allow to give simple theoretical explanation of such a complicated modes of breakup as "bag" and "claviform". This simplicity as well as uniformity of explanation of other modes at greater values of Weber number confirm the original hypothesis. Some reasons for causing "chaotic" type are also suggested.

Introduction

It is well known, that interaction of a gaseous flow of even low intensivity with drops, sprays and films can lead to atomization of liquid phase, which is important in various energetic machines as it speeds up essentially the evaporation, formation of homogeneous flammable mixture and thereby – ignition and heat release. Breakup of liquid drops is a key process in particular in high-speed gas-droplets systems, such as jet engine flows, detonative aerosols, where it proceeds at great values of Weber number We. Being the ratio of hydrodynamic forces to surface tension forces, We is the main criterion governing the mechanical processes of deformation and breakup of drop. Hence, difference in We values predetermine difference in modes of breakup of drop: for liquids with low viscosity at great We breakup occurs as intensive "stripping" and "catastrophic" modes, while at low We – as quite slow "bag", "claviform" and "chaotic". In various kinds of furnaces, which use the energy of injected liquid fuel, such as Diesel engines, combustors, etc., shattering proceeds at a relatively small We values and performs as a great preliminary deformation of a drop into a thin liquid disk, which then blows out into vast "bag" or "bag with a stamen" (fig. 1). The process finishes with rupture of thin film of bag which forms cloud of fine daughter droplets [1]. Viscosity can play important suppressive role only for liquids of high viscosity with Ohnesorge numbers $Oh > 0.1$: Joseph et al. [2] found experimentally for high-viscosity and viscoelastic drops that "bag" and "claviform" modes can perform at very high Weber numbers.

Breakup phenomena have been well enough studied experimentally in wide range of We (see for instance [3]–[5]). However, theoretical investigations have run across insuperable difficulties, associated with complexity of internal processes: extreme degree of drop deformation, complicated unsteady field of streamlining with unknown deformable surface of drop, onset of inertia mass forces due to drop acceleration, favorable conditions for development of hydrodynamic instabilities of drop surface. As a result, an adequate theoretical model, which

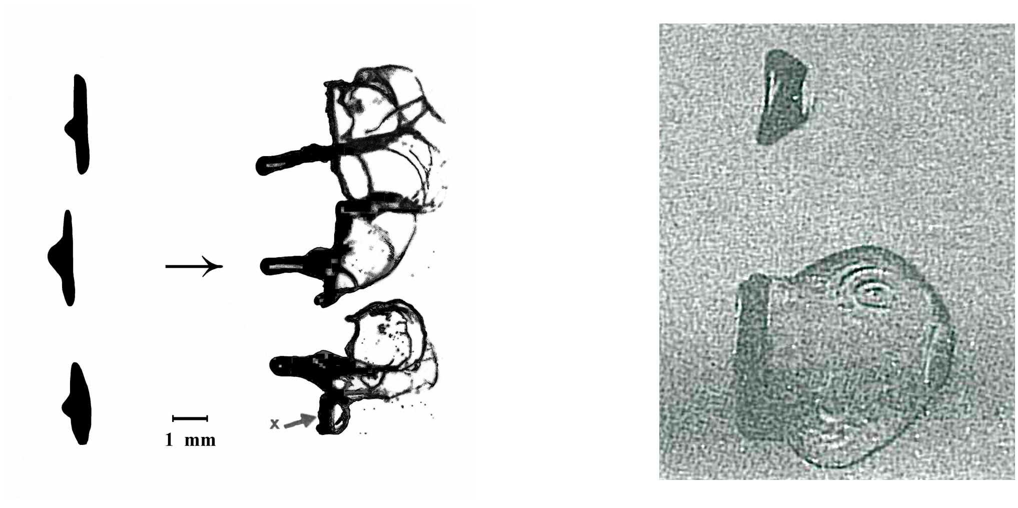

Figure 1 Experimental observations of breakup. Left: "claviform" mode, [6]; right: "bag" mode, [1].

would permit to obtain analytically reliable size distributions of torn off droplets, moments of their formation and to interpret qualitatively various types of breakup in wide range of flow conditions, intensive as well as weak, has not been elaborated yet in spite of numerous attempts.

Instability hypotheses were often used in investigations aimed on drop breakup. As ripples had been seen on a drop surface at experimental observations, both types of hydrodynamic instability – Rayleigh – Taylor and Kelvine – Helmholtz, – were enlisted as possible reasons of shattering to discover the breakup mechanism: the former has favorable conditions in vicinity of stagnation point, the latter – at the edge of a drop. Nevertheless, realization of the hypotheses was too rough and did not properly take into account all the important features of the process. For example, in comprehensive work of Engel [7] an application of experimental results of Lewis [8] for "fingers" penetration velocity $U_\mathrm{T} = 0.239(g\lambda)^{1/2}$ was combined with arbitrarily chosen radius of curvature of unstable disturbance, which corresponded to much smaller values of Weber number, than in her own experiments. Fishburne [9] has attractively combined mechanisms of boundary layer stripping and drop piercing by unstable Taylor-type disturbances, but the principle of superposition has been applied to non-linear stage of finite disturbance amplitude, which discredits obtained results. Moreover, it was shown later [10] that boundary layer stripping mechanism is inadequate as for the influence of liquid viscosity on drop breakup time t_b.

As far as unstable Taylor-type disturbances are concerned, their realization in the range of greater Weber numbers $\mathrm{We}=\rho_\mathrm{g} V_\infty^2 d_0/\sigma$ is doubtful (here d_0 is initial drop diameter, V_∞, ρ_g are the gas stream velocity and density). Indeed, as wavelength $\lambda_\mathrm{T} \sim \mathrm{We}^{-1/2} d_0$ is much smaller than d_0 for large We, the system of unstable disturbances must be a lengthy narrow penetrating channels. For example, when $\mathrm{We}=10^4$, channel's length-to-width ratio is $l/w=26$, assuming $l=d_0$. So, this channels are to be destroyed by intensive flows inside a drop, which are caused by permanent drop deformation and by the flow in liquid boundary layer, both flows have the velocity scale $U \sim \alpha^{1/2} V_\infty \mathrm{We}^{1/2}$, $\alpha=\rho_\mathrm{g}/\rho_l$ being the density ratio of the media. The velocity scale of Taylor disturbance penetration is $U_\mathrm{T} \approx \alpha^{1/2} V_\infty \mathrm{We}^{-1/4}$, which much less for $\mathrm{We} \geq 10^2$. Under this obstacle the possibility of Taylor unstable disturbances realization at greater We becomes unrealistic in general.

An investigation of non-viscid instability of flows in conjugated boundary layers on drop surface with a continuous linear approximation of real velocity profile was undertook in [11] for large values of We. The continuity was kept as an important property of the velocity profile, in contradistinction to other known investigations, where classic Kelvine – Helmholtz relationships based on tangential discontinuity of velocity profile were applied. It was shown in this work, that Kelvine – Helmholtz model can be used only when kinematic viscosity of liquid is greater than that of gas. Otherwise, the velocity deficit of gradient flow in the liquid boundary layer has essentially stabilizing effect, which leads to a mechanism of so-called "gradient" instability. Within the frames of this model the "stripping" mode of breakup is explained as periodic high-frequency dispersion of fine liquid particles from unstable part of drop surface, adjacent to the edge. Thus, a condition for existence of periodic unstable disturbances on drop surface was theoretically obtained [12], which coincides strictly enough with empirical criterion [13] for "stripping" mode of breakup:

$$\mathrm{We}\mathrm{Re}^{-0.5} \geq 0.3, \qquad 1)$$

where $\mathrm{Re} = \rho_\mathrm{g} V_\infty d_0 / \mu_\infty$ is Reynolds number for a drop. The elementary theory of stationary detonation waves in liquid aerosols was grounded on the basis of this model, which permitted to calculate the two-phase flow behind detonation front and to solve thus in closed form the general problem of heterogeneous detonation: to determine velocities of propagation of self-sustained detonation regimes [14].

The foundations of drop breakup theory for speedy flows were laid in [15]–[17] where the application of approximative analytical approach, based on mechanism of gradient instability, allowed to derive the governing differential equations of shattering process: equations of parent drop mass reduction and of torn off droplets quantity. Their integration provided the general relationships of the theory. Obtained theoretically in [15]–[17] distribution functions of torn off droplets by sizes, the law of motion of shattering in gas stream drop and the law of drop mass reduction (ablation law) gave the opportunity to describe quantitatively further processes of rapid acceleration and evaporation of the torn off droplet mist in a wake of a shattering drop at greater We.

At low We the problem related to the revealing of mechanism of a drop distortion remains still opened. In spite of importance of atomization, the mechanisms of breakup are still not well understood [18]. Engineering applications of simplest empirical models of breakup has led to a possibility of calculation of various properties of diesel engine performance [19, 20]. But there was none explanation given to the mechanism of generation of "bag", "bag with a stamen" and "chaotic" modes. The generation of "bag" can not be explained by a simple sagging due to action of free-stream dynamic pressure on liquid disk, or vortex, as was claimed in some papers [21]

– [23]. Moreover, these explanations contradict to the process of formation and stable existence of a "stamen" (fig. 1) at $30 < \mathrm{We} < 60$, which seems to be inexplicable, as well as generation of "threads" and "bubbles" at "chaotic" mode in range $70 < \mathrm{We} < 100$. Why does the "stamen" disappear at greater and lower values of We ?

In a slow flows, which do not satisfy to condition (1), periodic disturbances are stabilized on drop surface (or they have wavelengths greater, than the drop thickness d and can't "catch" the liquid disk edge), but aperiodic instabilities due to action of inertia forces of acceleration are still possible to effect. It is shown in present communication for non-viscid liquids, that in flows, which correspond to the range $5 \le \mathrm{We} \le 100$, the blowing of a "bag" and generation of a "stamen" may be explained as development of aperiodic unstable disturbances. This concept was first declared in [12] and empirically confirmed in [5]; to confirm it theoretically we will first construct a simplest model of flattened accelerating drop and will investigate the instability of liquid disk.

Formulation of the problem.

To construct the mathematical model let us adopt some assumptions. An experimental observations [1], [3]-[7], [10], [21] showed, that in gas flows with low We the drop easily takes the form of a thin disk, and the dimensionless lateral drop deformation $D = BB'/d_0$ exceeds values $D_{\max} \cong 2.5 \div 4.0$, so that the degree of drop flattening (ratio BC/BB', fig. 2) is well below 0.1. This blunt-body shape allows us to neglect by velocity of streamlining near front BB' and rear CC' parts of deformed drop. Mentioned above stability of streamlined edges BC and $B'C'$ to periodic disturbances reduces our problem to investigation of the aperiodic disturbances development on the front and rear parts of a liquid layer, which are exposed to the action of inertia forces of acceleration. An application of classic Rayleigh – Taylor theory at $\mathrm{We} \le 100$ requires its specifying, because the wavelength of most unstable disturbance, λ_{m}, becomes comparable with, and greater than the thickness d of a deformed drop. Therefore the disturbances at the front and rear sides of the layer begin to interact strongly. In this case the influence of rear part of drop CC' must be taken into account by setting an appropriate boundary conditions at a mathematical formulation of the problem. As far as influence of edges BC and $B'C'$ is concerned, it apparently may be neglected at rough approximation as insufficient, because extreme drop flattening makes this influence local. The fact that testifies to the favor of this assumption is that the aperiodic disturbances are not shifting along the surfaces of the layer, so, their development in a least degree depends on the edge conditions. Over the range $\mathrm{We} \le 100$ compressibility of the media may be neglected. As a weak-viscosity liquids are the most applicable, let us consider the case of non-viscid media.

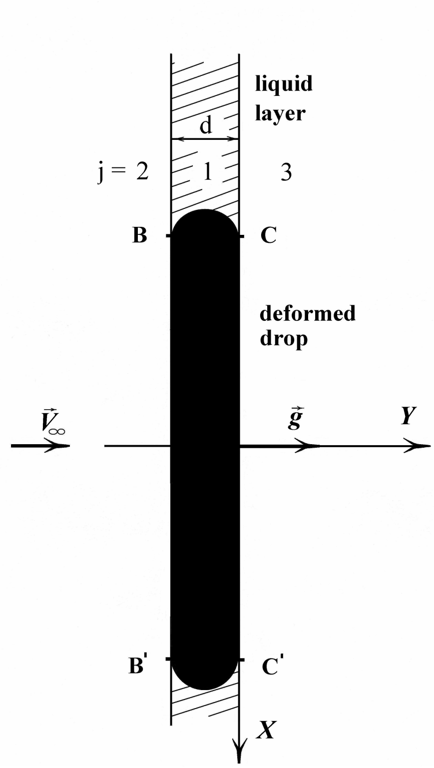

Fig. 2 Deformed drop and modeling liquid layer.

Let us carry out the instability investigation by the small perturbations technique for infinite in X-direction plane liquid layer, which is modeling the flattened drop (fig. 2). Let d and ρ_l be the thickness and density of the layer which is moving with drop acceleration $\vec{g}$ in the flow direction. In coordinate system (X,Y), which is rigidly connected to layer, the latter remains at rest. Let at $t=0$ small disturbances of "running-wave" type $\varepsilon_n = E_n(hx - i\omega t)$, $n = 1, 2$ arise on the front $y=-d$ and rear $y=0$ layer surfaces. Then the small disturbances of pressure p' and velocity components V_x', V_y' satisfy to the linearized equations of the hydrodynamics of ideal incompressible fluid, which have the partial solution of the form:

$$\left(p_j', V_{xj}', V_{yj}'\right) = \vec{A}_j \exp(\pm hy + ihx - i\omega t). \qquad 2)$$

Here $h = 2\pi/\lambda$ is wavenumber, ω is complex "frequency" of disturbance, $j = 1,2,3$ for liquid layer, $-d < y < 0$, and gas domains $(y < -d)$, $(y > 0)$, respectively; $\vec{A}_j$ are amplitude constants. Natural requirement for disturbances to be limited at infinity leaves in (2) sign "+" for domain "2" and "–" for domain "3".

The solutions in domains 1, 2, 3 must be conjugated now on separation surfaces $y = -d + \varepsilon_2$, $y = \varepsilon_1$ by setting the boundary conditions of ideal media interaction, which consist in coincidence of normal components of stresses (allowing for surface tension σ and for buoyancy effect in a field of inertia forces $-\rho\vec{g}$), and velocities of gas and liquid as well as velocity of separation surface. At $y=-d+\varepsilon_2$ they may be written as follows:

$$V'_{y1}-\frac{\partial\varepsilon_2}{\partial t}=0, \qquad V'_{y2}-\frac{\partial\varepsilon_2}{\partial t}=0, \qquad p'_2-p'_1+\sigma\frac{\partial^2\varepsilon_2}{\partial x^2}-(\rho_g-\rho_l)g\varepsilon_2=0\,. \tag{3}$$

Layer surface $y=0$ corresponds to the rear side of the drop, where the hydrodynamic conditions are too complicated to be formulated exactly due to intricate character of streamlining of deformed drop. To set a precise boundary conditions is therefore a knotty hydrodynamic problem. To extricate, let us consider two kinds of boundary conditions at $y=\varepsilon_1$:

1). "Free surface" conditions correspond to discontinuous regime of disk streamlining at a relatively high Reynolds numbers when the pressure just behind the liquid disk is small and action of the rear gas is negligible:

$$V'_{y1}-\frac{\partial\varepsilon_1}{\partial t}=0, \qquad p'_1+\sigma\frac{\partial^2\varepsilon_1}{\partial x^2}-\rho_l g\varepsilon_1=0\,. \tag{4}$$

2). For regular regime of streamlining at sufficiently small values of Reynolds number, when conditions are almost symmetric on both windward and leeward surfaces of a drop, it is natural to adopt the same hydrodynamic conditions at $y=\varepsilon_1$, as at $y=-d+\varepsilon_2$. These two kinds of boundary conditions are the opposite and one can expect that the real hydrodynamics on the rear drop surface are described by some intermediate conditions.

Consider the first case.

By substituting (2) into (3), (4) we obtain the system of five linear uniform equations with respect to amplitude constants $E_1, E_2, A_{11}, A_{21}, A_{12}$ (second subscript denoting the domain considered). The necessary condition of existence of non-trivial solution of this system yields following characteristic equation with respect to dimensionless frequency $z\equiv\omega d/V_\infty$:

$$z^4(1+\alpha\coth\Delta_1)-z^2\alpha\Delta_1^2\left((\alpha+\coth\Delta_1)H_1+\left(\frac{\alpha G}{\Delta_0}+H_2\right)\coth\Delta_1\right)+\alpha^2\Delta_1^4H_1\left(\frac{\alpha G}{\Delta_0}+H_2\right)=0\,, \tag{5}$$

where $\Delta_0=\sqrt{2}hd_0$, $\Delta_1=\sqrt{2}hd_1$, $H_1=\Delta_0/\mathrm{We}+G/\Delta_0$, $H_2=\Delta_0/\mathrm{We}-G/\Delta_0$, $G=gd_0/\alpha V_\infty^2$ – dimensionless drop acceleration. The values of wavenumber are multiplied here by the factor $\sqrt{2}$ at application of results for plane flow to regarded axisymmetric flow around the disk, because corresponding wavelengths for the two cases are related as $\lambda_{\mathrm{axi}}=\sqrt{2}\lambda_{\mathrm{plane}}$. Value $G=2$ was chosen in accordance with observed [3], [10] drop motion law $x_d=kt^2$ with approximately constant acceleration: $K\equiv kd_0/\alpha V_\infty^2=G/2=0.8\div1.1$.

Analysis of (5) shows that at $\Delta_0H_2+\alpha G<0$, i.e.

$$(hd_0)^2<(1-\alpha)\frac{G}{2}\mathrm{We} \tag{6}$$

there exists a pair of purely imaginary conjugated roots, one of them being unstable, $\mathrm{Im}(\omega)>0$, and it provides instability of aperiodic character:

$$z=i\Delta_1\sqrt{\frac{\alpha}{2}}\sqrt{\sqrt{F^2-\frac{4H_1(\alpha G+H_2\Delta_0)}{(1+\alpha\coth\Delta_1)\Delta_0}}-F}\,, \qquad F=\frac{(2\coth\Delta_1+\alpha)\Delta_0^2+\alpha G(1+\coth\Delta_1)\mathrm{We}}{(1+\alpha\coth\Delta_1)\Delta_0\mathrm{We}}\,. \tag{7}$$

The calculated dependence of increment of amplitude growth $\mathrm{Im}(\omega)$ on wavenumber shows the existence of disturbance h_m, which has maximum value of increment. Due to exponential dependence of amplitude on time it is natural to assume, that namely this fastest disturbance is dominant over all others and it realizes the mechanism of hydrodynamic instability on final nonlinear stage. The application of extremum condition $\partial\mathrm{Im}(\omega(\lambda))/\partial\lambda=0$ in the range of instability (6) for the case $\alpha\ll1$ of gas – liquid interaction yields the equation for Δ_{1m}:

$$3(\coth^2\Delta_{1m}-1)-\Delta_{1m}\frac{\coth\Delta_{1m}}{\sinh^2\Delta_{1m}}+\left(\frac{G\mathrm{We}}{\Delta_{0m}^2}\right)^2=\frac{\mathrm{We}}{\Delta_{0m}}\left(3\coth\Delta_{1m}-\frac{\Delta_{1m}}{\sinh^2\Delta_{1m}}\right)\sqrt{\left(\frac{\Delta_{0m}}{\mathrm{We}}\right)^2(\coth^2\Delta_{1m}-1)+\left(\frac{G}{\Delta_{0m}}\right)^2}\,. \qquad 8)$$

For short waves, when $\coth\Delta_1 \cong 1$, (8) gives the well-known values of wavenumber and period of growth of amplitude in e times for dominant wave for the case of semi-spaces:

$$\Delta_{0m}=\left(\frac{G\mathrm{We}}{3}\right)^{0.5}, \qquad \tau_{\mathrm{e.m.}}=\left(\frac{4G^3\mathrm{We}}{27}\right)^{-0.25}\frac{d_0}{\sqrt{\alpha}V_\infty}\,. \qquad 9)$$

Considering the second type of boundary conditions at $y=\varepsilon_1$, we obtain biquadrate characteristic equation, which to accuracy of order $O(\alpha)$ coincides with equation (5). Hence, for a gas-liquid systems, when $\alpha \ll 1$, we can expect, that hydrodynamics on the rear side of a drop has a weak influence on the development of unstable aperiodic disturbances at relatively small We, so, for small α the dominant disturbance is determined by (8).

Results and Discussion

The hypothesis and the idea of this paper is that in slow flows, when (1) is not satisfied, the mechanism of hydrodynamic aperiodic instability does work at the drop. The inertia forces of the accelerating liquid disk pull out "bag" and "bag with a stamen" by means of aperiodic unstable disturbances. This action naturally to connect with the dominant disturbance λ_m, being obtained from (8). For various dimensionless values of deformed drop thickness, $q \equiv d/d_0 = D^{-2}$, we have calculated with $G=2$ the wavelength, $\lambda_m(\mathrm{We})$, and the characteristic time interval, $\tau_{\mathrm{e.m.}}(\mathrm{We})=\mathrm{Im}^{-1}(\omega(\lambda_m))$, as functions of Weber criterion. The dependencies $L(\mathrm{We})=\lambda_m(\mathrm{We})/d_0$, $T_{\mathrm{e.m.}}(\mathrm{We})=\tau_{\mathrm{e.m.}}(\mathrm{We})/\tau_{\mathrm{ch}}$ show (fig. 3, 4), that at We<70 the influence of layer thickness is essential (here $\tau_{\mathrm{ch}}=d_0/\alpha^{1/2}V_\infty$ is characteristic time scale for the breakup process [3]). Thus, the drop deformation $D=q^{-1/2}$ plays an important role for the dynamics of instability development. In the limit $q \to 1$ (practically at $q \approx 0.7$) this influence disappear, and the dependencies tend to those for the case of semi-spaces.

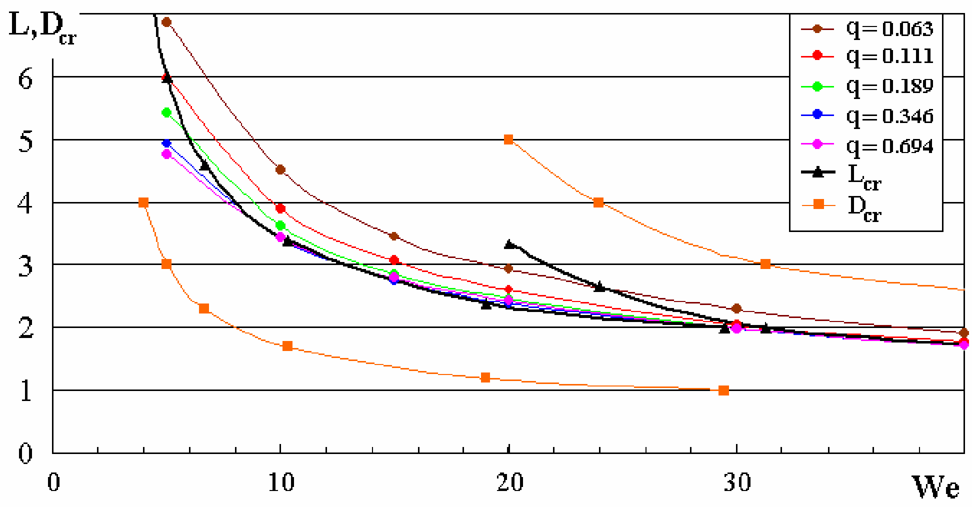

Figure 3 Dependencies $L(\mathrm{We})$, $L_{\mathrm{cr}}(\mathrm{We})$, $D_{\mathrm{cr}}(\mathrm{We})$ for various q

As Weber number decreases from ≈ 100 to ≈ 30 the reconstruction of $L(\mathrm{We})$ and $T_{\mathrm{e.m.}}(\mathrm{We})$ occurs from those given by (9) to new relationships inherent to a thin layer. Their approximations we can easily obtain by the asymptotic expansion of solutions of (7), (8) into series with respect to small parameter q:

$$\Delta_{0m} \cong (0.5)^{1/2} q^{1/3} G^{2/3} \mathrm{We}^{2/3}, \qquad T_{\mathrm{e.m.}} \cong \left(0.5\,q\,G^2\mathrm{We}\right)^{-1/2}. \qquad 10)$$

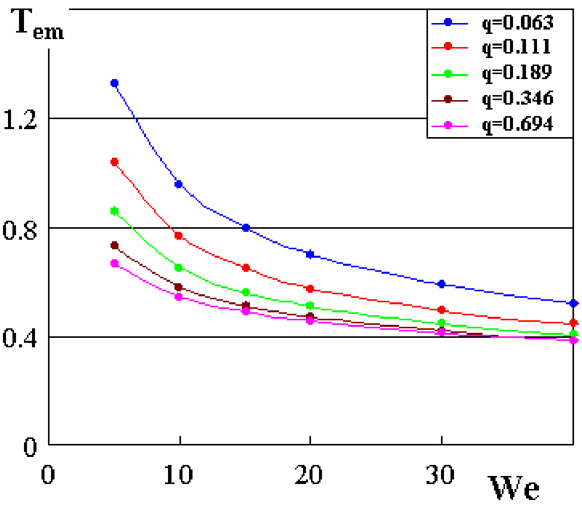

Figure 4 Dependencies $T_{\text{e.m.}}(\text{We})$ for various q

From the numerical solution of the dispersion relation (5) (fig. 3) it follows, that at $5 < \text{We} < 70$ the deformed drop is exposed to the action of unstable disturbances, which have wavelength greater, than initial drop diameter d_0, and is comparable with the lateral drop deformation Dd_0. Let us consider the situation, when one half-wavelength of dominant disturbance is approximately equal to the drop deformation: $\lambda_m / 2 = d_0 D$ (fig. 5, a). Then, the action of positive "phase" of disturbance at linear stage leads to a slow sagging of a middle part of liquid disk, while the edges go in opposite direction under action of a part of the negative "phase", which pulls it backward. This causes continuous (due to aperiodic character of instability) blowing out of generated cave, which at the non-linear stage becomes widened in lateral direction (it is typical of Rayleigh – Taylor instability [8]) and finally forms bag (fig. 5,a).

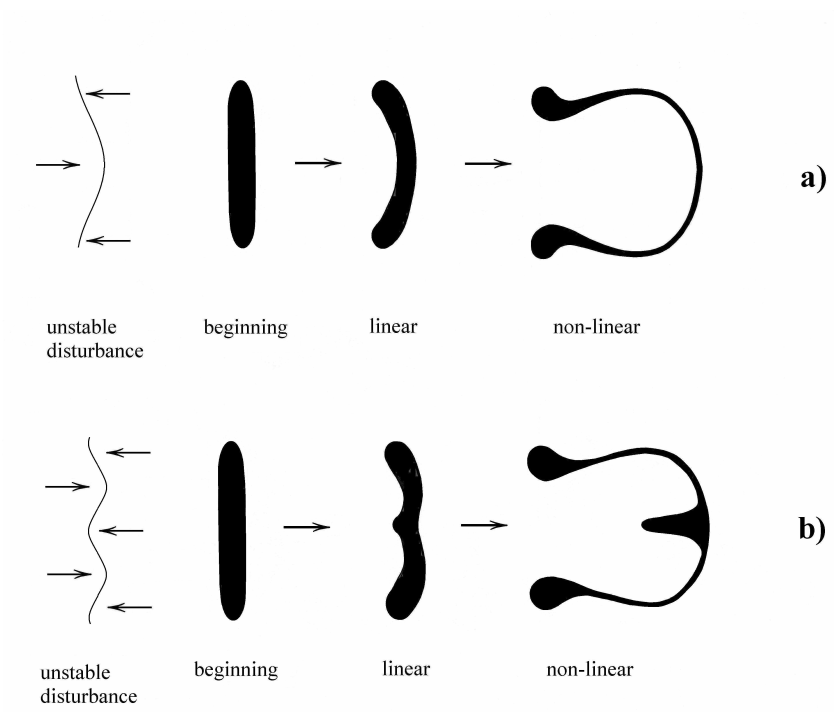

Figure 5. Scheme of action of aperiodic unstable disturbances at "bag" (***a***) and "claviform" (***b***) breakup modes.

Figure 3 shows, that the necessary condition $\lambda_m / 2 \leq d_0 D_{\max}$ at $D_{\max} = 3$ is satisfied when $\text{We} \gtrapprox 5$. But in order for the negative "phase" of the disturbance to catch the disk edge properly, it must have a little bit smaller wavelength, so, the condition for successive pulling out of a bag will be as follows:

$$\text{We}>\text{We}_{\text{cr1}} \cong 8 . \qquad 11)$$

Vice versa, when $\text{We}<\text{We}_{\text{cr1}}$ the condition $\lambda_m / 2 \geq BB'$ satisfies and a liquid disk can't be caught up by both "phases" of dominant disturbance simultaneously and therefore can't be expanded into a "bag". These two alternative situations can be seen for two droplets at experimental observations in fig. 1, right. The critical value (11) of We_{cr1} for long-wave aperiodic instability to work at a preliminary deformed drop agrees satisfactory with known empirical condition for general possibility of breakup phenomenon in gas flows ([1], [3]-[5], [10], [21], [24]), and approximately gives the lower limit for existence of "bag" mode of breakup in steady uniform gas streams for non-viscid drops, yet depending on the values of $D_{\max}$, G. In particular, critical value $\mathrm{We}_{\mathrm{cr1}}$ becomes somewhat greater at lesser drop deformation: $\mathrm{We}_{\mathrm{cr1}}=10$ at $D_{\max}=2.3$ ($q=0.189$, fig. 3).

The time interval for the dominant disturbance to perform ought to be proportional to characteristic time of its amplitude growth $\tau_{\mathrm{e.m.}}$. Results of the numerical solution of eqs (5), (8) show, that values of $T_{\mathrm{e.m.}}$ conform well enough to empirical values of total breakup time, $T_{\mathrm{b}} \cong 5 \div 6$, and do not contradict to known average time-values of elementary component processes [1], [3]-[5], [10], [24]. Namely, within the range $8<\mathrm{We}<30$, after the drop reaches the maximum degree of deformation $D_{\max} \cong 2.0 \div 3.0$ at $T_{\mathrm{d}} \cong 1.1 \div 1.6$, the dominant disturbance $\lambda_{\mathrm{m}}(\mathrm{We})$ comes into operation and approximately at $T = T_{\mathrm{d}} + T_{\mathrm{e.m.}}(\mathrm{We}) \cong 1.6 \div 2.6$ it turns to the non-linear stage. As experiments show [1], [4], [18], the bag diameter is approximately equal to $d_{\max}$; besides, linear stage terminates, when the disturbance' amplitude reaches the value of $\approx 0.4\lambda_{\mathrm{T}}$ [8]. Therefore, the time $T_{\mathrm{n}.l.}$ of bag blowing out at non-linear stage we can estimate as: $T_{\mathrm{n}.l.} \approx (D_{\max} - 0.4\lambda_{\mathrm{T}}/d_0)/U_{\mathrm{T}} \cong 1.0 \div 2.1$. The time for bag disintegration is calculated theoretically in [25] and it makes up to $T_{\mathrm{b.d.}} = 1.4 \div 2.4$. So, the final stage of breakup begins at $T_{\mathrm{b}} = \mathrm{T_d} + \mathrm{T_{e.m.}} + \mathrm{T}_{\mathrm{n}.l.} \cong 3.6 \div 3.7$ by rupture of bag film and finishes at $T_{\mathrm{b}} = 5.0 \div 6.0$ by the ring disintegration. Such an operation of aperiodic unstable disturbance constitutes the mechanism of "bag" breakup mode.

The length of the dominant wave λ_{m}, as well as $\tau_{\mathrm{e.m.}}$, decreases with We increasing. Now only a part of the disk surface BB' gets into sagging "phase" of disturbance. The remaining part gets into opposite "phase" and under action of surface tension forms massive rim at the disk edge, which is typical for "bag" mode of breakup (see fig. 1, right), and may contain up to 0.75 of the entire drop mass [1].

It appears the situation at further increasing of We, when three half-wavelengths are located on cross diameter of the disk (fig. 5, b): $3\lambda_{\mathrm{m}}/2 \approx d_0 D_{\max}$. In this case at the linear stage two half-wavelengths of positive "phase" generate an axisymmetric gas cave, which at the non-linear stage widens and forms bag, but one half-wavelength of negative "phase" at the axis of symmetry becomes sharpened (it is typical for Rayleigh – Taylor instability: [8]), and forms massive stamen (fig. 5, b). Figure 4 and the same concerns, as were given above for "bag" breakup, show that the overall breakup-time agreement with experimental observations is good enough, too. This is the mechanism of "claviform" mode of breakup. Figure 3 shows that condition $3\lambda_{\mathrm{m}}/2 < d_0 D_{\max}$ at $D_{\max} = 3$ satisfies, when

$$\mathrm{We} > \mathrm{We}_{\mathrm{cr}2} \cong 32, \qquad 12)$$

which agrees enough with lower empirical limit for "claviform" existence [24].

The analysis of the numerical results allowed to determine critical conditions for "bag" and "claviform" modes. They are represented in fig. 3 by curves $L_{\mathrm{cr}}(\mathrm{We})$ (black) and $D_{\mathrm{cr}}(\mathrm{We})$ (golden) for "bag" (left) and "claviform" (right) correspondingly. For example, when We=10, the minimum deformation needed for "bag" mode is $D = 1.7$, and minimum L is then 3.4 (see note before equation (13). When We=24, the minimum deformation needed for "claviform" mode is $D = 4.0$, and minimum L is 2.7. Besides, on the base of calculations of (10) the approximated relationship between the main parameters of the process can be obtained with the help of the least squares method: $\lambda_{\mathrm{m}} = 10.4 D_{\max}^{1/3} \mathrm{We}^{-0.58} d_0$.

At $\mathrm{We} \gtrapprox 60 \div 70$ there are several wavelengths dispose on cross diameter of liquid disk, which produce smaller bags ("bubbles") while penetrating into the disk. Thus, the value

$$\mathrm{We}_{\mathrm{cr}3} \cong 70 \qquad 13)$$

approximately gives upper limit of "claviform" mode. At the condition $\mathrm{We} \gtrapprox 70$ the appearance of periodic unstable disturbances with wavelengths less than the disk thickness d is possible on edge BC, as it follows from theory [12]. So, they can cause pulling out of liquid "threads" from the disk edge (see experimental observations: [24]). Besides, their characteristic times at $70 \lessapprox \mathrm{We} \lessapprox 100$ are comparable with obtained here values $\tau_{\mathrm{e.m.}}$ for aperiodic unstable disturbances. Apparently, simultaneous action of periodic and aperiodic mechanisms of instability generates "threads" and "bubbles", and forms "chaotic" mode of shattering at $70 \lessapprox \mathrm{We} \lessapprox 100$.

It is noteworthy, that values of $\mathrm{We}_{\mathrm{cr1}}$, $\mathrm{We}_{\mathrm{cr2}}$, $\mathrm{We}_{\mathrm{cr3}}$ depend on some other hydrodynamic factors, which can determine deformation rate and drop streamlining regime. For example, the presence of pressure gradient in gas flow will vary the value of dimensionless acceleration and in such a way will change the value of drop deformation achieved by time period $\tau_{\text{e.m.}}$. If at $\text{We}=\text{We}_{\text{cr1}}$ deformation did not reach the value $D_{\max} = \lambda_{\text{m}} / 2d_0$ till $t = \tau_{\text{e.m.}}(\text{We}_{\text{cr1}})$, unstable wave would unable to work and breakup process would not perform. It can then proceed at greater values of Weber number (when $\tau_{\text{e.m.}}$ is less), i.e., at these circumstances critical value We_{cr1} increases as was pointed out in experiments [26]. The scattering of observed experimentally We_{cr1}, We_{cr2}, We_{cr3} values may therefore be explained by such a reasons.

Summary and Conclusions

Taking into account some roughness of adopted assumptions about streamlining and weak influence of conditions at a disk edge on the disturbances at the middle part, one would expect, that results of instability treatment obtained here have too approximative character. Nevertheless, the critical values of Weber criterion given by (11)–(13), as a conditions for aperiodic disturbances to work, coincide good enough with known experimental boundaries of ”bag” and “claviform” modes of breakup for an inviscid drops. Suggested in present communication elementary theory allows to give simple visual explanation of preliminary and most complicated stage of these types of breakup. Following rupture of liquid film of the bag is of indubitable interest as final stage of breakup, which forms aerosol cloud of fine daughter droplets, and may be regarded as the aim of another investigation [25]. Possibly, the mechanism of rupture of liquid film of the bag, which is exposed to free stream, has instability nature too. Connecting results of present paper with those published earlier for greater We [12], [15]–[16], we can get general conclusion, that hydrodynamic instability of drop surface is indeed a universal mechanism, which constitutes the substance of shattering process at various modes of liquid drop breakup.

References

[1] Hanson A. R., Domich E. Y. and Adams H. S., *Phys. of Fluids* **6**: 1070–1080 (1963)

[2] Joseph D.D., Belanger J. and Beavers G.S., *Intern. Journ. of Multiphase Flow* **25**: 1263-1303 (1999)

[3] Reinecke W. G. and Waldman G.D., *Shock layer shattering of cloud drops in reentry flight,* AIAA Paper 152: 22 (1975)

[4] Chou W.-H., Hsiang L.-P. and Faeth G. M. *Dynamics of drop deformation and formation during secondary breakup in the bag breakup regime,* AIAA Paper 97-0797: (1997)

[5] Zhao H., Liu H.-F., Li W.-F. and Xu J.- L. Phys. of Fluids **22**: Doi: 10.1063/1.3490408 (2010).

[6] Simpkins P.G., *Nature: Physical Science*, **233**: 31-33 (1971)

[7] Engel O.G. *Journal of Researches of National Bureau of Standards* **60**: 245–280 (1958).

[8] Lewis D.J. *Proceedings of Royal Society of London Ser. A* **202**: 81-96 (1950)

[9] Fishburne B.D. *Acta Astronautica* **1**: 1267-1284 (1974)

[10] Gelfand B. E. *Present status and problems of investigations of detonation in liquid droplets–gas system* Ed. Dremin A. N. Chem. Phys. comb. and explos. proc. Detonation, A.S. USSR. Chernogolovka: 28–39 (1977).

[11] Aslanov S.K. and Girin A.G., *Proceedings of Ukrainian Academy of Sciences, Ser.A* **12**: 25-28 (1981)

[12] Girin A. G., *Journal of Engineering Physics and Thermophysics* **48**: 560-564 (1985)

[13] Rabin E., Schallenmuller A.R. and Lowhead R.B., *Displacement and shattering of propellant droplets* Sandia Corporation Laboratory Report, AFOSR-TR-60-75 (1960)

[14] Aslanov S. K. and Girin A. G., *Combustion, Explosions and Shock Waves* **24**: 475-482 (1988)

[15] Girin A. G., *Journal of Engineering Physics and Thermophysics* **84**: 262-269 (2011)

[16] Girin A. G., *Journal of Engineering Physics and Thermophysics* **84**: 872-880 (2011)

[17] Girin A. G., *Journal of Engineering Physics and Thermophysics* **84**: 1009-1015 (2011)

[18] Hosseinpour S., Binesh A. R. *Fuel* **88:** 799-805 (2009)

[19] Patterson M. A., Kong S. C., Hampson C. J. Reitz R. D. *Modelling of effects of fuel injection characteristics on diesel soot and NO_2 formation,* SAE Paper 940523 (1994)

[20] Patterson M. A., Reitz R. D. *Modelling the effects of fuel spray characteristics on diesel engine combustion and emission* SAE Paper 980131 (1998)

[21] Hinze J.O. *Applied Scientific Research* **A1**: 273-288 (1949)

[22] Tarnogrodzki A. Int. *Jour. of Mechanical Sciences* **43**: 883-893 (2001)

[23] O’Brian V. *Journal of Meteorology* **18** : 549-552 (1961)

[24] Gelfand B.E., Gubin S.A. and Kogarko S.M. *Jour. of Engin. Phys. and Thermophys.***27**: 877-882 (1974)

[25] Girin A. G., Ivanchenko Ye. A. *12th ICLASS 2012*, Heidelberg, Germany, Sept. 2-6, 2012, # 1226

[26] Gelfand B.E., Gubin S.A., Kogarko S.M. and Komar S.P. *Fluid Dynamics* **8**: 724-728 (1973)