

## Droplet Breakup Modelling in Spraying

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### Abstract

We study the problem of a liquid droplet in high speed trajectory through ambient gas. The fate of such a droplet is of importance to the overall spray characteristics, in the context of secondary breakup.

A theoretical analysis of the problem is presented, in which we examine an overall energy balance that includes surface and dissipation energies. Transformation between a droplet to its breakup products under trajectory conditions that include drag forces, is a spontaneous process that occurs at a characteristic finite time. The transformation involves reversible surface energy transitions, and irreversible viscous dissipation energy. Results propose a simple algebraic equation that shows the inter-relationships between the pertinent parameters, and constitutes the minimum essential conditions for the droplet breakup. Breakup times and travel distances are also evaluated.

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### Introduction

The droplet breakup mechanism is classified in the literature into several regimes according to the relative velocity between the droplet and the surrounding. Krzeczkowski [1] classified the droplet deformation into four different types using the Weber and Ohnesorge numbers, while Pilch and Erdman [2] modified the classification into five regimes, and defined a critical Weber number below which droplet breakup does not occur. Hsiang and Faeth [3] mapped the deformation and breakup regime map on a Weber - Ohnesorge coordinates, and this map will be used here. The number and names of the breakup regimes differ from one research group to another. Inamura et al. [4] presented a unique series of photographs showing the process progress of a droplet breakup in the bag- and umbrella-type breakup regimes, aiming to clarify how the airflow surrounding a droplet affects the breakup mechanism of the droplet in a uniform air-stream.

In the present paper, however, based on the recent study of Sher and Sher [5], we propose a single criterion for the minimum essential conditions for a droplet to breakup, thus defining the border between droplet deformation without breakup and the first mode of breakup mechanism.

### Analysis

Under certain circumstances, a single droplet that travels through ambient can undergo a spontaneous breakup process to yield travelling product droplets. Due to the ambient hydrodynamic drag forces, the travelling droplet is deformed, while the deformation extent and the deformation rate are strongly coupled to the magnitude of the drag force. The drag force itself depends mainly on the trajectory velocity and to a lower extent [6] on the droplet shape (deformation extent). Thus, when a drop is shot into ambient, deformation develops, which may lead to either droplet splitting (breakup) or alternatively to droplet further deceleration and reformation of its shape. The fate of the droplet depends on the droplet velocity, on the droplet thermo-physical properties, and on the ambient properties, through the time scales of the droplet's trajectory deceleration and deformation rate.

The droplet deformation extends its overall surface area, hence the surface energy, and this may induce a breakup process in which the deformed droplet extended surface energy  $E_s$  transforms into product droplets surface energy  $n \cdot e_s$ , while overcoming possible dissipation energy in the process,  $E_{diss}$ . In order to find the minimum essential conditions for droplet breakup, we consider the case of symmetrical breakup process. An energy balance over the process thus yields:

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$$E_s = n \cdot e_s + E_{diss}. \quad (1)$$

Following the symmetry assumption, we assume identical product droplets (minimum surface energy), where  $n$  is the number of product droplets. Next we calculate the droplet deformation and resulting deformed surface energy from the trajectory velocity while we assume potential flow around the droplet [7]:

$$E_s = \sigma \cdot 2\pi \int_0^\pi R^2(\theta) \sin \theta d\theta = \sigma \pi r_e^2 \left( \frac{3^2}{5 \cdot 2^6} We^2 + 4 \right) \quad (2)$$

Here  $We \equiv \frac{\rho_g U^2 r_e}{\sigma}$ ,  $r_e$  is the radius of a spherical droplet of the same volume as the deformed droplet:  $r_e \equiv \left( \frac{3V}{4\pi} \right)^{1/3}$ ,  $U$  is the droplet instantaneous trajectory velocity,  $\sigma$  is its surface tension,  $\rho_g$  is the ambient gas density, and following Meiron [6],  $R(\theta) = r_e \left[ 1 - \frac{3}{64} We (3 \cos \theta + 1) \right]$ .

The initial droplet surface energy,  $E_s$ , therefore, changes along the travelling time, as its velocity,  $U$ , changes due to the varying deformation-inducing drag force.  $E_s$  is the driving energy of the breakup process and since the breakup is a finite-time process,  $E_s$  is a time dependent parameter.

The surface energy of a single product droplet is therefore:

$$e_s = n^{-2/3} 4\pi r_e^2 \sigma \quad (3)$$

The dissipation energy is estimated by:

$$E_{diss.} = \mu_l \left( \frac{du}{dx} \right)^2 V \hat{t} = \mu_l \left( \frac{r_e / \hat{t}}{r_e} \right)^2 \cdot \frac{4}{3} \pi r_e^3 \hat{t} = \frac{4}{3} \pi r_e^3 \mu_l \frac{1}{\hat{t}} \quad (4)$$

Where the typical dimensions were depicted, namely:  $r_e$  is the characteristic length,  $\hat{t}$  is the characteristic time timescale, and  $u$  is the internal droplet characteristic velocity ( $u = r_e / \hat{t}$ ).

The equation of motion for the droplet during its trajectory until breakup is a simple momentum balance, relating acceleration and drag force [8]:

$$\frac{dU}{dt} = -\frac{3}{8} C_D \frac{\rho_g U^2}{\rho_l r_e} \quad (5)$$

Where  $C_D$  is the drag force coefficient, and  $\rho_l$  is the droplet's liquid density. We note that although drop drag coefficients vary when droplets are deformed, use of the original constant average coefficient have been effective for earlier considerations of drop motion [9]. Equation (5) is integrated between  $U_0$  at zero time – initial droplet velocity, and an arbitrary point in time,  $t$ , where the velocity is  $U$ . Velocities are expressed in terms of the corresponding Weber numbers. The result for  $t$ :

$$\frac{1}{t} = \frac{\frac{3}{8} C_D \frac{\rho_g U_0}{\rho_l r_e}}{\frac{\sqrt{We_0}}{\sqrt{We}} - 1} = \frac{3}{8} \frac{\sigma}{\mu_l r_e} C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \frac{\sqrt{We_0} \sqrt{We}}{\sqrt{We_0} - \sqrt{We}} \quad (6)$$

where the index 0 denotes initial values (at  $t = 0$ ), and the Ohnesorge number is:  $Oh \equiv \frac{\mu_l}{\sqrt{\rho_l r_e \sigma}}$ .

To evaluate the breakup scenarios, i.e. the trajectory time and velocity at breakup, the energy balance of eq. (1) is used, where  $e_s$  is evaluated by eq. (3), and  $E_s$  and  $E_{diss}$  are evaluated through eqs. (2) and eq. (4), respectively, at the characteristic timescale,  $\hat{t}$ . The typical timescale is being related to trajectory time from start to breakup, as evaluated by eq. (6). Equation (1) hence yields:

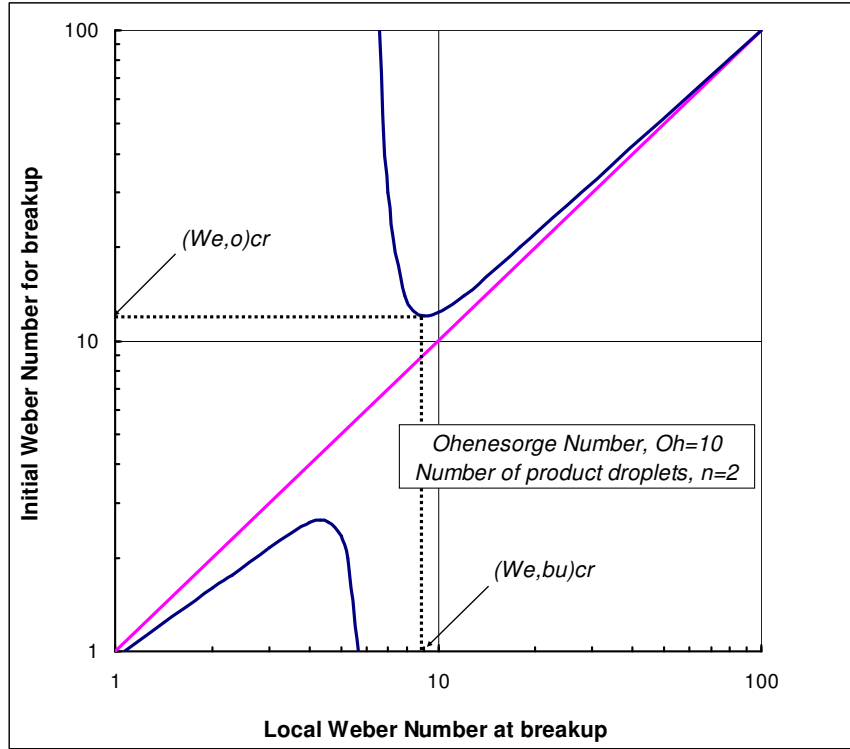
$$\frac{3^2}{5 \cdot 2^8} We_{bu}^2 = (n^{1/3} - 1) + \frac{1}{8} C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \frac{\sqrt{We_0} \sqrt{We_{bu}}}{\sqrt{We_0} - \sqrt{We_{bu}}} \quad (7)$$

In this equation (eq. 7),  $We_{bu}$  is the Weber number at breakup, whereas  $We_0$  is the initial Weber number, at start of trajectory.

## Results and Discussion

For the minimum essential conditions for the droplet breakup, we consider  $n=2$ . In this connection, Steisch [6] noticed that for low Weber numbers near the critical value, "the droplet executes an oscillation and may breakup into two new droplets of approximately equal size". Plotting eq. (7) (fig. 1) and looking at the upper branch, in which  $We_0 > We_{bu}$  (since the droplet decelerates), reveals that there is a minimum  $We_0$  (namely:  $We_{0,cr}$ ) for which a  $We_{bu}$  exists. This means that only above that minimum  $We_0$ , breakup does occur. This critical  $We_0$  is evaluated by finding that minimum, namely:

$$\left. \frac{d(\sqrt{We_0})}{d(\sqrt{We_{bu}})} \right|_{cr} = 0 \quad (8)$$



**Figure 1:** Initial Weber number,  $We_0$ , vs. Weber number at breakup,  $We_{bu}$ . [5]

Solution for the critical initial Weber number,  $We_{0,cr}$ , can be found for two asymptotic cases. One case is where critical surface energy is much larger than that of the expected breakup products, namely:  $(n^{1/3} - 1) \ll \frac{3^2}{5 \cdot 2^8} We_{bu,cr}^2$ . This means that at critical breakup conditions, the initial surface energy is predominantly viscously dissipated. For that case the solution of eq. (8) would be:

$$We_{0,cr} = \left[ \frac{5 \cdot 2^{13}}{3^5} C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \right]^{2/3} = \left( \frac{4}{3} \right)^2 We_{bu,cr} \quad (9)$$

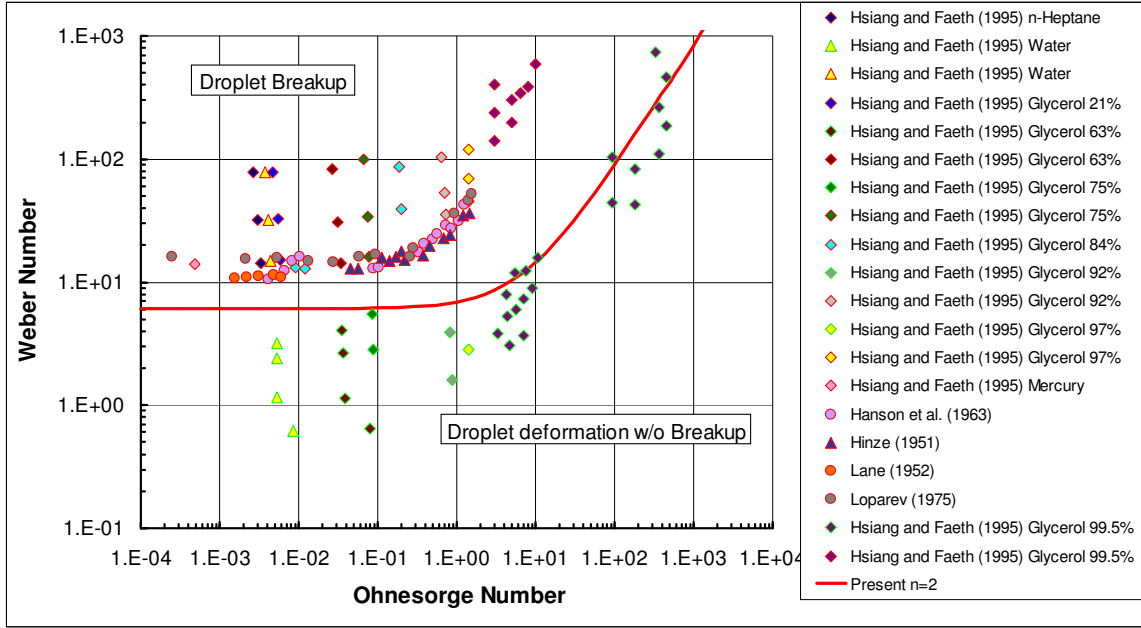
Another case is where viscous dissipation is negligible, namely:  $C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \rightarrow 0$ . This means that at critical breakup conditions the initial surface energy is almost completely converted to products surface energy. Breakup would therefore be rapid, as there is almost no retarding dissipation. For that case the solution of eq. (8) would be:

$$We_{0,cr} = \frac{\sqrt{5} \cdot 2^4}{3} \sqrt{(n^{1/3} - 1)} \rightarrow We_{bu,cr} \quad (10)$$

Combining the two asymptotic solutions, for high and low viscosity, i.e. eqs. (9) and (10), respectively, yields:

$$We_{0,cr} = 11.93 \sqrt{(n^{1/3} - 1)} + 30.51 \left( C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \right)^{2/3} \quad (11)$$

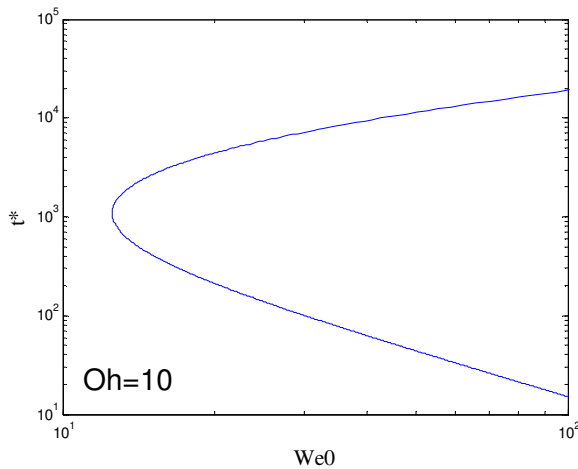
Eq. (11) is, in effect, a relation of the critical initial Weber number of a droplet for breakup, as a function of the Ohnesorge number. This relation is plotted in fig. 2, and is compared to available results.



**Figure 2:** Droplet deformation and droplet breakup regime map. The present results are for  $n = 2$ , an average  $C_D = 0.5$ , and  $\rho_g / \rho_l = 0.003$ . [5]

Figure 2 shows the droplet deformation and droplet breakup regime map. The green data points represent droplet deformation without breakup and the other data points (red framed) represent droplet breakup. The solid curve represents the hereby suggested theory, showing the minimum conditions for droplet breakup. It seems that the present criterion fits quite well any available experimental data without any calibration or fitting parameter. Of particular interest is the prediction of the well known constant critical  $We$  number at low Ohnesorge numbers ( $We_{ocr} = 6$ ), and the increasing effect of the Ohnesorge numbers at high values in a linear log-log scale manner. It is important to note that while the droplet breakup mechanism may vary from vibrational, bag, stripping or other mechanism, the minimum essential conditions for breakup are expected to be independent of it. Also, while breakup can occur for  $n$ 's larger than 2, minimum conditions for breakup are expected for  $n=2$ .

Time for breakup can be evaluated from eq. (6), at breakup Weber of eq. (7). Plotting it in fig. 3, shows a possible bi-modal behavior, where for any initial Weber above the critical one, two breakup scenarios could take place, at different breakup times. It is worth noting here that our analysis shows points in time where breakup is energetically possible, yet may not necessarily happen at one or any of these points.

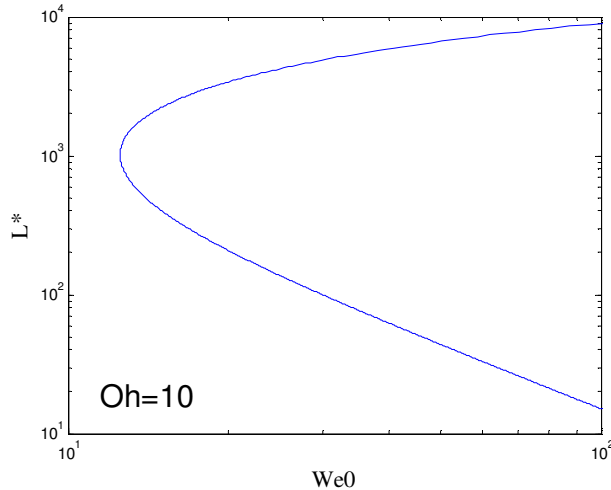


**Figure 3:** Breakup time vs. initial Weber number. Non-dimensional time is:  $t^* = \frac{U_0 t}{r_e}$ .

Travel distance until breakup could be evaluated accordingly:

$$L^* \equiv \frac{L}{r_e} = \frac{1}{r_e} \int_0^t U dt = \frac{1}{\frac{3}{8} C_D \frac{\rho_g}{\rho_l}} \ln \left( \frac{3}{8} C_D \frac{\rho_g}{\rho_l} t^* + 1 \right) \quad (12)$$

Plotted in fig. 4, the two possible modes of breakup can be seen, where for any initial Weber above the critical one, two breakup scenarios could take place, at different travel distances.



**Figure 4:** Travel distance to breakup vs. initial Weber number.

Further study is directed at relating these breakup times and lengths of a single droplet, to the complex problem of spray distribution.

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