

Numerical studies of multiple droplet impacts onto a flat substrate

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Abstract

We numerically study multiple droplet impingement. The numerical method consists of a CLSVOF (coupled level set and volume-of-fluid) framework, CIP-CSL (constraint interpolation profile-Conservative Semi-Lagrange) method, VSIAM3 (volume/surface integrated average based multi-moment method) and a CSF (continuum surface force) model. The numerical framework can robustly simulate multiple droplet impingement.

Introduction

Spray impingement on solid surfaces are of importance for many industrial processes: direct fuel injection in internal engines, gas turbines, spray cooling of steel sheets, spray coating, spray painting and agricultural sprays [1]. Many experimental and theoretical studies have concentrated on single drop impacts. Prevailing models extrapolate the results of single droplet impact onto dry rigid walls to the case of spray-wall interaction by a superposition of many individual droplets. However, recent research has shown that this limited approach neglects several important interactive effects such as the influence of deposited film on secondary spray; the effect of film fluctuation on impacting droplets; the effect of multiple droplet interactions as well as the interaction of splash crowns and jets with other impacting drops. Therefore, the interaction of multiple drop impacts has to be considered when considering spray impingement.

In this paper, we employ an approach using a fixed grid and use the CLSVOF (Coupled Level Set and Volume of Fluid) formulation [6], which uses both the level set method [4, 5] and the VOF (Volume of Fluid) method [2, 3]. In this formulation, the VOF method deals with interface motion and the level set method is used for surface tension and wettability computations. In this paper, the THINC/WLIC method [7, 8] is used as a VOF method. The THINC/WLIC method is easy to implement and enough accurate. For the flow calculation, we employ a finite volume framework. The CIP-CSL method [9, 10] is used as the conservation equation solver. Although finite volume methods usually deal with only the cell average as the variable, the CIP-CSL method uses both the cell average and the boundary value as variables. By using both values (moments), a parabolic interpolation function is constructed in a cell, and the boundary value and the cell average are updated based on the parabolic function. For multi-dimensional cases, dimensional splitting is used [11]. VSIAM3 [11] is a fluid solver which can be combined with the CIP-CSL methods. For the surface tension force, we use the CSF model [12]. For contact angle implementation, a method by Sussman [13] is used, because of its simplicity.

Numerical method

Interface capturing based on CLSVOF method

In our numerical framework, the interface is tracked by a VOF function evolved by the THINC/WLIC method, and the level set function is constructed based on the interface indicated by the VOF function. Here we should note that VOF methods guarantee conservation when the divergence-free condition ($\nabla \cdot \mathbf{u} = 0$) is precisely satisfied on staggered grids. If the velocity field does not satisfy the divergence-free condition, conservation is not satisfied even though a VOF type method is used.

The THINC/WLIC method

The THINC/WLIC method is a type of VOF method. The VOF function is advected by

$$\frac{\partial \chi}{\partial t} + \nabla \cdot (\mathbf{u}\chi) - \chi \nabla \cdot \mathbf{u} = 0, \quad (1)$$

here \mathbf{u} is the velocity, χ is the characteristic function. The cell average of χ is the VOF function $C_{i,j}$ (axisymmetric case)

$$C_{i,j} = \frac{1}{|\Omega_{i,j}|} \int_{\Omega_{i,j}} \chi r dr dz. \quad (2)$$

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The $C_{i,j}$ is evolved by an approximation using a dimensional splitting algorithm as follows:

$$C_{i,j}^* = C_{i,j}^n - \frac{r_{i+1/2,j} F_{r,i+1/2,j}^n - r_{i-1/2,j} F_{r,i-1/2,j}^n}{r_{i,j} \Delta r} + C_{i,j}^m \frac{r_{i+1/2,j} u_{r,i+1/2,j} - r_{i-1/2,j} u_{r,i-1/2,j}}{r_{i,j} \Delta r} \Delta t, \quad (3)$$

$$C_{i,j}^{n+1} = C_{i,j}^* - \frac{F_{z,i,j+1/2}^* - F_{z,i,j-1/2}^*}{\Delta z} + C_{i,j}^n \frac{u_{z,i,j+1/2} - u_{z,i,j-1/2}}{\Delta z} \Delta t, \quad (4)$$

with

$$F_{r,i+1/2,j} = - \int_{z_{i,j-1/2}}^{z_{i,j+1/2}} \int_{r_{i+1/2,j}}^{r_{r,i+1/2,j} - u_{r,i+1/2,j} \Delta t} \chi_{is,j}(r, z) dr dz, \quad (5)$$

$$F_{z,i,j+1/2} = - \int_{z_{i,j+1/2}}^{z_{i,j+1/2} - u_{z,i,j+1/2} \Delta t} \int_{r_{i-1/2,j}}^{r_{i+1/2,j}} \chi_{is,j}(r, z) dr dz. \quad (6)$$

Here $F_{r,i+1/2,j}$ and $F_{z,i,j+1/2}$ are the advection fluxes for the r and z directions, respectively. The is and js are

$$is = \begin{cases} i & \text{if } u_{r,i+1/2,j} \geq 0 \\ i + 1 & \text{if } u_{r,i+1/2,j} < 0 \end{cases} \quad (7)$$

and

$$js = \begin{cases} j & \text{if } u_{z,i,j+1/2} \geq 0 \\ j + 1 & \text{if } u_{z,i,j+1/2} < 0. \end{cases} \quad (8)$$

The fluxes can be computed by the THINC/WLIC method or the VOF/PLIC method. The details of the THINC/WLIC method used in this paper are given in [14, 7, 8]. We use the THINC/WLIC method, because its implementation is easier than that of the VOF/PLIC method and the results are almost the same.

The level set method (CLSVOF)

The level set function ψ (signed distance function) is constructed from the interface indicated by the VOF function by a method [15] which uses the fast marching method [16] and an iterative re-initialization scheme proposed by Sussman et al. [5] (referred to hereafter as Sussman's method). The level set function ψ within Δh (where Δh is the grid spacing) from the interface indicated by the VOF function is computed by the fast marching method, solving the Eikonal equation:

$$|\nabla \psi| = 1. \quad (9)$$

Other ψ further from the interface are calculated by Sussman's method, while ψ computed by the fast marching method is fixed. The Sussman's method solves the following problem to a steady state

$$\frac{\partial \psi}{\partial \tau_1} = S(\psi)(1 - |\nabla \psi|), \quad (10)$$

where, τ_1 is artificial time and $S(\psi)$ is a smoothed sign function:

$$S(\psi) = \frac{\psi}{\sqrt{\psi^2 + \varepsilon^2}}. \quad (11)$$

To reduce the iteration number of (10), we also solve the level set equation

$$\frac{\partial \psi}{\partial t} + \mathbf{u} \cdot \nabla \psi = 0, \quad (12)$$

before the calculation of (10). We just use a first order upwind method for (12). This is a kind of preconditioner to make ψ^n approach ψ^{n+1} .

The density (color) function ϕ_d which is used to define the physical properties for different materials, such as density and viscosity can be generated as a smoothed Heaviside function

$$\phi_d = H_\alpha(\psi), \quad (13)$$

with

$$H_\alpha(\psi) = \begin{cases} 0 & \text{if } \psi < -\alpha \\ \frac{1}{2}[1 + \frac{\psi}{\alpha} + \frac{1}{\pi}\sin(\frac{\pi\psi}{\alpha})] & \text{if } |\psi| \leq \alpha \\ 1 & \text{if } \psi > \alpha, \end{cases} \quad (14)$$

where 2α represents the thickness of the transition region between the liquid phase and the gas phase. In this paper, $\alpha = \Delta x$ was used. The density function is set as $\phi_d = 1$ for the liquid and $\phi_d = 0$ for the gas. The density ρ and the viscosity coefficient μ are calculated by

$$\rho = \rho_{liquid}\phi_d + \rho_{air}(1 - \phi_d), \quad (15)$$

$$\mu = \mu_{liquid}\phi_d + \mu_{air}(1 - \phi_d), \quad (16)$$

where ρ_{liquid} and ρ_{air} are the densities of liquid and air, and μ_{liquid} and μ_{air} are the viscosities of liquid and air.

Governing equations of fluid

We use a finite volume formulation so that we use the following governing equation of an integral form,

$$\int_{\Gamma} \mathbf{u} \cdot \mathbf{n} dS = 0, \quad (17)$$

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{u} dV + \int_{\Gamma} \mathbf{u}(\mathbf{u} \cdot \mathbf{n}) dS = -\frac{1}{\rho} \int_{\Gamma} p \mathbf{n} dS + \frac{1}{\rho} \int_{\Gamma} (2\mu \mathbf{D}) \cdot \mathbf{n} dS + \frac{\mathbf{F}_{sf}}{\rho} + \mathbf{g} \quad (18)$$

where \mathbf{u} is the velocity, \mathbf{n} the outgoing normal vector for the control volume Ω with its surface denoted by Γ , ρ the density, p the pressure, \mathbf{D} the deformation tensor ($\mathbf{D} = 0.5(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$), \mathbf{F}_{sf} the surface tension force and \mathbf{g} the acceleration due to the gravity. Our formulation is fully conservative for cells not containing the interface, and is only approximately conservative in cells containing the interface. Equations (17) and (18) are solved by a multi-moment method based on the CIP-CSL method and VSIAM3.

We use a fractional step approach [17]. (18) is split into three parts as follows:

$$\mathbf{u}^{t+\Delta t} = f^{NA2}(f^{NA1}(f^A(\mathbf{u}^t))), \quad (19)$$

1. advection part (f^A):

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{u} dV + \int_{\Gamma} \mathbf{u}(\mathbf{u} \cdot \mathbf{n}) dS = 0 \quad (20)$$

2. non-advection part 1 (f^{NA1}):

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{u} dV = \frac{1}{\rho} \int_{\Gamma} (2\mu \mathbf{D}) \cdot \mathbf{n} dS + \frac{\mathbf{F}_{sf}}{\rho} + \mathbf{g} \quad (21)$$

3. non-advection part 2 (f^{NA2}):

$$\int_{\Gamma} \mathbf{u} \cdot \mathbf{n} dS = 0, \quad (22)$$

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{u} dV = -\frac{1}{\rho} \int_{\Gamma} p \mathbf{n} dS, \quad (23)$$

The advection part and non-advection parts are solved by the CIP-CSL method and VSIAM3, respectively.

Contact angle implementation

To impose the contact angle, we used a method developed by Sussman [13]. An important advantage of this method is that we do not need to locate the position of the triple point in the subgrid explicitly. Contact angle is taken into account by extrapolating the liquid interface represented by the level set function as well as the VOF function into the solid as shown in Fig. 1.

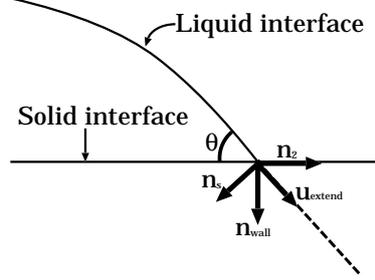


Figure 1. Contact angle implementation. The dashed line represents the imaginary liquid interface in the solid. The contact angle is taken into account by the imaginary liquid interface represented by the level set function.

The liquid interface is extrapolated by solving the extension equation

$$\frac{\partial \psi}{\partial \tau_2} + \mathbf{u}_{extend} \cdot \nabla \psi = 0, \quad (24)$$

here τ_2 is the artificial time. In this work, $\Delta \tau_2 = 0.5 \Delta x$ is chosen. \mathbf{u}_{extend} is the extension velocity and is computed as follows.

$$\mathbf{u}_{extend} = \begin{cases} \frac{\mathbf{n}_{wall} - \cot(\pi - \theta) \mathbf{n}_2}{|\mathbf{n}_{wall} - \cot(\pi - \theta) \mathbf{n}_2|} & \text{if } c < 0 \\ \frac{\mathbf{n}_{wall} + \cot(\pi - \theta) \mathbf{n}_2}{|\mathbf{n}_{wall} + \cot(\pi - \theta) \mathbf{n}_2|} & \text{if } c > 0 \\ \mathbf{n}_{wall} & \text{if } c = 0, \end{cases} \quad (25)$$

here

$$\mathbf{n}_{wall} = (0, -1), \quad (26)$$

$$\mathbf{n}_1 = -\frac{\mathbf{n}_{ls} \times \mathbf{n}_{wall}}{|\mathbf{n}_{ls} \times \mathbf{n}_{wall}|} \quad (27)$$

$$\mathbf{n}_2 = -\frac{\mathbf{n}_1 \times \mathbf{n}_{wall}}{|\mathbf{n}_1 \times \mathbf{n}_{wall}|} \quad (28)$$

$$c = \mathbf{n}_{ls} \cdot \mathbf{n}_2. \quad (29)$$

Here θ is the contact angle. The extension equation is simply solved by using a bi-linear interpolation.

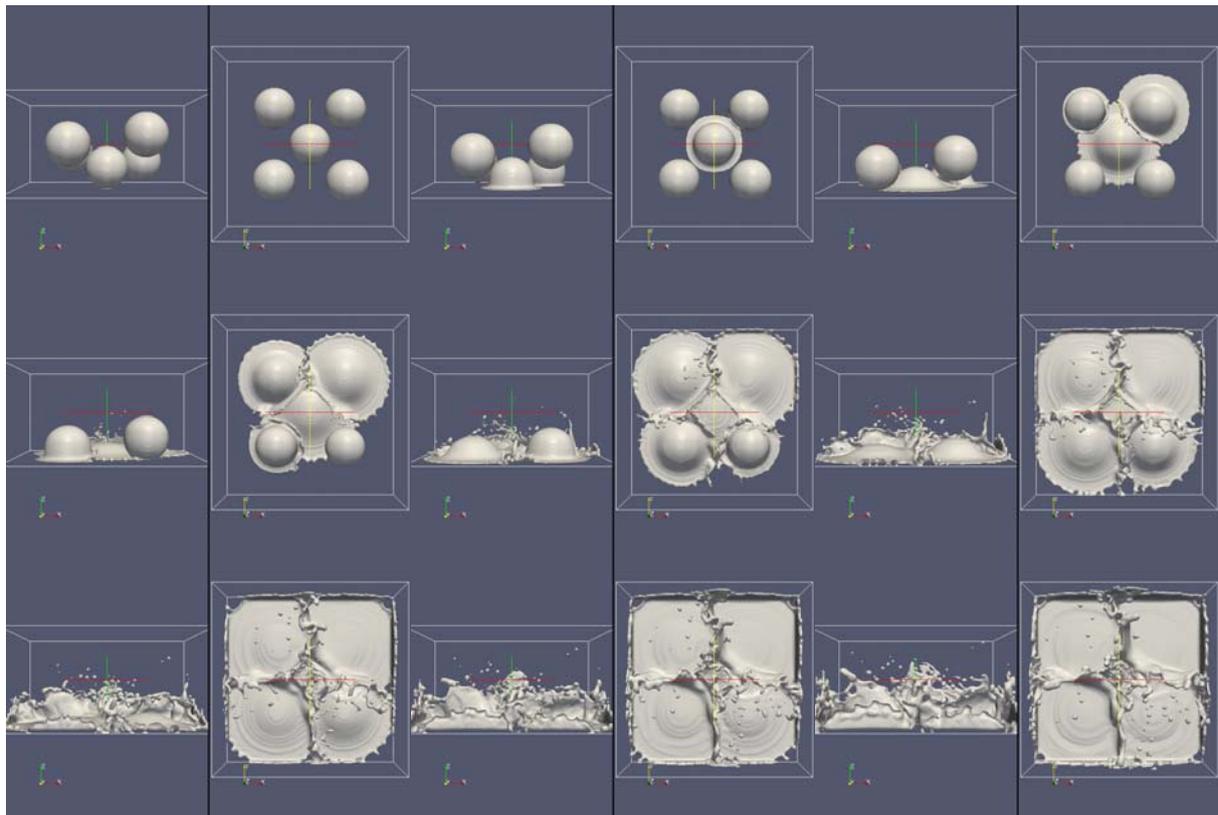


Figure 2. Snapshots of five droplets impacting. The time difference between snapshots is 0.2 [ms].

Numerical results

We conducted a numerical simulation of five droplets impact onto a dry surface as shown in Fig. 2. We used the densities $\rho_{liquid} = 1000 \text{ kg/m}^3$, $\rho_{air} = 1.25 \text{ kg/m}^3$, viscosities $\mu_{liquid} = 1.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$, $\mu_{air} = 1.82 \times 10^{-5} \text{ Pa}\cdot\text{s}$, surface tension $\sigma = 7.2 \times 10^{-2} \text{ N/m}$, gravity 9.8 m/s^2 , initial droplet diameter $D = 2.28 \text{ mm}$ and impact speed 3 m/s . The liquid was distilled water. The initial height of each droplets are different. The numerical formulation can robustly simulate multiple droplets impact onto a dry surface and capture lamella structures among impacting droplets.

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