Energy conversion during the splash

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Abstract

In this study, a theoretical study of splashing droplet is presented based on the energy conversion during the splash. The obtained results indicate that the non-dimensional crown height increases nonlinearly with increasing the impact velocity, but decreases slightly with the non-dimensional film thickness. Theoretical predictions properly estimate the maximum crown height in the case of a splash in isolation, i.e. single drop impact, whereas slightly underestimate in the case of a splash in spray impact conditions. This study can be valuable for future modelling of spray impact.

Introduction

In an overall effort to model the impact of liquid sprays onto rigid walls, the splashing phenomena plays an important role in determining the velocity and size distribution of ejected droplets from the wall as well as the ejected mass fraction, see e.g., Coghe et al. (1999), Cossali et al. (1997), Cossali et al. (1999), Kalantari and Tropea (2005), Kalantari and Tropea (2006d), Mundo et al. (1997), Mundo et al. (1998), Sivakumar and Tropea (2002).

In practice, increasing the number of splashing droplets in spray impact phenomena can decrease the quality of coated or painted surfaces. A large number of parameters and variables can influence the splashing phenomenon; physical properties of droplet fluid: viscosity, surface tension and density, impact parameters: impact velocity, flux density of impacting droplets, i.e. frequency of impacting droplets, and droplet trajectory, and target characteristics (rigid wall: dry or wetted wall (surface roughness, wall temperature), liquid layer (film thickness, surface roughness). From the listed parameters, two of them are very important in determining the onset of splashing: surface roughness and average depth of accumulated liquid film on the wall, e.g. splashing takes place faster for rough surfaces as postulated by Mundo et al. (1998) and Range and Feuillebois (1998). Therefore, the ratio of average wall roughness to the average primary droplet size ($\bar{\varepsilon}^* = \bar{\varepsilon}/d_b$, where $\bar{\varepsilon}$ is the average of roughness of the target surface) should be considered if rough or structured surfaces are used. Josserand et al. (2005) investigated the triggering of splashing by using a small obstacle on a dry-solid surface. Also the ratio of the average liquid film thickness accumulated on the wall to the average primary droplet size ($\bar{h}^* = \bar{h}/d_b$, where $\bar{h}$ is the average film thickness) must be considered in the case of accumulated wall film, see e.g., Kalantari and Tropea (2007), Cossali et al. (1997), Mundo et al. (1997), Rioboo et al. (2003).

The present work provides a theoretical model for the maximum crown height of a splashing droplet, compared with the available experimental data. Fundamental study of the splashing phenomenon for single drops and for drops in a spray can be very valuable for future modeling of single droplet and spray impact.

Theoretical approach

During the splash, an uprising crown-like thin liquid sheet develops at the kinematics discontinuity position (point “B”: a point between the spreading droplet and unperturbed wall film with very high velocity and film thickness gradient). This crown-like sheet is bounded with a free end rim due to the surface tension effect, which generates finger-like jets disintegrating into the secondary droplets. A sketch of a single droplet splashing on a liquid layer is illustrated in Fig.1.

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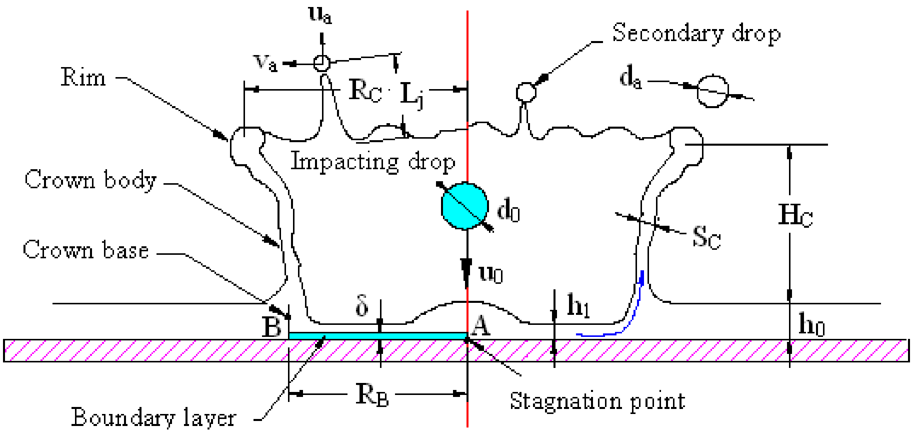

Figure 1: Sketch for structure of a splashing liquid droplet.

In this study an energy conservation approach is considered for estimating the maximum crown height during the crown development. The energy conservation links the total energy of the impacting droplet and splashing crown. Total kinetic and surface energy of the impacting droplet onto liquid film is

$$E_0 = \underbrace{\frac{1}{12}\pi\rho d_0^{\,3} u_0^{\,2}}_{Kinetic} + \underbrace{\sigma\pi d_0^{\,2}}_{Surface} \tag{1}$$

During the droplet spreading, kinetic energy dissipation takes place at the boundary layer of the spreading droplet between the stagnation point and the kinematics discontinuity point. Energy dissipation in the boundary layer of a spreading droplet is estimated similar to a methodology derived by Pasandideh-Fard et al. (1996), generalized for a splashing droplet as

$$E_{diss} = \int_0^{t_C}\int_\Omega \Phi\, d\Omega\, dt \approx \Phi\Omega t_C \tag{2}$$

where Ω is the volume of boundary layer of the spreading droplet which energy dissipation takes place in it, Φ is the viscous dissipation function that can be expressed in a simple form of $\alpha\mu(u/\delta)^2$; in which α is a constant value, and δ is the thickness of the boundary layer defined as:$\beta\, d_0/\sqrt{Re}$; β is a constant coefficient. Total energy dissipation can be then expressed in the form

$$E_{diss} = \pi(\alpha/\beta)\rho u_0^2 d_0 R_B^2 \frac{t_C^*}{\sqrt{\mathrm{Re}}} \tag{3}$$

where R_B is the crown base radius and t^*_C is dimensionless time measured from the impingement to the maximum crown height. Based on the measurements obtained by Cossali et al. (1997), it can be considered that at the instant of the maximum crown height, i.e. $t^*_C \cong H^*_C/0.15$, Eq. 3 yields in the form of

$$E_{diss} \cong \frac{\pi}{10}\rho u_0^2 d_0 R_B^2 \frac{H_C^*}{\sqrt{\mathrm{Re}}} \tag{4}$$

Here H^*_C is non-dimensional maximum crown height measured from the wall film surface to the rim centerline (Fig. 1). In Eq.4, the value of $0.15\alpha\cdot\beta^{-1}$ is found to be approximately 10 based on the measurement data used in this study.

Gravitational potential energy of the spreading lamella, crown body and crown rim is expressed in the approximated form

$$E_g \cong \underbrace{\frac{1}{2}\pi\rho g(h_0 - h_1)^2 R_B^{\,2}}_{Lamella} + \underbrace{\frac{3}{2}\pi\rho g(R_B + R_C)S_c H_C^{\,2}}_{Crown} + \underbrace{2\pi^2\rho g R_C r_R^{\,2} H_C}_{Rim} \tag{5}$$

where R_C is crown upper radius, h_0 and h_1 are undisturbed film thickness and thickness of the spreading droplet , respectively.

Potential surface energy of the lamella, crown body and crown rim is

$$E_\sigma \cong \underbrace{2\pi\sigma(h_0 - h_1)R_B}_{Lamella} + \underbrace{2\sigma\left[2\pi(\frac{R_B + R_C}{2})H_C\right]}_{Crown} + \underbrace{4\pi^2\sigma R_C r_R}_{Rim} \tag{6}$$

With considering $R_C = R_B + H_C \tan\theta_C$ and $\tan\theta_C = 1 - 4{h_0}^*$ from Fedorchenko and Wang (2004), the second term of Eq.6, i.e. potential surface energy of the crown body can be expressed as

$$\mathrm{E}_{\sigma-\mathrm{crown}} = 2\sigma\left\{2\pi\mathrm{H}_\mathrm{C}\left[\mathrm{R}_\mathrm{B} + \frac{1}{2}\mathrm{H}_\mathrm{C}\left(1 - 4\mathrm{h}_0^*\right)\right]\right\} \qquad ; \text{for } {h_0}^* < 0.25 \tag{7}$$

$$\mathrm{E}_{\sigma-\mathrm{crown}} = 4\pi\sigma\mathrm{R}_\mathrm{B}\mathrm{H}_\mathrm{C} \qquad ; \text{for } {h_0}^* \geq 0.25 \tag{8}$$

Here several parameters inside the obtained equations should be determined; first a relation between the crown base radius and the crown height at the instant of the maximum crown evolution is required. To obtain this relationship, consider a general form of the time history for the crown base radius in the form of (see e.g. Yarin and Weiss, 1995).

$${R_B}^* \sim (t^* - \tau_0)^{0.5} \tag{9}$$

where τ_0 is a constant value in the range $\tau_0 \in [0, \xi]$; ξ is a small constant value corresponding to the initial condition of the splash. A relation between $R^*{}_B$ and $H^*{}_C$ at the instant of the maximum crown height can be obtained by substituting $\tau_0 = 0$, $t^*{}_C = H^*{}_C/0.15$ at the instant of the maximum crown height, and a average constant coefficient equal to 2.2 for the crown base radius from the experimental observations of Cossali et al. (1999) in the (9), yielding

$${R_B}^* \cong 5.68 H_C^{*0.5} \tag{10}$$

This relationship will be used in simplification of the several obtained expressions, e.g., the gravitational potential energy, or potential surface energy.

To estimate the crown thickness at the instant of the maximum crown height, experimental data from Coghe et al. (1999) is used in this study which presented in Fig. 4. It is shown in this figure that the crown thickness at the instant of maximum crown height is distributed around the value ${S_C}^* \approx 0.22$ independent of the impact Weber number and the wall film thickness. This value will be used in this approach.

An approximate value for the radius of the rim can be considered based on the experimental observation of Range and Feuillebois (1998) in the form of

$$\frac{\mathrm{r}_\mathrm{R}^*}{\mathrm{R}_\mathrm{C}^*} \cong \frac{1}{20} \tag{11}$$

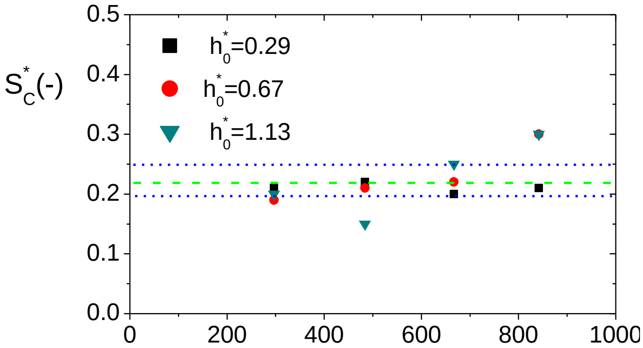

Figure 2: Crown thickness at the instant of the maximum crown height as a function of impact Weber number for various wall film thicknesses.

Energy balance ($E_0 = E_{diss} + E_g + E_\sigma$) at the maximum crown height (H^*_{Cmax}) for $h_0{}^* \geq 0.25$ yields

$$\sum_{n=0}^{6} A_n \left(H^*_{Cmax}\right)^{\frac{n}{2}} = 0 \tag{12}$$

$$A_0 = -\left(\frac{We}{12}+1\right) \qquad A_1 = \sqrt{30D^*_f} \qquad A_2 = \frac{15}{4}\left(\frac{We}{Fr}\right)D^{*2}_f + \frac{3\pi}{2} \qquad A_3 = 11 \qquad A_4 = \frac{3}{4}\left(\frac{We}{\sqrt{\mathrm{Re}}}\right)$$

$$A_5 = 0.55\left(\frac{We}{Fr}\right) \qquad A_6 = \frac{3\pi}{80}\left(\frac{We}{Fr}\right)$$

Which We, Re, and Fr are dimensionless impact parameters defined as: $We = \rho u^2 d_0/\sigma$; $Re = \rho u d_0/\mu$; $Fr = u^2/g d_0$, also $D_f = h_0(1 - h_1/h_0)$.

Results and Discussion

Numerical solution of the obtained theoretical results in the case of single isolated drops (Eq.12), indicate that the non-dimensional crown height increases nonlinearly with increasing the impact velocity, see Fig.3. On the other hand, the non-dimensional crown height decreases slightly with the non-dimensional film thickness as illustrated in Fig.4, corresponding to the wall film thickness varied in the range 0.25 <$h_0{}^*$ < 1.

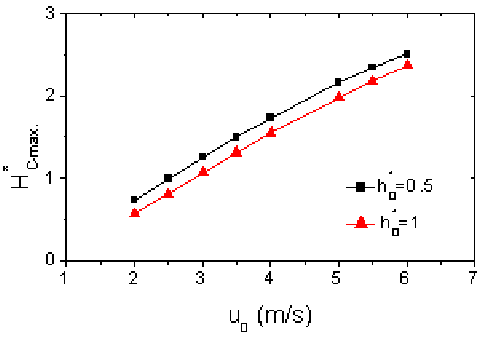

Figure 3 : Maximum non-dimensional crown height as a function of impact velocity for two different non-dimensional film thicknesses, d_0=3 *mm*.

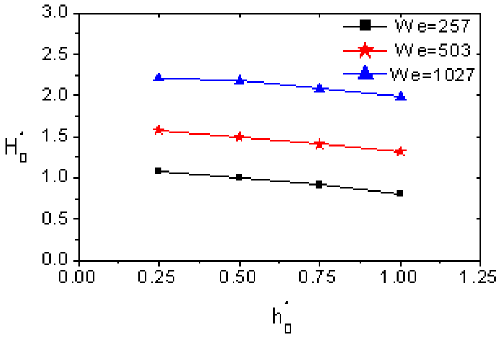

Figure 4: Influence of non-dimensional film thickness on the maximum non-dimensional crown height.

Results of this theoretical approach for predicting the maximum crown height Eq.12 is presented in Figs. 5 and 6 compared to experimental measurements for a single drop impacting onto a stationary liquid film (Fig.5) and for a single droplet of a water spray impacting onto an oscillating film created on the wall due to spray impact (Fig.6).

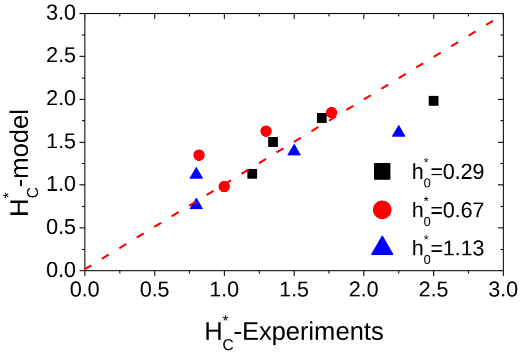

Figure 5: Prediction of the maximum non-dimensional crown height estimated from Eq. (12) as a function of the experimental measurements obtained by Cossali et al. (1997).

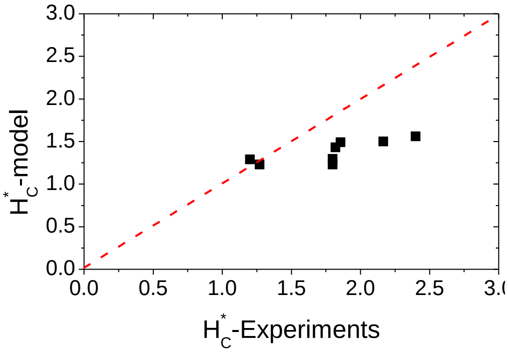

Figure 6: Maximum non-dimensional crown height estimated from Eq. (12) as a function of the experimental results.

Summary and Conclusions

The theoretical equation for the maximum crown height is developed based on the energy conservation method. The theoretical prediction is then compared with the available measurement results for single drop impact and spray impact conditions. Theoretical predictions properly estimate the maximum crown height in the case of a splash in isolation, i.e. single drop impact, whereas slightly underestimate in the case of a splash in spray impact conditions. Perhaps in the case of a spray impact, velocity fluctuations inside the wall film cases such differences. Results obtained in this study indicate that the maximum non-dimensional crown height increases linearly with the Weber number before the impact in spray impact phenomena.

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