Fragmentation under scaling symmetry and stochastic modeling of a highly turbulent flow with spray

M. Gorokhovski*
* Laboratoire de Mecanique des Fluides et Acoustique Ecole Centrale de Lyon
France

Abstract
This review concerns complexity in primary atomization of spray and its stochastic simulation. The approach is based on fragmentation under scaling symmetry. Along with physical examples, discussion starts with the formulation of the mathematical background and analysis of the statistical universalities of such fragmentation. Our interest is in the question of how, despite the many degrees of freedom in dynamic production of liquid fragments, the universal distribution of length scales may come about, leading to fractal properties at the highest spatial resolution. The resulting statistical universalities are used in the construction of stochastic processes for the evolution of length scales in atomizing liquid jets, with control parameters arising from the presumed breakup mechanism. Examples of application include primary air-blast and pressure-assisted atomization. Emphasis is placed on coupling of such a stochastic simulation with LES for turbulent gas flow in the vicinity of an injector. The final point of discussion concerns the assessment and applicability of this approach in engineering computations.

Introduction

The equations governing an unsteady, incompressible, immiscible, two-fluid system are the Navier-Stokes equations, which were recently applied to primary atomization modeling (review is given in [1]):

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\rho \nabla \cdot \boldsymbol{u}\,\boldsymbol{u} - \nabla p + \mu \nabla^2 \boldsymbol{u} + \sigma\, \delta(\boldsymbol{x} - \boldsymbol{x}_f(t))\,\boldsymbol{k}; \qquad \nabla \cdot \boldsymbol{u} = 0; \tag{1}$$

where $\boldsymbol{u}$ is the velocity, μ the dynamic viscosity, ρ the density, p denotes the pressure, $\delta(\boldsymbol{x}-\boldsymbol{x}_f(t))$ is the Dirack delta function, $\delta(\boldsymbol{x}-\boldsymbol{x}_f(t)) = \prod_{i=1}^{3} \delta(x_i - x_{f,i}(t))$, σ the constant surface tension coefficient, $\boldsymbol{k} = \nabla \cdot \boldsymbol{n}\boldsymbol{n}$ is the vector of interface curvature, and $\boldsymbol{n}$ is the interface normal vector. The source term in (1) is non-zero only if the considered control volume is visited by the phase interface, which is described by a location $\boldsymbol{x}_f(t)$. For a high Reynolds number, in the framework of approach proposed in [2] for the group-theoretical model of turbulence, one can obtain the renormalized form of this equation

$$\frac{\partial \langle \boldsymbol{u} \rangle_\Delta}{\partial t} = \mathrm{LES} + \left(\frac{1}{4\pi \Delta^2} \right)^{3/2} e^{-\frac{\sum_{i=1}^{3} (x_i - x_i(t))}{4\Delta^2}} \frac{\sigma_{tur}}{\bar{\rho}} \langle \boldsymbol{k} \rangle_\Delta; \qquad \nabla \cdot \langle \boldsymbol{u} \rangle_\Delta = 0 \tag{2}$$

where LES denotes the right-hand of LES-equations averaged by the Gauss filtering, with Favre weight [3]; $\langle \boldsymbol{k} \rangle_\Delta$ is the curvature of interface smoothed over filter thickness Δ, and the turbulent surface tension coefficient is given here by:

$$\frac{\sigma_{tur}}{\bar{\rho}} = C\,\Delta^3 \langle S_{ik} \rangle_\Delta \langle S_{ik} \rangle_\Delta\;; \quad \langle S_{ik} \rangle_\Delta = \frac{1}{2}(\nabla_i \langle \boldsymbol{u_k} \rangle_\Delta + \nabla_k \langle \boldsymbol{u_i} \rangle_\Delta). \tag{3}$$

One of the problems in (2) is that free surface geometry $\boldsymbol{x}_f(t)$ is, in principle, unknown, and at a high Reynolds number, it may behave randomly, with extremely strong fluctuations. From other side, there is a fragmentation theory, which characterizes evolution in time of distribution of a typical length-scale in a system of breaking particles. The simplest but however intriguing model is to view fragmentation as a multiplicative process in which the sizes of children are fractions of the size of the parent. When breaking events are uncorrelated, such a fragmentation is referred to as fragmentation under scaling symmetry.

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* Corresponding author: Mikhael.Gorokhovski@ec-lyon.fr

Results and Discussion

Our talk consists in three parts. The first part is devoted to analyses of statistical universalities that arise over time during fragmentation under scaling symmetry at constant frequency [4]. The explicit expression of particle-size distribution obtained from the evolution kinetic equation shows that with increasing time, the initial distribution tends to the ultimate steady-state delta function through at least two intermediate universal asymptotics. The earlier asymptotic is the well-known log-normal distribution of Kolmogorov (1941). This distribution is the first universality and has two parameters: the first and the second logarithmic moments of the fragmentation intensity spectrum. The later asymptotic is a power function (stronger universality) with a single parameter that is given by the ratio of the first two logarithmic moments. At large times, the first universality implies that the evolution equation can be reduced exactly to the Fokker-Planck equation instead of making the widely-used but inconsistent assumption about the smallness of higher than second order moments. At even larger times, the second universality shows evolution towards a fractal state with dimension, which can be identified by presuming of the mechanism of breakup.

Identification of this parameter (according to observations from Lasheras's, Hopfinger's, Villermaux's and Arcomanis's scientific groups) and development of stochastic model of spray formation in the vicinity of the air-blast and pressure assisted atomizers will be described in the second part, discussing assessment of modeling by comparison with measurements. In the model, the 3D configuration of continuous liquid core is simulated by spatial trajectories of specifically introduced stochastic particles. The stochastic process is based on assumption that the exiting continuous liquid jet is depleted in the framework of statistical universalities of a cascade fragmentation under scaling symmetry. The stochastic spray formation model is linked with the LES computation of the gas flow by introducing the immersed boundary method on the basis of spatial distribution of the probability of finding the non-fragmented liquid jet in the near-to-injector region. Some details can be found in [5-7].

The third part is devoted to recent results on fragmentation under scaling symmetry but with the breakup frequency decreasing with decreasing radius of particle: $v_{bu} = c\,r^{\beta}$; $\beta > 0$. To this end, the renormalized form of the fragmentation equation is first shown, and advantages of this form against classical integral form of the fragmentation equation are shown (for example, this form contains explicitly the mass-flux of cascade process of fragmentation in the space of length r). In the case, when the spectrum of fragmentation is a power function, the analytical self-similar solution is shown. Surprisingly this solution goes at large times to the generalized Weibull distribution, which is often found in measurements. Example of determining scaling β, and perspectives of application of distributions, when breakup frequency is decreasing with decreasing radius of particle, are discussed, targeting to the spray formation stochastic modelling.

References

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