ICLASS 2012, 12th Triennial International Conference on Liquid Atomization and Spray Systems, Heidelberg, Germany, September 2-6, 2012

Ballistics of Evaporating Spray in a Wake of a Shattering Drop

A. G. Girin
Odessa National Maritime University, Odessa, Ukraine
Club21@ukr.net

Abstract

Mathematical model of evaporation dynamics of a mist in a wake of a shattering in speedy flow drop is elaborated on a base of obtained earlier distribution function for stripped droplets by sizes. Stripped droplets are considered as a multi-velocity continuum and a system of differential equations of two-phase polydisperse spray dynamics is composed. At one-dimensional spatial approximation of flowfield the mathematical problem is formulated and solved in a closed form in a dynamic 3D space. A detailed calculation of the ballistics of an evaporating spray, generated in the wake of a kerosene drop fragmented by air stream, is performed. The spray internal structure is investigated as related to the dynamic process of spray formation. Evolution of the dispersive characteristics of liquid-phase jet of the spray is studied. In the case considered the stabilization of jet is discovered and the range of a jet is found. Analysis of the processes of jet and vapor cloud formation is done and their structures are described. By rough estimations, intensification of liquid transfer into vapor phase due to shattering leads to over-riching and cooling of the air-vapor combustible mixture in a wake, which increases essentially the induction period of ignition.

Introduction

Drop atomization in a speedy flows proceeds intensively and produces aerosol cloud of great number of tiny daughter droplets. This starts following processes of motion and evaporation of stripped mass, which influences greatly on a two-phase flow, especially in the case of reactable mixture, such as detonative systems and flows in jet engines, where it determines the mixing kinetics of oxidizer with fuel vapors. Calculations show [1] that shattering causes the growth of total surface of liquid phase by 2-3 orders, which together with rapid evaporation of finest stripped daughter droplets leads to increasing in rate of liquid mass transfer to gaseous phase by 5-7 orders. An investigation of processes in a wake of a shattering drop runs across obstacles which are caused by the lack of knowledge about stripping kinetics: sizes, quantity, moments of tearing-off and distribution of stripped droplets by sizes are unknown. This didn't allow to elaborate a mathematical model which would be able to predict in details stripping kinetics and evolution of stripped mass with due regard to dependencies on variable values of droplet radius and velocity of streamlining. For example, evaporation of mist, accelerated by air stream, when distribution function is unknown, was studied in [2] with arbitrary chosen isotropic structure of monodisperse spray.

Evaporation and combustion of multi-droplet systems were comprehensively studied by W. Sirignano with co-workers [3]–[5]. Influence of droplet regression, deceleration of the flow due to drag of the droplets, internal circulation inside droplet, variable properties, non-uniform surface temperature, and the effect of surface tension were included into mathematical model. The transient flame shape, surface temperature, relative velocity and burning rate were studied for slow and middle initial velocity of a stream, droplet spacing, and ambient temperature and pressure. A complete review of droplet vaporization and burning is given by Sirignano [6].

However, the mathematical models of drop evaporation, which were developed in the papers cited, can not be easily applied directly to spray in a wake of drop, shattering in a speedy flows, because: 1) total number of daughter droplets can exceed $10^5 \div 10^7$; 2) stripped mass is essentially non-stationary polydispersed spray which is fed back by the parent drop; 3) polydispersed spray is essentially multi-velocity system; 4) the strength of source of daughter droplets varies continuously; 5) velocity of flow past droplets can exceed $10^3 \div 1.5 \cdot 10^3 m/\sec$. Spatial structure of a wake spray has transient complicated character since every droplet has its own values of velocity, radius, evaporation rate, and distances between droplets vary due to non-uniform two-dimensional field of accelerations. Apparently, the construction of mathematical model of a wake spray requires special approaches, and reasonable simplifications must be done at elementary processes description. Therefore, a one-dimensional model of two-phase mixture motion is considered here as first-order approximation.

Internal structure of a wake spray can not be set up a priori but must be calculated as dynamic process of spray formation. So, model must be elaborated, which would be able to reflect the evolution of stripped droplets distribution by sizes, caused by processes of their acceleration and evaporation. Aerodynamic factor being of great significance, since initial values of Reynolds number for daughter droplets are large enough; in our calculations they varied over the range 300 to 1500. To our mind, it is more important thus in high-speed flows to take into account for the convection enhancement of the evaporation than for such peculiarities of evaporation, as non-uniform surface temperature, circulation flow inside tiny droplet etc., because in aspiration for all the features of evaporation kinetics to be accounted for we would have mathematical model too large and calculation procedure too bulky. So, calculations would become insuperable and modeling must be facilitated. In presented calculations with simplified evaporation kinetics of a spray each variant required $\approx 100h$ of time on Pentium based computer. Therefore, pointed above peculiarities were omitted in our analysis and evaporation kinetics was simplified up to Ranz – Marshall model, where specifics of spray thermal and mass interaction with hot gas flow was taken into account by using the evaporation constant value known directly from the experiment measurements and applied already for hot flows in rocket chambers by Lambiris and Combs [7].

A key element of a spray model should be distribution function of stripped droplets by sizes which is determined by a mechanism of droplets tearing-off. Since early investigations the correspondence of "stripping" mode of drop breakup to conditions behind shock and detonation front was well-established [8]–[11]. Thus, in many papers concerning the problem of detonation the "boundary layer stripping" model was used, which was grounded on the concept of breakup process as continuous stripping of thin liquid boundary layer from drop's equator. However, essential qualitative contradiction was found later between this concept and experimental results, which concerns an inadequate influence of liquid viscosity on drop breakup times [12], [13].

In presented paper the concept of quasi-continuous high-frequency periodic dispersion of daughter droplets from unstable part of parent drop surface due to action of gradient instability in conjugated boundary layers [14], [15] is applied. Numerical application of this model permitted to calculate two-phase flow in stationary zone behind the front of self-sustained detonation wave in aerosol and to solve thus the main problem of heterogeneous detonation – to find velocities of self-sustained regimes of detonation and their properties [16]. The foundations of drop breakup theory for speedy flows were laid in [1], [17], [18], where the application of approximative analytical approach, based on mechanism of gradient instability, allowed to derive the governing differential equations of shattering process: equations of parent drop mass reduction and of torn off droplets quantity. Their integration provided all the necessary relations of the theory of shattering process. Obtained theoretically in [1], [17], [18] distribution functions of stripped droplets by sizes $f_{\text{n.p.}}(r,t)$, law of motion of shattering in gas stream parent drop $x_{\text{p.d.}}(t)$ and law of parent drop mass reduction $m_{\text{p.d.}}(t)$ (ablation law) give the opportunity to describe quantitatively further processes of rapid acceleration and evaporation of mist of stripped droplets in a wake of a shattering drop. The mathematical model of two-phase polydisperse liquid-vapor spray, which is built on this basis, and some elementary results of numerical investigation of evaporation ballistics and dynamic formation of a wake spray are presented below.

Mathematical Model of Liquid-Vapor Spray Dynamics

Essential difficulties were noted in experimental observations of a spray formation at drop shattering [8], [11], [19], as well as in the similar phenomena of the development of fuel flare thrown out from the atomizer [20], and of normal injection of liquid jet into a high-speed gas flow [21]. They are caused by essential transiency, great density and number of generated droplets and their finest sizes. Therefore, the progress in spray formation studying can be connected with the development of advanced mathematical models. The presented model here gives a possibility to spread light inside the internal transient aerodynamics of evaporating mist in a wake of shattering drop.

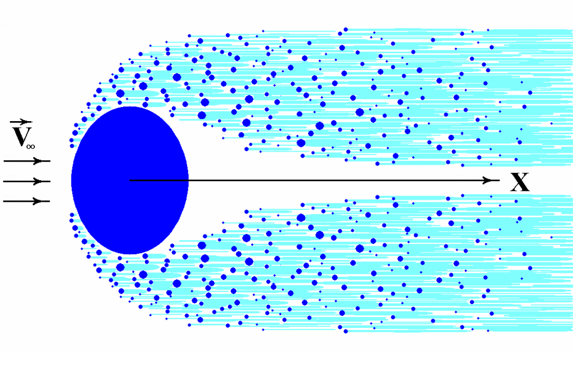

Figure 1 Spray in a wake of a shattering drop; dark-blue – liquid phase, light-blue – vapors

Difficulties for mathematical modelling of evaporation ballistics of a wake spray can be seen on a schematic image of processes considered (fig. 1), as the finest droplets, as well as the largest, are distributed everywhere in the spray. Each cross-section contains droplets of various sizes and velocities and puffs of vapors, which were produced at different moments and are therefore at different stages of diffusion in ambient gas stream. Polydispersity of spray is the main reason of the complexity. Indeed, evaporation rate depends on size of a droplet, besides, it depends also on a relative velocity of flow past droplet, which in turn is defined by acceleration, depending on droplet size too. Polydispersity makes the spray multi-velocity and makes it necessary to regard droplets radii r as independent variable for description of fields of their velocities and evaporation rates. Thus, concentrations of mixture components are essentially non-stationary, non-uniform and are defined by the rates of both evaporation and acceleration.

To include polydispersity in the model properly, a new approach to modeling the aerodynamics of evaporating polydisperse sprays is presented here. With regard to the great number and density of stripping quasicontinuously droplets we suggest considering stripped droplets altogether as a fluid continuum, neglecting by their collisions. At first-order approximation let us consider the two-phase flow as a one-dimensional motion in direction OX of a spray axis of symmetry. To take into account the enhancement of evaporation by gas flow we introduce into consideration the transient distribution function of daughter droplets by sizes $f_{\text{n.d.}}(r,x,t)$ as it exactly reflects the entire evolution of stripped droplets in time t, in space x, and in sizes r. By parentage it coincides with the distribution function of parent drop: $f_{\text{n.d.}}(r,0,t)=f_{\text{n.p.}}(r,t)$ at parent drop location $x=0$.

The parent drop is considered as located in spray origin $x=0$ source of daughter droplets of strength $\dot{F}_{\text{s}}(r,t)=\dot{f}_{\text{n.p.}}(r,t)$. They are moving in OX -axis direction with velocity $w_{\text{d}}(r,x,t)$. As well, each daughter droplet is a moving point source of vapor of strength $\dot{F}_{\text{v}}(r,t)$, and they altogether form the distribution of a vapor mass $m_{\text{v}}(x,t)$ in a wake. To describe evolution of $f_{\text{n.d.}}$ in (r,x,t)-space let us enlist an equation of dispersed fuel, which was derived by Williams [22] on a base of analogy with motion of fluid continuum. It describes density changing of the distribution of daughter droplets quantity $f_{\text{n.d.}}$ in axis OX direction, which proceeds with velocity $dx_{\text{d}}/dt=w_{\text{d}}(r,x,t)$ due to droplets acceleration by gas flow, and in r-axis direction – due to evaporation, which proceeds with a rate $u_{\text{d}}=dr/dt$, determined by the evaporation law. The equation of evolution must be supplemented with equation of motion of daughter droplets continuum and with equation of vapor influx in a spray volume, evaporation law accounting for intensification due to streamlining of daughter droplets. So, both vapors and droplets phases are considered as continuum fluids in dynamic (r,x,t)-space.

Thus, two-phase one-dimensional multi-velocity flow in a wake spray of a shattering drop is described by three dimensionless functions $f_{\text{n.d.}}(\tilde{r},x',\tau)$, $W_{\text{d}}(\tilde{r},x',\tau)$, $M_{\text{v}}(\tilde{r},x',\tau)$, which are the solutions of *the system of differential equations of spray dynamics:*

$$\begin{cases} \dfrac{\partial f_{\text{n.d.}}}{\partial\tau}+\dfrac{\partial}{\partial\tilde{r}}\left(U_{\text{d}}f_{\text{n.d.}}\right)+\dfrac{\partial}{\partial x'}\left(W_{\text{d}}f_{\text{n.d.}}\right)=0; & 1)\\ \dfrac{\partial W_{\text{d}}}{\partial\tau}+W_{\text{d}}\dfrac{\partial W_{\text{d}}}{\partial x'}=\dfrac{3}{2}\dfrac{\sqrt{\alpha}\,C_{\text{d}}}{B_1\tilde{r}}\dfrac{(1-W_{\text{d}})^2}{2}; & 2)\\ \dfrac{\partial M_{\text{v}}}{\partial\tau}+\dfrac{\partial M_{\text{v}}}{\partial x'}=\dot{F}_{\text{v}}f_{\text{n.d.}}\Delta\tilde{r}\,. & 3)\end{cases}$$

where $\tilde{r}=r/(R_{\text{p0}}B_1)$, $x'=x\sqrt{\alpha}/2R_{\text{p0}}$, $W_{\text{d}}=w_{\text{d}}/V_\infty$, $U_{\text{d}}=2u_{\text{d}}/(\sqrt{\alpha}B_1V_\infty)=-\Lambda\text{Nu}_{\text{d}}/16\tilde{r}$, $M_{\text{v}}=m_{\text{v}}/m_{\text{p0}}$ $\tau=t/t_{\text{ch}}$, $\Lambda=2\lambda/\left(B_1^2\sqrt{\alpha}V_\infty R_{\text{p0}}\right)$ is dimensionless evaporation constant, $t_{\text{ch}}=2R_{\text{p0}}/\sqrt{\alpha}V_\infty$ is characteristic time of shattering process, $\alpha=\rho_\infty/\rho_l$, $\mu=\mu_\infty/\mu_l$, $B_1=0.51\pi\alpha^{1/3}\mu^{-2/3}\text{Re}_{\text{p0}}^{-0.5}$ has a sense of scaling parameter for sizes of stripped droplets, R_{p0}, Re_{p0} – initial parent drop radius and Reynolds number. It is assumed in (3), that vapors accelerate instantly to velocity of stream V_∞ due to negligible mechanical non-equilibrium of gas components, and it is neglected in (3) by parent drop evaporation.

The strength of a spray source was set up as a boundary condition $\dot{f}_{\text{n.d.}}(\tilde{r},0,\tau)_{\text{s}}=\dot{f}_{\text{n.p.}}(\tilde{r},\tau)$ in parent drop location $x=0$ as a function, that was found in [17]. It was shown in [17], that $f_{\text{n.p.}}(\tilde{r},\tau)$ depends on the ratio h of rates of two competing processes – mass efflux due to dispersion and relaxational reducing of relative velocity of gas stream and parent drop. In the case $h=1$ when they are equal it has a simple form

$$f_{\text{n.p.}}(\tilde{r},\tau)=\frac{1-\exp(-A\tau)}{A\tilde{r}^2}\frac{B_2\sin^3\varphi_0(\tilde{r})}{(8-2.5\tilde{r}^2\cos\varphi_0(\tilde{r}))} \qquad 4)$$

and is valid for the range $\tilde{r}_{\min}<\tilde{r}<\tilde{r}_{\max}$, where A is characteristic (initial) rate of mass efflux, $B_2=0.15\alpha^{-7/6}\mu^{7/3}\text{Re}_{\text{p0}}^{3/2}$ has a sense of scaling parameter for quantity of stripped droplets, $\varphi_0(\tilde{r})$ is an inverse with respect to $\tilde{r}(\varphi)$ function in equation of curve of integration $\tilde{r}(\varphi,0)=const$ on (φ,τ)- plane.

At right boundary $\tilde{r}=\tilde{r}_{\max}$ of the domain considered a condition of droplets absence $f_{\text{n.d.}}=0$ was introduced. The left boundary $\tilde{r}=0$ is irregular line of system (1)–(3), so, usual in gasdynamic problems method of boundary shifting to small distance $\tilde{r}_{l\text{.b.}}$ was applied, and condition of entire evaporation of droplets

$f_{\text{n.d.}}\big|_{\tilde r_{l.\text{b.}}}=0$ $M_{\text{v}}=(\tilde r_{l.\text{b.}}B_1)^3$ was set up at $\tilde r_{l.\text{b.}}=0.01\tilde r_{\max}$. The condition $f^{+}_{\text{n.d.}}=f^{-}_{\text{n.d.}}$ of reflected disturbances absence was set at top boundary of domain of calculation $x=l_{\text{t}}$. Values of W_{d}, M_{v} at $x'=0$ were set to zero.

The dependence of drag coefficient C_{d} on droplets velocities and sizes was taken in Kurten's form $C_{\text{d}}=24/\text{Re}_{\text{d}}+6/\text{Re}_{\text{d}}^{0.5}+0.28$, which is valid for $\text{Re}_{\text{d}}<4\cdot10^3$; the initial Reynolds number for daughter droplets Re_{d0} never exceeding values of $1.5\cdot10^3$ in calculations. Convective increase of evaporation rate due to a droplet streamlining by a speedy flow was taken into account by using evaporation law in Ranz – Marshall's form

$$\frac{dr}{dt}=-\frac{\lambda \text{Nu}_{\text{d}}}{16r}, \qquad \text{Nu}_{\text{d}}=2+0.53\text{Re}_{\text{d}}^{0.5} \qquad 5)$$

with empirical value of evaporation constant for kerosene $\lambda=2.7\cdot10^{-6}m^2/\text{sec}$, which was used by Lambiris, Combs [7] in conditions inherent to rocket chamber hot flow, Nu_{d} being Nusselt number for a streamlined daughter droplet. The strength of point source of vapor in (3) is then $\dot F_{\text{v}}=3/16\tilde r\Lambda B_1^3\text{Nu}_{\text{d}}$.

Formulated in this way non-stationary two-dimensional problem for system (1)–(3) with source function (4) was solved numerically. As mathematical properties of system (1)–(3) have not been studied yet, universal Lax – Vendroff finite-difference scheme of second-order accuracy was chosen for calculations. The three-point smoothing procedure was applied in x-direction to $f_{\text{n.d.}}$ alone to neutralize the oscillations, which appeared because $f_{\text{n.d.}}$ has discontinuities along trajectories of $\tilde r_{\min}$ - and $\tilde r_{\max}$ -droplets. Value $k_{\text{sm}}=0.9995$ for smoothing coefficient was enough to hold magnitude of the relative error $\varepsilon(\tau)=(m_{\text{v}}(\tau)+m_l(\tau)-m_{\text{s}}(\tau))/m_{\text{s}}(\tau)$ of total mass balance in within $-2\%<\varepsilon<4\%$ interval, where m_{v} is current vapor mass in spray, m_l is current mass of liquid in spray and m_{s} is current stripped mass. Without smoothing ($k_{\text{sm}}=1$) the unstable disturbances began to grow soon after first interaction of droplets trajectories with left boundary. The stability of calculations was provided by small enough step of integration $\Delta\tau=2\cdot10^{-5}$, while spatial steps were $\Delta\tilde r\approx\Delta\tilde X=3\cdot10^{-3}$.

General Properties of Spray Evaporation

As source function (4) is self-similar, the range of stripping droplets sizes $\tilde r_{\min}<\tilde r<\tilde r_{\max}$ does not vary in time and coincides with a basic range $\tilde r_{l0}=\sqrt{3.2}<\tilde r<\tilde r_{r0}=\sqrt{3\pi}$ [1]. This circumstance becomes a necessary condition for liquid-phase jet of spray to be stabilized to certain moment $\tau_{\max}$. In general case $h\neq1$ the basic range sets up only initial distribution, produced by parent drop, and then, as shattering proceeds, the bounds of distribution range shift decreasing at $h>1$ and increasing at $h<1$ [1]. At $h=1$ the range of droplets sizes has therefore the least width, and values of the mean diameters d_{ij} of source distribution $f_{\text{n.p.}}$ are close each other.

There were three variants of values of input definitive parameters calculated for kerosene drop shattering in air stream of density $\rho_\infty=3.85kg/m^3$, which are listed in ***Table 1***. Conditions for the first and third being corresponded to flows behind shock and detonation waves, second – in rocket chambers. Values of main output parameters are given in ***Table 2***. Values of B_1, B_2, which define sizes and quantity of droplets in source distribution coincide for the second and third variants, therefore these results are identical in dimensionless variables, so,

Table 1. **Values of definitive input parameters,** GI **– criterion of gradient instability**

	$V_\infty, m/\text{sec}$	$2R_{\text{p0}}, m$	Re_{p0}	GI	B_1	B_2	t_{ch}, sec
Variant 1:	10^3	10^{-3}	$9.17\cdot10^4$	636	$5.1\cdot10^{-3}$	$4.01\cdot10^6$	$14.4\cdot10^{-6}$
Variant 2:	10^2	10^{-3}	$9.17\cdot10^3$	20.1	$1.61\cdot10^{-2}$	$1.27\cdot10^5$	$144\cdot10^{-6}$
Variant 3:	10^3	10^{-4}	$9.17\cdot10^3$	201	$1.61\cdot10^{-2}$	$1.27\cdot10^5$	$1.44\cdot10^{-6}$

Table 2. **Values of main output parameters**

	$2r_{\min}, m$	$2r_{\max}, m$	$\text{Re}_{\text{d0}}(r_{\max})$	$t_{\max}$, sec	l_{j}, m	N
Variant 1:	$9.2\cdot10^{-6}$	$15.6\cdot10^{-6}$	$1.42\cdot10^3$	$16.8\cdot10^{-6}$	$1.05\cdot10^{-2}$	$6.0\cdot10^5$
Variant 2:	$28.8\cdot10^{-6}$	$49.4\cdot10^{-6}$	$4.53\cdot10^2$	$234\cdot10^{-6}$	$1.16\cdot10^{-2}$	$1.9\cdot10^4$
Variant 3:	$2.88\cdot10^{-6}$	$4.94\cdot10^{-6}$	$4.53\cdot10^2$	$2.34\cdot10^{-6}$	$1.16\cdot10^{-3}$	$1.9\cdot10^4$

only the first and second variants will be compared. Values of final for shattering process parameters $\tilde{r}_{\min}$, $\tilde{r}_{\max}$, N, which characterize source-produced distributions, are initial for further processes of spray formation.

The general regularities are shown in fig. 2 as dependencies of total stripped mass $M_s(\tau)=1-M_p(\tau)$ (*curve1*) and total mass $M_v(\tau)$, evaporated from daughter droplets (*curves 2*), on time. They show that the intensification of evaporation in hot streams behind shock and detonation waves is substantial, so the greater part of fuel is presented in vapor phase. Maximum current value of liquid phase mass (*curves 3*) in the first variant $M_{l1}=0.07$ was reached at $\tau_{l1}\approx 0.45$, while in the second – $M_{l2}=0.10$ at $\tau_{l2}\approx 0.66$, respectively. At any moment the most part of mass of liquid phase is located in the vicinity of parent drop and decreases sharply along spray axis. The mass rate of evaporation $\dot{M}=dM_v/d\tau$ (*curves 4*) exceeds maximum values to the moments $\tau_{v1}\approx 0.50$ and $\tau_{v2}\approx 0.72$, increasing before due to growth of mass stripped,

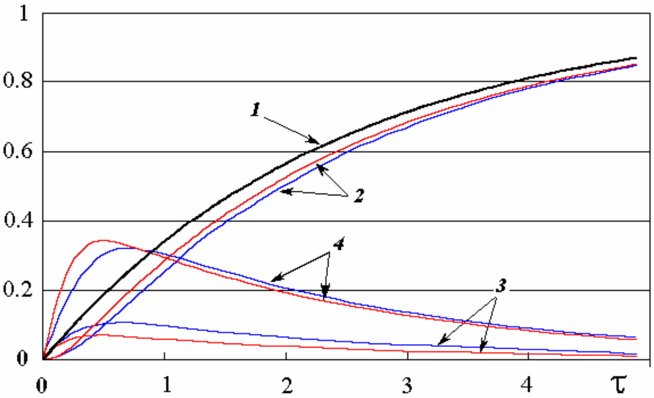

Fig. 2. variation of masses in time. Red – ***variant 1***, blue – ***variant 2***.

and decreasing after because of vanishing (entire evaporation) of droplets, which begins at $\tau_{\min 1}=0.46$ and $\tau_{\min 2}\approx 0.65$ respectively. As it follows from the next sections, moments $\tau_{\min}$ and $\tau_{\max}$ of entire evaporation of droplets of minimum $\tilde{r}_{\min}$ and maximum $\tilde{r}_{\max}$ in distribution radii, which were torn off at $\tau=0$, are the characteristic moments in the process of formation of liquid-phase jet of spray.

Formation of Liquid-Phase Jet

In the considered case $h=1$ formation of liquid-phase jet of stable length in a wake of shattering drop takes place, and three stages of this process were revealed, which are divided by time moments $\tau_{\min}$, $\tau_{\max}$. The process is illustrated in fig. 3 on $(\tilde{r}, \tilde{X})$-plane, where $\tilde{X}=x/R_{p0}$. Trajectories of $\tilde{r}_{\min}$- and $\tilde{r}_{\max}$-droplets at any moment make a lateral bounds of a jet. They were build upon fields of parameters $W_d(\tilde{r},x',\tau)$, $\mathrm{Re}_d(\tilde{r},x',\tau)$, $\mathrm{Nu}_d(\tilde{r},x',\tau)$, calculated on solutions of system (1)–(3). Their endings at $\tilde{r}=0$ (points ***A***, ***B***) correspond to vanishing of $\tilde{r}_{\min}$- and $\tilde{r}_{\max}$-droplets, which were torn off at $\tau=0$; for the *variant 1* that occurred at $\tau_{\min 1}=0.46$, $\tau_{\max 1}=1.17$; for the *variant 2* – $\tau_{\min 2}=0.65$, $\tau_{\max 2}=1.63$.

Position of set of droplets from diapason $\tilde{r}_{\min}<\tilde{r}<\tilde{r}_{\max}$, which were torn off at $\tau=0$, makes a top bound of jet, which is marked by green lines ***1-6*** at $\tau_1=0.16$, $\tau_2=0.31$, $\tau_3=0.46$, $\tau_4=0.63$, $\tau_5=0.80$, $\tau_6=0.97$. The tip of the jet (green arrows) is sharpen because each moment it consists of smallest droplets of jet, while astern and middle parts of jet are widened, which testify to most polydispersity of jet at these cross sections.

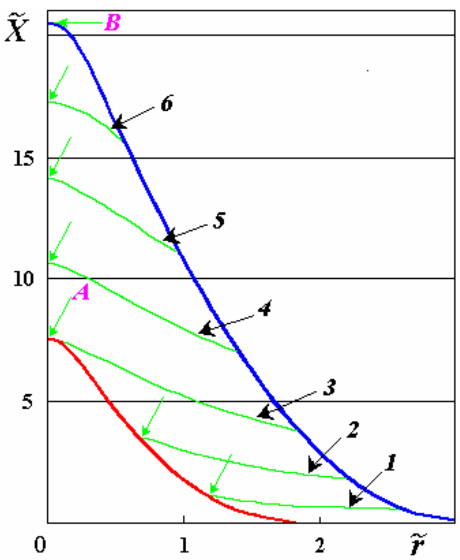

Figure 3 Formation of liquid jet: trajectories $\tilde{r}_{\min}$-droplets (red), $\tilde{r}_{\max}$ (blue); ***variant 1***

The first stage $0<\tau<\tau_{\min}$ of jet formation is characterized by rapid jet lengthening till it reaches the length of path l_d of $\tilde{r}_{\min}$-droplet (point ***A***), and by rapid growth of polydispersity (jet width in axis $\tilde{r}$-direction). At the same time the sharp front of vapor cloud is formed. On the second stage $\tau_{\min}<\tau<\tau_{\max}$ entire evaporation of droplets of radii $\tilde{r}_{\min}<\tilde{r}<\tilde{r}_{\max}$, that were torn off at $\tau=0$, takes place along ***A – B***. As a result, rates of lengthening and of polydispersity growth decrease, and rate of vapor production $\dot{M}_{\text{v}}$ exceeds maximum value at the beginning of second stage. The vapors density in vapor cloud and the rate of droplets vanishing $\dot{n}_{\text{c}}$ also exceed their maximums at the end of the second stage. On the third stage $\tau_{\max} < \tau$ the droplets of the same sizes from the range $\tilde{r}_{\min} < \tilde{r} < \tilde{r}_{\max}$ vanish, and this is the necessary condition for stabilization of length of jet l_{j} and its dispersive characteristics, while the rate of vapor production $\dot{M}_{\text{v}}$ slowly decreases due to decreasing of current total droplets quantity in the jet n_{c} .

The standard calculation graph of jet length dependence $L_{\text{j}} = l_{\text{j}} / R_{\text{p0}}$ on time is shown in fig. 4. The dependence is close to quadratic in the first stage due to nearly quadratic law of motion of finest droplets, which compose tip of the jet. On the second stage, when the tip consists of various new droplets, it is close to linear. After $\tau_{\max}$, the jet length remains unchangeable. Stabilization proceeds successively along jet from astern part to the tip, which is confirmed by analysis of dispersive parameters given in the next section. The droplet path length of characteristic radii are: $l(\tilde{r}_{\min 1}) \approx 7.5 R_{\text{p0}}$, $l(\tilde{r}_{\max 1}) \approx 20.5 R_{\text{p0}}$ and $l(\tilde{r}_{\min 2}) \approx 9.0 R_{\text{p0}}$, $l(\tilde{r}_{\max 2}) \approx 24.3 R_{\text{p0}}$; the value $l(\tilde{r}_{\max})$ is the stable jet length l_{st} (range of jet).

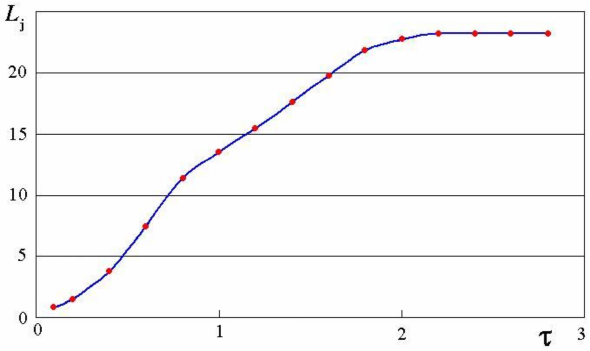

Figure 4 Variation of length of liquid jet of spray in time $L_{\text{j}}(\tau)$ for ***variant 2***

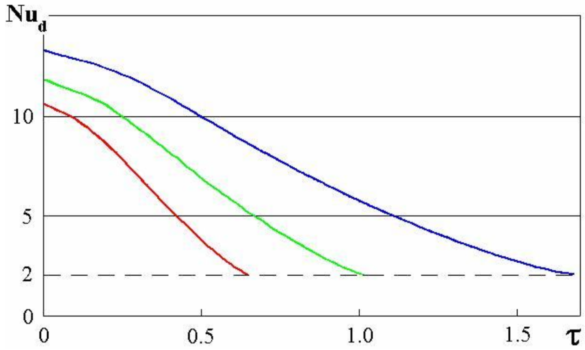

Figure 5 Dependencies $\text{Nu}_{\text{d}}(\tau)$ for droplets of radii $\tilde{r}_{\min}$ (red), $\tilde{r}_{\max}$ (blue) and Sauter mean (green); ***variant 2***

Neither the droplets' lifetimes nor the droplets' path lengths relate as squares of their initial diameters because convection enhancement of evaporation depends on droplet size. It manifests in a slower decreasing of the Nusselt number for more inertial coarse droplets of spray (Fig. 5) and in a consequently greater velocity of streamlining. Due to this effect, the intensification of evaporation for $\tilde{r}_{\max}$ -droplet is equivalent to a 11% growth in evaporation constant when compared to $\tilde{r}_{\min}$ -droplet.

Dispersive Parameters of Spray

The dispersive characteristics of liquid-phase jet in a wake of shattering drop – the mean diameters $d_{ij}(x,t)$, – and their evolution in space and time can be easily calculated in frames of suggested model. As the processes considered are essentially non-stationary, parameters of two kinds were considered: mean diameters $\Omega_{ij}(x) = (2 R_{\text{p0}} B_1)^{-1} d_{ij}(x, t_{\text{c}})$ are calculated in each cross section x and characterize spatial structure of jet at fixed current moment t_{c} , while $D_{ij}(t) = (2 R_{\text{p0}} B_1)^{-1} \int_0^{l_{\text{j}}} d_{ij}(x,t) dx$ are calculated for the totality of droplets at each t and describe temporal changing of dispersity of entire spray.

The calculated dependencies $D_{ij}(\tau)$ are given in fig. 6 for ***variant 1*** and they illustrate the stabilization of dispersive properties of spray; here and in each group in fig. 7 the curves are located in the following order (from top to bottom):

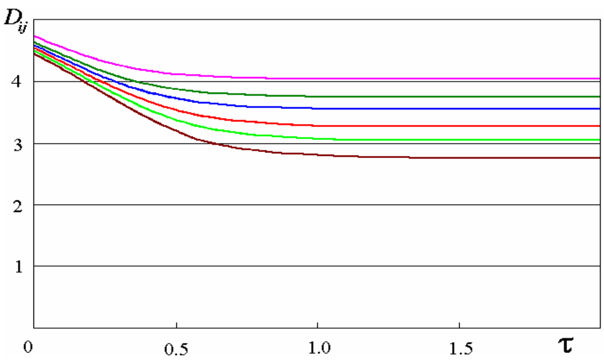

Figure 6 Dependencies $D_{ij}(\tau)$ for ***variant 1***: D_{43} – crimson, D_{32} – dark-green, D_{31} – blue, D_{30} – red, D_{20} – light-green, D_{10} – brown

$D_{43}, D_{32}, D_{31}, D_{30}, D_{20}, D_{10}$. Their initial values are: $D_{43}=4.74, D_{32}=4.64, D_{31}=4.59, D_{30}=4.54$, $D_{20}=4.50, D_{10}=4.45$, while final – $D_{43}=4.04$, $D_{32}=3.75, D_{31}=3.55, D_{30}=3.25, D_{20}=3.03$, $D_{10}=2.74$. Sharp initial decreasing of D_{ij} is caused by rapid evaporation and acceleration of droplets. Growth of divergence of curves means growth of polydispersity of jet due to non-homogeneity of fields of evaporation and acceleration rates. In the time interval $\tau_{\min}<\tau<\tau_{\max}$ the rate of this growth decreases, and soon after $\tau_{\max 1}=1.17$ each D_{ij} remains still. The polydispersity of jet in final state is much greater then in initial, produced by source.

The dependencies $\Omega_{ij}(\tilde{X})$ for ***variant 2*** are given in fig. 7 for different moments and they show the evolution of liquid jet. At every moment the set of $\Omega_{ij}(\tilde{X})$ composes the bunch of curves. At the beginning the bunch is narrow (black), that testifies to low polydispersity of jet. To the moment $\tau_{\min}$ (green) tip of jet descends down to $O\tilde{X}$ -axis, after that lengthening of jet continues until $\tau_{\max}$. At the same time the part of jet, where stabilization has finished, increases too (points ***a–e***). Each curve and a group as a whole tend to their limit as $\tau \to \tau_{\max}$, which approximately coincide with a group shown at $\tau_6=2.27$ (red). Soon after $\tau_{\max}$ all parts of $\Omega_{ij}(\tilde{X})$ are stabilized and remain still. It ought to be noted, that near and after $\tau_{\max}$ the part of the group that is adjacent to tip bears corrugations, which can be seen at $\tau_6=2.27$.

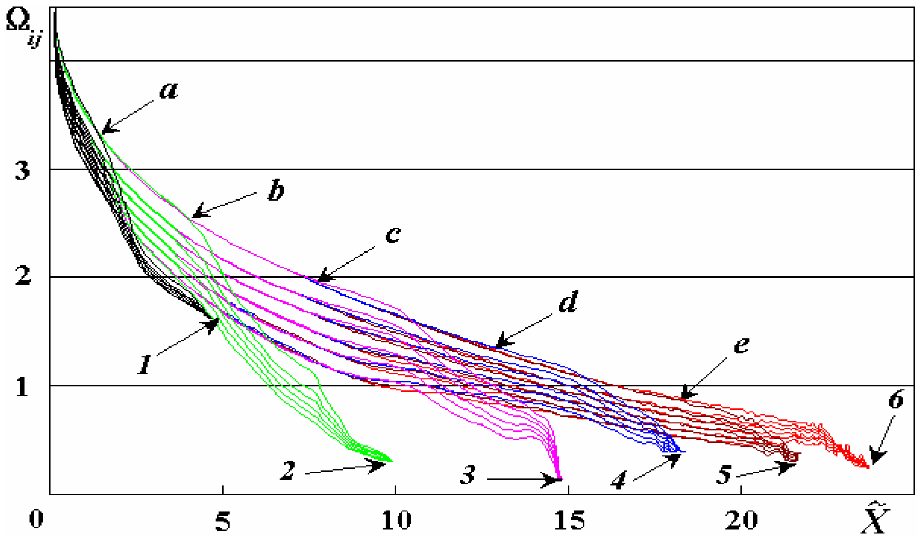

Figure 7 $\Omega_{ij}(\tilde{X})$ are given at $\tau_1=0.43$ (black), $\tau_2=0.71$ (green), $\tau_3=1.14$ (crimson), $\tau_4=1.42$ (blue), $\tau_5=1.73$ (brown), $\tau_6=2.27$ (red); ***1–6*** – locations of tip of jet; ***a–e*** – right-point bounds of stabilized part of jet

Formation of Vapor Cloud

The model allows to calculate the mean values of parameters in each cross section of a spray. Let us assume that the gas components occupy a stream tube of cross area $S_{\text{t.c.}}=\pi(2.5R_{\text{p0}})^2$. Dynamics of vapor cloud formation is illustrated by fig. 8, where the profiles of dimensionless mean density of vapors $\tilde{\rho}_{\text{v.m.}}=\rho_{\text{v.m.}}/\rho_l$ along spray axis are presented for ***variant 1*** at $\tau=0.25;\ 0.46;\ 0.66;\ 0.96;\ 1.26;\ 1.56;\ 1.86;\ 2.16;\ 2.46;\ 2.76;\ 3.06$. At the beginning the capacity of source (4) is highest, therefore intensification of evaporation due to rapid growth of liquid surface is so large, that wave of vapor appears, which has

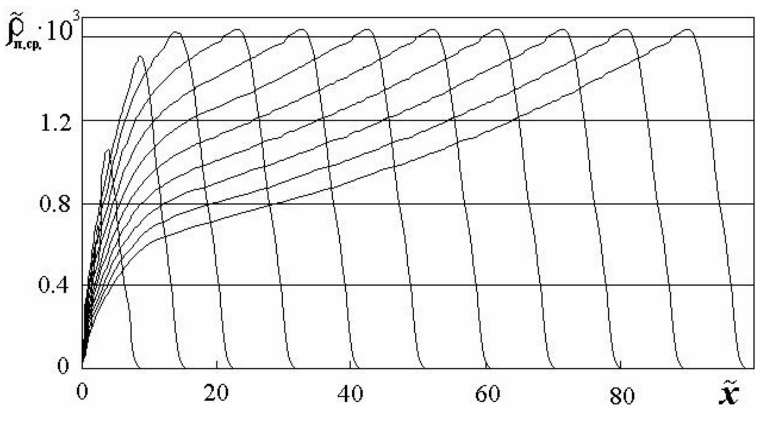

Figure 8 Profiles of $\tilde{\rho}_{\text{v.m.}}(\tilde{X})$

sharp front similar to that of blast wave. After the contact is lost with a liquid-phase jet this wave drifts convectively, keeping its form invariable. Gradual weakening of $\dot{F}_s$ leads to generation of rarefied wave, so, at distances from front to parent drop $x_f > 100 R_{p0}$ distribution tends to "triangular" form, which is inherent to blast waves too. The general shape of $\tilde{\rho}_{v.m.}(\tilde{X})$ is similar to that, obtained by Aggarwal et al. [2].

These data show that fuel-air mixture in a wake spray is substantially over rich in average, as a vapor density several times exceeds the stoichiometric value. Vapor oversaturation leads to cooling of the combustible mixture. An application of a heat balance equation for the process of mixing a vapor mass of m_v at initial temperature of $373°K$, with an air mass $m_a = 3.85 m_v$ at the stream temperature behind the shock front $T_\infty = 1200°K$, gives, when accounting for latent heat of evaporation, a mixture temperature value of $T_{mix} = 900°K$, which means that ignition delay may increase by $1.5 \div 2.0$ orders.

Conclusions

The problem of aerodynamics of an evaporating mist in the wake of a shattering drop behind shock or detonation wave is solved theoretically in closed form at one-dimensional approximation. Presented mathematical model of evaporation ballistics of sprays allows to investigate the formation dynamics of liquid-phase jet and vapor cloud in the wake of a shattering drop, given the parent drop radius and physical properties of gas – liquid system. The previously obtained distribution function was applied as a source function. It pertains to a class of gas – liquid systems with $h = 1$, $h \sim \alpha^{-2/3}\mu^{1/3}$. For such systems the settling of stable structure in liquid-phase jet of spray is revealed by analyzing the dispersive characteristics of jet and the trajectories of stripped droplets on ($\tilde{r}$, X')-plane. Estimations show, that the combustible mixture is over-rich on average in the majority of the generated vapor cloud because of the rapid liquid surface growth due to shattering and evaporation enhancement by a hot, speedy gas stream. Intense evaporation causes the cooling of the combustible mixture, which may increase the induction period delay of chemical reactions in combustion systems. The model presented here is not limited to heterogeneous detonation, but can be applied with some modifications to other kinds of furnaces.

Bibliography

[1] Girin A. G. *Journal of Engineering physics and Thermophysics* **84**: 872–880 (2011).

[2] Aggarwal S. K., Tong A. Y. and Sirignano W. A. *Amer. Inst. Aeron. & Astron. Journ.* **22**: 1448– (1984).

[3] Prakash, S. and Sirignano, W.A. *International Journal of Heat & Mass Transfer* **23:** 885-895 (1980).

[4] Imaoka, R.T. and Sirignano, W.A. *International Journal of Heat & Mass Transfer* **48**: 4342–4353 (2005)

[5] Wu, G. and Sirignano, W.A. *Combustion Theory and Modelling* **15**: 227–243 (2011).

[6] Sirignano, W.A. *Fluid Dynamics and Transport of Droplets and Sprays*, second ed. Cambridge University Press (2010).

[7] Lambiris S, Combs L.P. *Steady-state combustion measurements in a LOX/RP-1 rocket chamber* Progr. Astron. Rocketry. Ed. S.S. Penner, F.A. Williams. Acad. Press. N.-Y.–L.} **6**: 269 (1962).

[8] Engel O.G. *Journal Researches National Bureau of Standards* **60**: 245–280 (1958).

[9] Ragland, K. W., Dabora, E. K., and Nickolls J. A. *Physics of Fluids* **11**: 2377–2388 (1968).

[10] Ranger, A. A. and Nickolls, J. A. *Amer. Inst. Aeron. & Astron. Journ.* **7**: 285–290 (1969).

[11] Reinecke, W.G. and Waldman, G.D. *Shock Layer Shattering of Cloud Drops in Reentry Flight* AIAA Paper **152**: 22 (1975).

[12] Morell G. NASA TN D-677 (1961).

[13] Gelfand B. E. *Present status and problems of investigations of detonation in liquid droplets – gas system* Ed. Dremin A. N. Chemical physics of combustion and explosion processes. Detonation, Acad. Sci. USSR, Inst. Chem. Phys. Chernogolovka. : 28–39 (In Russian) (1977).

[14] Aslanov SK, and Girin AG *Proceedings of Academy of Sciences of UkrSSR, ser. A* **12**: 25–28 (1981).

[15] Girin AG. *Journal of Engineering physics and Thermophysics* **48**: 560–564 (1985).

[16] Aslanov SK, and Girin AG *Combustion, Explosion and Shock Waves* **24**: 475–482 (1988).

[17] Girin AG. *Journal of Engineering physics and Thermophysics* **84**: 262–269 (2011).

[18] Girin AG. *Journal of Engineering physics and Thermophysics* **84**: 1009–1015 (2011).

[19] Kauffman, C.W. and Nicholls, J.A. *Amer. Inst. Aeron. & Astron. Journ.* **9**: 880–885 (1971).

[20] Baev V.K., Bazhaikin A.N., Boldyrev I.V., Buzukov A.A., et al. *Combustion, Explosion and Shock Waves* **15**: 20 (1979).

[21] Less DM, and Schetz J. A. *American Institute of Aeronautics and Astronautics Journal* **24**:1979 (1986)

[22] Williams FA. *Combustion theory* Addison–Wesley Publ. Comp. Palo-Alto – London (1964).