Application of dimensional analysis in estimating the wall film thickness created by a liquid spray impact

D. Kalantari*, C. Tropea**
*Dep. of Mech. Eng., University of Sari (SANRU), P.O.Box 578, Sari, Iran
**Institute of Fluid Mechanics and Aerodynamics, Center of Smart Interfaces, Technische Universität Darmstadt, Germany
dkalantari2000@gmail.com and ctropea@sla.tu-darmstadt.de

Abstract

This study presents a theoretical investigation on the formation and spreading of a liquid film on a flat and rigid wall due to spray impact. The new model for predicting the average film thickness is a function of mean Reynolds number of the impacting drops, flux density of the impacting droplets, and the average drop diameter. The theoretical derivation exhibits good agreement with the measured data in the thin film condition. Experimentally the average film thickness accumulated on the wall and the spray has been measured using a high-speed CCD camera and a dual-mode phase Doppler instrument, respectively.

Introduction

With the process of spray/wall interaction, it has become apparent in recent years that the liquid film formed on the wall plays an important role in determining the velocity and size of ejected droplets as well as the deposited mass fraction, see e.g., Bai et al. (2002), Cossali et al. (1999). Nevertheless, the formation of a wall film is often neglected in spray impact models, although prediction of average film thickness and average velocity is very important for many industrial applications, especially for those involving spray cooling systems or for fuel injection sprays onto heated walls, since the wall film significantly affects the efficiency of heat transfer on the surface. In some applications, it is desirable to eliminate the deposited film on the wall as far as possible, e.g. in internal combustion engines, whereas in other cases a maximum deposition is required, e.g. in spray coating, spray painting or agricultural sprayers. On the other hand, the induced fluctuations in the liquid layer formed on the rigid walls may decrease the quality of coated or painted surfaces. The average film thickness can also affect the properties of the secondary spray, splashing threshold, ejected mass and number of secondary droplets.

Numerous models exist regarding the formation of the wall film generated by an impacting liquid spray, see e.g., Stanton and Rutland (1996), Lee et al. (2001), and Bai and Gosman (1996), Ahmadi-Befuri et al. (1996), Kalantari and Tropea (2006e). The model of Stanton and Rutland (1996) solves the continuity and momentum equations for a 2-D film flow over an arbitrary solid surface using the Euler method. This model considers many physical effects such as shear forces and dynamic pressure of impacting droplets, but neglects the Laplace (capillary) pressure arising from curvature of the air-liquid film interface.

In the present study, a new expression is given based on the dimensional analysis for predicting the average film thickness accumulated on the wall due to a liquid spray impact. The theoretical derivation for the average film thickness then compared with the measured data in the thin film condition, i.e., ($\bar{h}/d_{10b} < 1$, where $\bar{h}$ is the average film thickness and d_{10b} is the mean drop diameter before impact).

Theoretical approach

In the impingement region of an inertial spray, the average film thickness created on the wall depends on the several parameters of the impacting spray ; normal and tangential component of impact velocity $\bar{u}_b$ and v_b , volume flux density of impacting spray ($\dot{q} = q/A$; "q" and "A" to be volume flux of the impacting spray(m^3/s) and the reference area over which flux is measured), volume-averaged diameter of impacting droplets (d_{30b}) defined by $d_{30b} = \left(\sum_{i=1}^{N} {d_i}^3/N\right)^{1/3}$, density ($\rho_L$) and dynamic viscosity of the liquid (μ), as well as the boundary condition of the target (e.g., flat or curved target surface or target surface completely covering with spray); average target surface roughness ($\bar{\varepsilon}$) and target size (D). A general expression for the average film thickness can be written as

$$\bar{h} = \Psi\left(\bar{u}_b, \bar{v}_b, d_{30b}, \dot{q}_b, \rho_L, \mu, \bar{\varepsilon}, D_{spray}, D\right) \tag{1}$$

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* Corresponding author: dkalantari2000@yahoo.com

where, D_{spray} is the diameter of the effective impinging spray on the target defined as: $D_{spray} = 2x_{Nozzle}\ \tan(\alpha/2)$, α is the spray cone angle. The parameters $\bar{u}_b$, v_b , $\dot{q}_b$, d_{30b} and D_{spray} vary with nozzle pressure and nozzle height above the target. The three first parameters can be combined into an impact Reynolds or Weber number depending on the dimensional analysis output. Volume flux of impacting spray (q) depends on the atomizing pressure (P) with a power law; $q \propto P^{0.5}$ (Bernoulli's equation).

For characterizing the average film thickness accumulated on the wall due to a normal spray impact (negligible tangential velocity component of the impacting droplets) and for a spray covering the target surface (i.e., $D_{spray}/D>1$), variables involving in the film thickness (1) can be reduced to

$$\bar{h} = \Psi(\bar{\mathrm{u}}, \mathrm{d}_{30\mathrm{b}}, \dot{\mathrm{q}}_{\mathrm{b}}, \rho_{\mathrm{L}}, \mu, \bar{\varepsilon}) \tag{2}$$

Using the dimensional analysis with considering the d_{30b}, $\bar{u}_b$ and ρ_l as repeating variables, the Π-numbers can be written as

$$\begin{cases} \Pi_1 = (d_{30b})^{a1}(\bar{u}_b)^{b1}(\rho_l)^{c1}\bar{h} = M^0L^0T^0 \\ \Pi_2 = (d_{30b})^{a2}(\bar{u}_b)^{b2}(\rho_l)^{c2}\mu = M^0L^0T^0 \\ \Pi_3 = (d_{30b})^{a3}(\bar{u}_b)^{b3}(\rho_l)^{c3}\dot{q}_b = M^0L^0T^0 \\ \Pi_4 = (d_{30b})^{a4}(\bar{u}_b)^{b4}(\rho_l)^{c4}\bar{\varepsilon} = M^0L^0T^0 \end{cases} \tag{3}$$

Equating the exponents of two sides of each Π-number and Solving explicitly the obtained equations for a_1, b_1, …, c_4, one obtains

$$\begin{cases} \Pi_1 = (d_{30b})^{-1}\bar{h} \\ \Pi_2 = (d_{30b})^{-1}(\bar{u}_b)^{-1}(\rho_l)^{-1}\mu \\ \Pi_3 = (\bar{u}_b)^{-1}\dot{q}_b \\ \Pi_4 = (d_{30b})^{-1}\bar{\varepsilon} \end{cases} \tag{4}$$

Therefore based on the Π-theorem, the functional relationship for the obtained Π-numbers can be written in the form of

$$\left(\frac{\bar{h}}{d_{30b}}\right) = f\left(\frac{1}{Re_b}, \frac{\dot{q}_b}{\bar{u}_b}, \frac{\bar{\varepsilon}}{d_{30b}}\right) \tag{5}$$

Practically it is more convenient to use d_{10b} instead of d_{30b} for non-dimensionallizing the target surface roughness in the form of $\bar{\varepsilon}^* = \bar{\varepsilon}/d_{10b}$. For further developing the obtained relationship (5) and trying to remove the Re number from within the bracket, an asymptotic solution of a single droplet impact will be presented in the following section for an inertial impact condition.

An asymptotic solution of the average wall film thickness for a relative sparse but inertial spray (i.e. $We_b/\sqrt{Re_b} \gg 1$**)** can be obtained from result of a single drop impact onto a flat-rigid surface. considering that $We_b \gg \sqrt{Re_b}$ and $We_b >> 12$, which is valid for most inertial spray/wall interactions**,** theoretical results of Pasandideh-Fard et al., 1996 indicate that the impact of a single droplet onto a flat, rigid wall produces a splat with the maximum dimensionless diameter of $\xi \cong 0.5{Re_b}^{0.25}$.

Considering the mass balance for a spreading droplet at its maximum diameter, and assuming a disk shape yields: $h^* = (2/3)\xi^{-2}$, where h^* is the dimensionless film thickness formed on the wall defined as $h^* = h/d_b$. Substituting $\xi \cong 0.5{Re_b}^{0.25}$ into this expression yields

$$h^* \cong 2.67{Re_b}^{-1/2} \tag{6}$$

The dependency of h^* on ${Re_b}^{-1/2}$ in the case of a single drop impact for inertial impact condition indicates that we can extract the Reynolds number outside of Eq. (5) in the form

$$\left(\frac{\bar{h}}{d_{30b}}\right) = {Re_b}^{-1/2} f\left(\frac{\dot{q}_b}{\bar{u}_b}, \frac{\bar{\varepsilon}}{d_{10b}}\right) \tag{7}$$

It is interesting to note that based on the work of Roisman et al. (2005), the thickness of the lamella of a spreading liquid droplet on the flat and rigid surface (h) is expressed in the form of
$h^* \propto {Re_b}^{-1/2}$ for high impact Reynolds numbers ($Re_b>2000$). Based on this result and the results obtained in

this study, Eq. (8) for dimensionless film thickness on the wall, we may emphasize Reynolds number as a scaling parameter of film hydrodynamics.

$$\bar{h} = \alpha d_{30b} {Re_b}^{-1/2} f\left(\frac{\dot{q}_b}{\bar{u}_b}, \frac{\bar{\varepsilon}}{d_{10b}}\right) \tag{8}$$

One simple form of the (8) for a negligible surface roughness can be written as

$$\bar{h} = \alpha d_{30b} {Re_b}^{-1/2} \left(\frac{\dot{q}_b}{\bar{u}_b}\right)^{\gamma} \tag{9}$$

where α and γ are constant values found to be 4 and -0.5, respectively. These constants have been found based on the measured data in this study for a water spray impacting onto a stainless steel target with 5 mm in diameter, negligible surface roughness and normal impact condition.

In Fig.1 predictions of the dimensional analysis (Eq. 9) for the average film thickness is presented together with the many individual measurements as a function of impact Reynolds number. In this figure each individual average film thickness ($\bar{h}$) is normalized by the volume averaged droplet diameter before impact (d_{30b} **)**. The results presented in this figure indicate good prediction of the average film thickness obtained from dimensional analysis (Eq. 9).

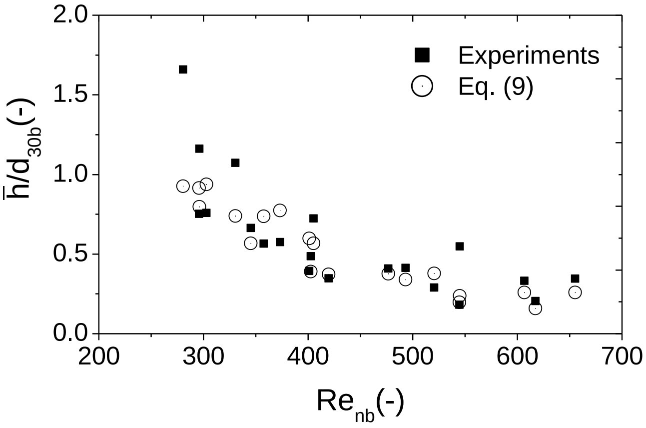

Figure 1: Prediction by dimensional analysis for the average film thickness.

Summary and Conclusions

This paper presents a new theoretical model for predicting the average film thickness as a function of mean Reynolds number of the impacting drops, flux density of the impacting droplets, and the average drop diameter. The theoretical derivation for the average film thickness exhibits good agreement with the measured data in the thin film condition. Results obtained in this study indicate a significant influence of the Reynolds number on the average film thickness accumulated on the wall due to liquid spray impact. As illustrated above, the mean film thickness varied in the range 8µm < $\bar{h}$< 107µm, corresponding to an impingement Weber number in the range 10 < We_{nb} < 165, based on the normal velocity component. The expression based on dimensional analysis (9) has been derived for a general spray impact condition and has no principle limitations. However applicability of this expression for higher Reynolds numbers and other experimental conditions should be examined in future studies. The only important condition in using this expression for film thickness is that the entire target surface should be exposed to the impacting spray.

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